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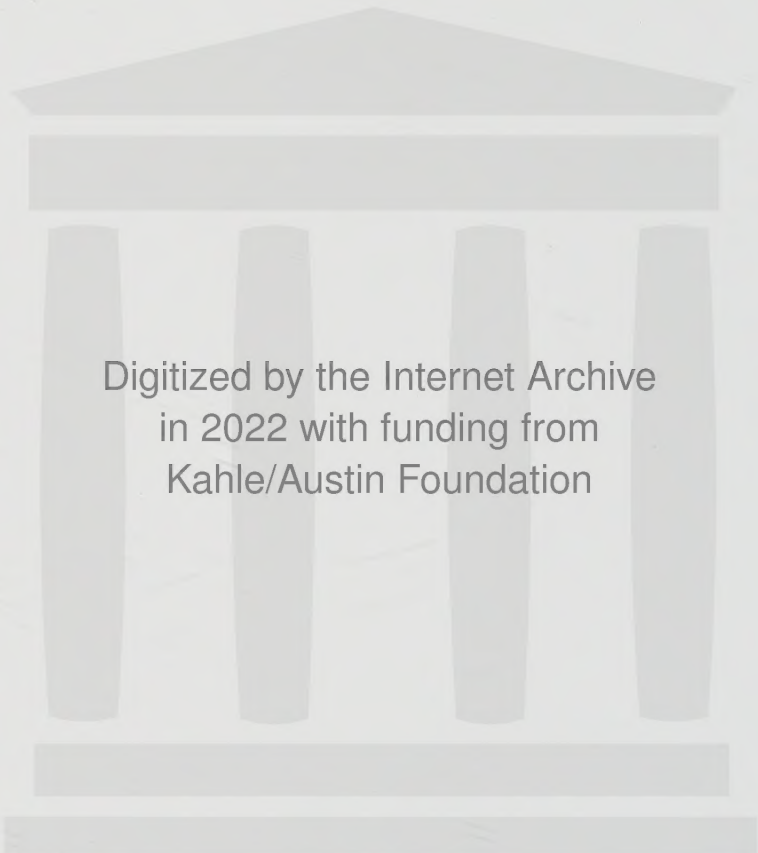
The Assurance Magazine, and Journal of the Institute of Actuaries, 1855

Vol. 5



by
Institute of Actuaries

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THE

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AND

JOURNAL

OF THE

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VOL. V.

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NOTICES.

We have availed ourselves of a favourable opportunity to insert in this Volume Mr. Peter Gray's paper on the Construction of Survivorship Assurance Tables, believing that our readers would not wish it to be omitted, although it has appeared in another shape. Under the like impression, we reprint Mr. Edmonds' Remarks on the Laws of Mortality and Sickness of the Labouring Classes of England, notwithstanding that they have already made their appearance in the pages of the *Lancet*.

The Five Tables, commencing at page 180, have been computed and placed at our disposal by Mr. Shaw, whose solution of a problem appears at page 152. We present them to our readers, as likely to be useful in many ways, particularly in checking similar values obtained by approximation. We believe their accuracy may be relied on, although the number of digits retained in the D column is, for convenience sake, a good deal restricted.

ERRATUM.

PAGE 106, line 7.

For

DR.	CR.
$(p'+\phi)_{x+n}(1+A'_{x+n})$	$p'_x(1+A'_{x+n})$

read

DR.	CR.
$(p'_{x+n}+\phi_x)(1+A'_{x+n})$	$(p'+\phi)_x(1+A'_{x+n})$.

THE

ASSURANCE MAGAZINE,

AND

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OF THE

INSTITUTE OF ACTUARIES.

On the Summation of a Compound Series, and its application to a Problem in Probabilities. By Bishop Terrot.

[Extracted, by permission, from the *Transactions of the Royal Society of Edinburgh.*]

The series proposed for solution in the following paper is—

$$\left.\begin{array}{l}(\overline{m-q}.\overline{m-q-1}\ldots\ldots\overline{m-q+p}+1)\times(1.2.3\ldots\ldots q)\\ +(\overline{m-q-1}.\overline{m-q-2}\ldots\overline{m-q+p})\quad\times(2.3.4\ldots q+1)\\ \qquad\vdots\\ +(p.p-1\ldots\ldots1\times(m-\overline{p+q}+1.m-\overline{p+q}+2\ldots.m-p)\end{array}\right\}.\ (A)$$

The law of this series is manifest. Each term is the product of two factorials—the first consisting of p, and the latter of q, factors; and in each successive term, the factors of the first factorial are each diminished by one, and those of the latter increased by one.

Let there be a series, $X_n Y_1 + X_{n-1} Y_2 + \ldots\ldots\ldots X_1 Y_n$ where, $Y_2 = Y_1 + \Delta_1$, $Y_3 = Y_2 + \Delta_2 = Y_1 + \Delta_1 + \Delta_2$, and so on. Then the series =

$$\begin{array}{r}X_n\ \times Y_1\\ +X_{n-1}\times\overline{Y_1+\Delta_1}\\ +X_{n-2}\times\overline{Y_1+\Delta_1+\Delta_2},\ \&c.\end{array}$$
$$=\Sigma X_n\times Y_1+\Sigma X_{n-1}\times\Delta_1+\Sigma X_{n-2}\times\Delta_2+,\ \&c.$$

where ΣX_n means the sum of all the terms of X from X_1 to X_n inclusive.

Let us then, in the first place, take the differences of the second factorials—

$$-(1.2.3.\ldots\ldots q)+(2.3.4.\ldots q+1)=(2.3.4.\ldots\ldots q).q$$
$$-(2.3.4.\ldots q+1)+(3.4.5.\ldots q+2)=(3.4.5.\ldots q+1).q, \text{\&c.\&c.}$$

Hence the sum of the whole series =

$$\left.\begin{array}{l} \Sigma(m-q.m-q-1.\ldots\ldots m-\overline{p+q}+1).1.2.3.\ldots q-1.q \\ +\Sigma(m-q-1.m-q-2\ldots\ldots m-\overline{p+q}).2.3.4.\ldots q.q \\ +\Sigma(m-q-2.m-q-3.\ldots m-\overline{p+q-1}).3.4.5.\ldots q+1.q \\ \qquad\qquad \text{\&c.} \qquad\qquad\qquad \text{\&c.} \end{array}\right\} \text{(B)}$$

Integrating then each line separately, we have the sum

$$\left.\begin{array}{l} =\frac{q}{p+1}.m-q+1.m-q.\ldots\ldots m-\overline{p+q}+1\times 1.2.3.\ldots\ldots q-1 \\ +\frac{q}{p+1}.m-q.m-q-1.\ldots\ldots m-\overline{p+q}\times 2.3.4.\ldots\ldots\ldots q \\ +\frac{q}{p+1}.m-q-1.m-q-2..m-\overline{p+q}-1\times 3.4.5.\ldots\ldots q+1 \\ \qquad\qquad \text{\&c.} \qquad\qquad\qquad \text{\&c.} \end{array}\right\} \text{(C)}$$

If again we treat this form as we have done the original, by taking the differences of the second factorials as they now stand, and again integrating, we reproduce the sum in the form

$$\left.\begin{array}{l} \frac{q.q-1}{p+1.p+2}.m-q+2.m-q+1\ldots m-\overline{p+q}+1\times 1.2.3.\ldots q-2 \\ +\frac{q.q-1}{p+1.p+2}.m-q+1.m-q.\ldots\ldots m-\overline{p+q}\times 2.3.4\ldots\ldots q-1 \\ \qquad\qquad \text{\&c.} \qquad\qquad\qquad \text{\&c.} \end{array}\right\} \text{(D)}$$

It appears, then, that we may continue this differentiation on the one side q times, and integration on the other $q+1$ times; and that at each succeeding operation an additional next lower factor will be introduced into the numerator of the fractional coefficient, and an additional next highest into the denominator. And after q differentiations, the last factorials will all become unity; and, the middle factorial having acquired an additional higher factor at each of $q+1$ integrations, we have for the sum of the series—

$$\left.\frac{q.q-1.q-2.\ldots.1}{p+1.p+2.\ldots.p+q+1}\times\overline{m+1.m.\ldots\ldots m-\overline{q+p}+1}\right\}. \quad \text{(E)}$$

II.

The problem in probabilities to which the foregoing summation is applicable, is the following:—

Suppose an experiment, concerning whose inherent probability of success we know nothing, has been made $p+q$ times, and has succeeded p times and failed q times, what is the probability of success on the $\overline{p+q+1}|^{\text{th}}$ trial?

This problem is interesting, because it tends to the discovery of a rational measure for those expectations of success which constitute the motive for a large portion of human actions. The force of such expectations commonly depends, not upon reason, but upon temperament; and according as a man is naturally sanguine or the reverse, so in all the contingencies of life does he overestimate or underestimate the chances in his favour.

It would be going much too far to think that we can give an algebraic formula, by the application of which a man may in every practical case correct his natural tendency to error, and arrive at a strictly rational amount of expectation. All that we can say is, that experience has led dispassionate men to come to nearly the same conclusion as the mathematician; for while he asserts the probability of success to be $\frac{p+1}{p+q+2}$, they act upon the supposition that the probabilities of success and failure are proportioned to the number of experienced cases of success and failure: and when either p or q is a large number, that is, when the experience is great, the conclusion and the supposition coincide.

In order to realize the problem, we shall use the ordinary illustration, and suppose that a bag contains m balls in unknown proportions of black and white, but all either black or white; that p white and q black balls have been drawn, and that it is required to find the probability of drawing a white at the $\overline{p+q+1}|^{\text{th}}$ drawing.

The problem, as thus stated, admits of four varieties.

1. m may be given, and the balls drawn may have been replaced in the bag.
2. m may be given, and the balls drawn not replaced.
3. m may be infinite or indefinite, and the balls replaced.
4. m may be infinite or indefinite, and the balls not replaced.

Of these, the 3rd is the only case which I find solved in the treatises which I have consulted. I propose to solve the 2nd case, and therein the 4th; and, in conclusion, to make an attempt at the solution of the 1st case.

To render the observed event, that is, the drawing of p white and q black balls (or E) possible, the original number of whites may have been any number from $m-q$ to p inclusive, and the number of blacks any number from q to $m-p$.

Let us call the hypothesis of $m-q$ white and q black, H_1, and $m-q-1$ white and $q+1$ black, H_2, &c. Then H_1 gives for probability of E $\dfrac{m-q\,.\,m-q-1\,.\,.\,.\,.\,m-q-p+1\times 1.2.3.\,.\,.\,.\,q}{m\,.\,m-1\,.\,.\,.\,.\,.\,.\,m-q-p+1}$;*
or, calling the denominator A,

$$\left.\begin{aligned} H_1 \text{ gives } &\frac{1}{A}.m-q.m-q-1.\,.\,.\,.\,.m-q-p+1\times 1.2.3\,.\,.q \quad (\alpha)\\ H_2 \text{ gives } &\frac{1}{A}.m-q-1.m-q-2.\,.\,.\,.m-q-p\times 2.3.4.\,.\,.\,.q+1\ (\beta)\\ H_3 \text{ gives } &\frac{1}{A}.m-q-2.m-q-3.\,.\,.\,.m-q-p-1\times 3.4\,.\,.q+2\ (\gamma)\\ &\text{\&c.} \qquad\qquad\qquad\qquad \text{\&c.} \end{aligned}\right\}(F)$$

Now, $\alpha+\beta+\gamma$, &c., by the former proposition (E),

$$=\frac{1}{A}\cdot\frac{q.q-1.\,.\,.\,.\,.\,.\,.\,.1}{p+1.p+2\,.\,.\,.p+q+1}\times m+1.m.\,.\,.\,.m-p-q+1;$$

$\therefore$ probability of $H_1=\dfrac{\alpha}{\alpha+\beta+\gamma,\ \&c.}$

$$=\frac{p+1.p+2\ .\,.\,.\,.\,.\,.p+q+1}{m+1.m.\,.\,.\,.m-p-q+1\times 1.2.3\,.\,.q}\times(m-q.m-q-1\,.\,.\,.m-q-p)$$
$$+(1\times 1.2.3.\,.\,.\,.q).$$

But the probability of a white at $\overline{p+q+1}|^{\text{th}}$ drawing on H_1 is $\dfrac{m-p-q}{m-p-q}$; $\therefore$ probability of white derived from H_1 is

$$\left.\begin{aligned} &\frac{p+1.p+2.\,.\,.\,.p+q+1}{m+1.m.\,.\,.\,.m-p-q\times 1.2.3.\,.\,.\,.q}\times(m-q.m-q-1\,.\,.\,.m-q-p)\\ &\qquad\qquad\times(1.2.3\,.\,.\,.q)\end{aligned}\right\}(G)$$

So probability from H_2

$$=\frac{p+1.\,.\,.\,.p+q+1}{m+1.m.\,.\,.\,.m-p-q\times 1.2.3\,.\,.\,.q}\times(m-q-1.m-q-2.\,.\,.\,.$$
$$m-q-p-1)\times(2.3\,.\,.\,.\overline{q+1}).$$

And so for all the other hypotheses in succession.

Now this series, omitting for the present the consideration of the fraction which is a factor common to them all, is a series of the same form as that summed in the last proposition, only that now $p+1$ must be substituted for p.

* The coefficient (U of Galloway's *Treatise*), expressing the number of different ways in which p white and q black balls can be combined in $p+q$ trials, is here omitted. This is immaterial, as it disappears in the expression $\dfrac{\alpha}{\alpha+\beta+\gamma,\ \&c.}$

We have therefore the whole probability of a white at $\overline{p+q+1}|^{\text{th}}$ drawing

$$\left.\begin{aligned} &= \frac{p+1.p+2\ldots p+q+1}{m+1.m\ldots m-p-q\times 1.2\ldots q} \times \frac{1.2\ldots q}{p+2\ldots p+q+2} \\ &\qquad \times m+1.m\ldots m-p-q = \frac{p+1}{p+q+2} \end{aligned}\right\} \quad \ldots \text{(H)}$$

Note.—It may be worth observing, that had we summed the original series in Prop. 1 upwards instead of downwards, we should have got for a first factor $\frac{1.2.3\ldots p}{q+1.q+2\ldots p+q+1}$, which must therefore $= \frac{1.2.3\ldots q}{p+1.p+2\ldots p+q+1}$. And that these fractions are equal may be proved independently; for if we divide each by $1.2.3\ldots p \times 1.2.3\ldots q$, we have on both sides the same quotient $\frac{1}{1.2.3\ldots p+q+1}$.

There now remains for solution only the first case of the problem in chances—that is, to find the probability of drawing a white ball, when m the number of balls is given, and p white and q black have already been drawn and returned.

The main object in this case is to sum the series

$$\overline{m-1}|^{p} \times 1^{q} + \overline{m-2}|^{p} \times 2^{q} \ldots\ldots 1^{p}.\overline{m-1}|^{q} \quad \ldots \text{(I)}$$

This may be done much as in the preceding case, by taking the successive differences of the right hand factors till the differences vanish, and multiplying the successive terms of the last or $\overline{q+1}|^{\text{th}}$ row of differences into the $\overline{q+1}|^{\text{th}}$ summation of the successive terms of the series $(1+2^{p} \ldots + \overline{m-1}|^{p}) + (1+2^{p} \ldots + \overline{m-2}|^{p})$, &c.

This may be sufficiently explained by going through the operation in a low particular case. Let $p=2$, $q=3$. Then the series, written perpendicularly, is

$\overline{m-1}\|^2 \times 1$	$\Sigma_1\overline{m-1}\|^2 \times 1$	$\Sigma_2\overline{m-1}\|^2 \times 1$	$\Sigma_3\overline{m-1}\|^2 \times 1$	$\Sigma_4\overline{m-1}\|^2 \times 1$
$\overline{m-2}\|^2 \times 8$	$\Sigma_1\overline{m-2}\|^2 \times 7$	$\Sigma_2\overline{m-2}\|^2 \times 6$	$\Sigma_3\overline{m-2}\|^2 \times 5$	$\Sigma_4\overline{m-2}\|^2 \times 4$
$\overline{m-3}\|^2 \times 27$	$= \Sigma_1\overline{m-3}\|^2 \times 19$	$= \Sigma_2\overline{m-3}\|^2 \times 12$	$= \Sigma_3\overline{m-3}\|^2 \times 6$	$= \Sigma_4\overline{m-3}\|^2 \times 1$
$\overline{m-4}\|^2 \times 64$	$\Sigma_1\overline{m-4}\|^2 \times 37$	$\Sigma_2\overline{m-4}\|^2 \times 18$	$\Sigma_3\overline{m-4}\|^2 \times 6$	
$\overline{m-5}\|^2 \times 125$	$\Sigma_1\overline{m-5}\|^2 \times 61$	$\Sigma_2\overline{m-5}\|^2 \times 24$	$\Sigma_3\overline{m-5}\|^2 \times 6$	
&c.	&c.	&c.	&c.	

The value of the different sigmas is easily found by the method of finite differences.

Generally, since the differences of 1^q, 2^q, 3^q, &c., always vanish in the $\overline{q+1}|^{\text{th}}$ line and after the q^{th} term of it, the general expression is

$$\Sigma_{q+1}\overline{m-1}|^p + d_2\Sigma_{q+1}\overline{m-2}|^p \ldots . d_q\Sigma_{q+1}\overline{m-q}|^p;$$

d_1, d_2, 3_3, &c. signifying the 1st, 2nd, 3rd, &c. terms of the $\overline{q+1}|^{\text{th}}$ row of differences.

This summation may be applied to find the probability in the case now under consideration, for it expresses the $\alpha+\beta+\gamma$, &c. of the preceding case. Applying it as we did the value of $\alpha+\beta+\gamma$, &c., there found, we shall find the probability of a white ball at the $\overline{p+q+1}|^{\text{th}}$ trial to be

$$\frac{\Sigma_{q+1}\overline{m-1}|^{p+1} + d_2\Sigma_{q+1}\overline{m-2}|^{p+1} \ldots . d_q\,\Sigma_{q+1}\overline{m-q}|^{p+1}}{m(\Sigma_{q+1}\overline{m-1}|^p + d_2\Sigma_{q+1}\overline{m-2}|^p \ldots . d_q\Sigma_{q+1}\overline{m-q}|^p} \quad . \quad . \quad . \quad (\text{K})$$

If m be infinite, the expression becomes

$$\frac{(1+d_2+\ldots .d_q).\Sigma_{q+1}m^{p+1}}{m(1+d_2\ldots .d_q).\Sigma_{q+1}m^p} = \frac{\Sigma_{q+1}m^{p+1}}{m\Sigma_{q+1}m^p}.$$

But if x be a quantity varying between the limits 0, x,

$$\frac{\Sigma_1 m^{p+1}}{m\Sigma_1 m^p} = \frac{\int_x^0 x^{p+1}dx}{x\int_x^0 x^p dx} = \frac{p+1}{p+2}\cdot\frac{x^{p+2}}{x.x^{p+1}}.$$

And by continuation,

$$\frac{\Sigma_{q+1}m^{p+1}}{m\Sigma_{q+1}m^p} = \frac{p+1\,.\,p+2\ldots p+q+1}{p+2\,.\,p+3\ldots p+q+2} = \frac{p+1}{p+q+2} \quad . \quad . \quad (\text{L})$$

We have thus found the probability in every case of the problem; the 2nd and 4th at H, for the result, being independent of m, must be true for an infinite as well as for a finite number. The 1st case is solved at K, and the 3rd at L.

Observations upon the Sickness and Mortality experienced in Friendly Societies. By Henry Tompkins.

[Read before the Institute, 26th June, 1854, and ordered by the Council to be printed.]

THE Institute of Actuaries is devoted to the consideration of all questions relating to assurance; and as life assurance occupies a prominent place in the daily occupations of many of its members,

it is not surprising that papers on almost every conceivable topic in connection therewith have been read before it. There are, however, various other kinds of assurance well worthy of inquiry, and among these may be classed fire, marine, fidelity, and, last but not least, when we consider the number of persons affected by it, sickness assurance. It is true this last kind of assurance is almost confined to the working classes: they have no wealthy boards of managers; they have no man of science on their staff to direct their proceedings; and they are conducted by individuals who often possess no qualifications for such occupations. For all this, we must admit that every Friendly Society is a mutual Assurance Company; and, compelled by necessity, these Societies have taken up the most difficult and complicated branch of assurance. Not many actuaries have been much occupied with the subject, for it is a branch which does not sufficiently reward those who grapple with its difficulties; we are therefore under the greater obligation to those who have attempted it. Up to the present time, however, I am afraid that the success has not equalled the endeavour.

It must be admitted that the law of sickness is a very difficult one to discover, on account of so many external circumstances complicating its action. Some even have been of opinion that no such law exists, because it has baffled so many endeavours to develope it. It is far more difficult to obtain facts relating to sickness than to death—because, in the first place, death has one common meaning among all classes; but it is not so with the word 'sickness.' Also sickness may be, and often is, feigned; but it is not so with mortality. A death also occurs but once to an individual; but sickness attacks an individual at various periods in his life, and at each time with a different degree of intensity and duration. All these and other causes tend to complicate the subject; but I think there is no doubt that in this, as in all other matters, if we obtain a sufficient quantity of facts, the laws regulating their action can be discovered.

The ratio of sickness has never been laid down with a very near approach to accuracy until the publication of Mr. Finlaison's late Report upon the *Sickness in Friendly Societies*. It has, I think I shall prove, been enormously overstated in all the works heretofore published on the subject. Whether this opinion will be shared by others, must depend upon their view of what appears to me to be conclusive.

On a former occasion I pointed out the great discrepancy between various authorities as to the amount of premiums required

for the same amount of sickness assurance. The variation was so unexpectedly great that it called to mind the line—

"Who shall decide, when doctors disagree?"

This however was said to have been caused by the very nice discrimination which the accuracy of the data obtained enabled us to make. Thus, it was supposed that a table of premiums for an allowance of £1 per week in sickness, for a Society established at Clapham, would, for substantial reasons, differ from a table for a like assurance for a Society at Greenwich—that there was an ascertained difference between the sickness to be experienced by a Society composed of butchers and another composed of tallow-chandlers. It is true there are works which have attempted to lay down such nice distinctions in the law of sickness; but if the *general* sickness has been erroneously laid down in these works, it follows that all *particular* cases must be equally or even more erroneous. The discrepancies pointed out by me had, therefore, no foundation in the exquisite accuracy of the data made use of.

Tables of premium for sickness assurance are generally calculated to extend up to 70 years of age, or even throughout the whole of life. This mode of proceeding appears to me to be highly inconvenient, since a sickness assurance is for the purpose of supplying the deficiency which occurs to a working man by the stoppage of his daily labour by disease; and as men generally cease to obtain their living by work at or before 60 years of age, it appears that a sickness assurance should cease at that period, because it is not applicable to a state of circumstances which should be met by an annuity. There is, however, another reason why such assurances should cease at 60 years of age—which is, that even now, so far as I am aware, we have no satisfactory data for the ages above 60 on which such sickness tables can be computed. The data given in the Report, for ages above 60, which appear to be correct, have not yet been corroborated, which I hold to be necessary in all cases. Those which have been hitherto adopted for such advanced ages are always too high; and as the premiums for these ages are partly paid in the more youthful periods of life, it has resulted that tables have been computed which have never, or rarely, been adopted by the classes for whom they were intended. Even popular prejudices frequently have a remote connection with fact; and when I state that out of 1600 Societies which during four years applied for the protection of the law, and were informed that the certificate of an actuary was an indispensable preliminary, 700 decided to be without such pro-

tection rather than obtain the certificate, we may well inquire what popular prejudice was the cause of this? I think it was mainly because the parties interested considered, and I believe with some reason, that their liabilities were estimated by an exaggerated standard.

The circulation of incorrect data has induced the notion that Benefit Clubs in general are insolvent; but when we measure their liabilities by such exaggerated standards as I shall endeavour to show have been put forth, it is astonishing that any were not included in the insolvent list. In illustration of this we may inquire, what would be the result if the liabilities of Life Assurance Companies were estimated to include the debts of the assured as well as the special sums assured to their representatives? Very few, if any, would be solvent; yet such is the way, with the help of a few extreme cases, in which it has been attempted to be proved that Friendly Societies in general are insolvent. It is true that some may be in that state; but before sentence be pronounced, we should be certain that we know the law.

I now pass to a comparison showing the results as regards sickness in Friendly Societies, arrived at by Mr. Neison, Mr. Ratcliff, and Mr. Finlaison, Jun., in their several works on this subject. I choose these, because Mr. Neison's *Vital Statistics*, and Mr. Ratcliff's *Sickness among the Odd Fellows*, were the two best works extant on the subject; and Mr. Finlaison's is the most recent. Everyone has, however, observed the want of uniformity in statistical documents, and of the impossibility in most cases of instituting any comparison between them.

In these three works there are several particulars in which they are all differently arranged; and we may be, in some respects, instituting a comparison of unlike things. These discrepancies are the cause of a deplorable loss of time; and, on the other hand, the surest method of giving an impetus to science is to perfect its language and to adopt uniform notations, so as to comprise the largest number of ideas, and to bring together a greater multitude of facts from which to deduce relations and laws. Taking however these works as we find them, and collating the results, we find that the annual amount of sickness, on the average, experienced by each individual member of a Friendly Society, is as follows. It ought to be mentioned, however, that Mr. Finlaison excludes all miners, colliers, and sailors from his "general" class, on account of their special risks, while the other two writers admit them into that category.

AVERAGE SICKNESS PER ANNUM TO EACH PERSON,
EXPRESSED IN DAYS.

Age.	Finlaison.	Ratcliff.	Neison.	Age.	Finlaison.	Ratcliff.	Neison.
18	6·8296	3·81	5·76	53	12·9882	15·97	16·56
19	6·9055	3·95	5·82	54	13·4090	17·37	17·70
20	6·8835	4·09	5·88	55	13·9455	18·98	18·93
21	6·9058	4·37	5·92	56	14·6288	20·78	20·27
22	6·8931	4·61	5·96	57	15·2382	22·86	21·96
23	6·8619	4·81	6·01	58	16·2291	25·22	24·00
24	6·8245	4·98	6·06	59	17·4330	27·86	26·40
25	6·8298	5·10	6·12	60	18·7348	30·79	29·16
26	6·8379	5·19	6·18	61	20·1717	33·98	32·27
27	6·8973	5·27	6·24	62	21·8270	36·86	36·33
28	6·9497	5·39	6·29	63	23·4850	39·43	41·35
29	6·9441	5·50	6·36	64	25·4208	41·68	47·32
30	6·9122	5·62	6·37	65	27·3564	43·61	54·25
31	6·8626	5·75	6·41	66	29·7893	45·23	62·13
32	6·8003	5·90	6·65	67	32·3540	48·39	70·48
33	6·8364	6·04	7·11	68	35·4239	53·07	79·28
34	6·9650	6·20	7·77	69	39·0656	59·28	88·55
35	7·1424	6·35	8·65	70	43·6153	67·02	98·27
36	7·4108	6·52	9·74	71	48·1807	76·29	108·45
37	7·6396	6·72	10·51	72	53·6783	84·54	118·76
38	7·8335	6·96	10·96	73	59·3229	91·77	129·15
39	7·9846	7·23	11·10	74	63·2823	97·99	139·66
40	8·2135	7·55	10·92	75	66·8123	103·19	150·26
41	8·4035	7·89	10·41	76	71·4833	107·38	160·97
42	8·5896	8·28	10·12	77	77·3931	113·06	170·16
43	8·7587	8·72	10·02	78	82·9178	120·29	177·82
44	9·0516	9·19	10·14	79	90·9886	129·06	183·97
45	9·3392	9·70	10·46	80	97·6832	139·38	188·58
46	9·5990	10·26	10·98	81	102·8451	151·28	191·68
47	10·0575	10·87	11·57	82	106·0442	160·84	195·34
48	10·5149	11·52	12·22	83	106·3780	168·02	199·56
49	11·0123	12·22	12·94	84	99·4289	172·83	204·35
50	11·4868	12·97	13·72	85	..	175·25	209·70
51	12·0145	13·77	14·57	86	..	177·20	215·62
52	12·4766	14·77	15·51	87	..	..	217·69

From an examination of the preceding columns, I have come to the conclusion that both Mr. Neison and Mr. Ratcliff, as well as others who have preceded them, have allowed claims made on account of *superannuation* to come into consideration where claims for *sickness* only should have been admitted, and that therefore their measures of the duration of sickness are of very little value. With regard to Mr. Neison's work, I find that he does not refer to any such claims having been eliminated; and it is certain that a

very great number of them are entered as *sickness* in the returns made, and especially at the advanced ages. Mr. Ratcliff, however, states in his book that he had only the means of testing the accuracy of the *mortality;* and though the Odd Fellows have, I believe, no fund expressly called a superannuation fund, yet there is no doubt that they have both committed the same error, viz., that of confounding claims on account of sickness with claims on account of superannuation, which are very different. I have summed up the sickness experienced between ages 18 and 50 with the following result, viz.; according to

Sum of the Sickness per Individual.	Finlaison.	Ratcliff.	Neison.
	Days.	Days.	Days.
From age 18 to 50	260	230	281

Thus far the results corroborate the general accuracy of each.

The difference between the two observations, and the returns, is, that Mr. Ratcliff gives nearly one day's sickness per year to each individual *less* than the returns, while Mr. Neison shows about 15 hours in a year *more* than the returns. After the age of 50, the results are widely different—and, I believe, for the reasons I have stated with reference to the superannuation claims.

Sum of the Sickness per Individual.	Finlaison.	Ratcliff.	Neison.
	Days.	Days.	Days.
From age 50 to 60	147	208	205
„ 60 „ 70	299	469	610
„ 70 „ 80	712	1063	1528

By admitting all cases of sickness, so called (including superannuation), the following are the results which have been arrived at—viz., that Messrs. Neison and Ratcliff both show that at age 60 EVERY man is ill on an average *one* month in a year; at 70, EVERY man has *two months* and a half's illness in every year; at 75, it is said to increase to nearly *four months;* at 80, to *five months;* at 85, to *six months, or half every man's time;* and at 87, to *seven months in the year!* and this to every man. If our aged and worn-out workpeople can really withstand such terrible attacks of *sickness* and live, they must certainly be possessed of more than

ordinary British toughness. The truth however is, that a very great proportion of the sexa, septa, and octo-genarians who are members of Friendly Societies receive allowances on account of superannuation, being at the same time in very good health. It is a remarkable fact, that that celebrated actuary Dr. Price estimated, upon theory only, the average sickness to each individual, with a very close approximation to the truth, if we consider the Report to be the standard of it. His estimate is as follows:—

Under	32	years of age	.	.	.	.	7·58	days
From	32	years to	42	years	.	.	.	9·47	„
„	43	„	51	„	.	.	.	11·37	„
„	51	„	58	„	.	.	.	13·26	„
„	58	„	64	„	.	.	.	15·17	„

The last line shows that Dr. Price had no notion of the incredible amount of sickness which affects the old people of the working class. This discovery has been reserved for more modern times.

With regard to the sickness stated in the Report, all claims which were not for *sickness* were eliminated, and the results given are believed to be those which relate to sickness and to that alone. The results, however, differ so widely from some previous writers, that another observation with regard to persons of advanced ages is a desideratum. Until that however is obtained, we must be content with these conflicting data, unless the weight of probability on the one side appears to outweigh that on the other.

I now pass to the interesting question of the mortality experienced in Friendly Societies. The first glance shows it to be very low, being on the whole number only about 1·26 per cent. per annum. The annual mortality per cent. of London is about 2·5; but in its population are included children, cripples, paupers, insane persons, and others with enfeebled powers; but in Friendly Societies there are no such persons, or at least there are none who were so on their admission. They are all able-bodied adults. The operation of the "temporary selection" of many of the assured lives from a class which, on the whole, is placed in circumstances favourable to the prolongation of life, may in some degree account for the small mortality experienced. It is well known that a large portion of the members of these Societies leave them, or become removed from them in various ways beside by death. This makes a large number of the assurances temporary; and as the selection is made from the *élite* of the working classes, we may reasonably expect that a low rate of mortality prevails.

NUMBER OUT OF WHICH ONE WILL DIE.

Age.	Ratcliff.	Finlaison.	Nelson.	Age.	Ratcliff.	Finlaison.	Nelson.
18	181	149	161	51	54	64	66
19	173	141	152	52	49	61	62
20	166	135	148	53	46	58	59
				54	43	53	56
21	153	133	147	55	41	49	53
22	140	133	147				
23	137	137	145	56	39	45	50
24	132	141	144	57	36	42	47
25	129	141	143	58	34	40	45
				59	31	40	42
26	127	141	141	60	28	38	40
27	126	137	139				
28	124	132	137	61	26	37	38
29	122	128	134	62	24	35	35
30	120	130	132	63	22	34	33
				64	20	30	31
31	118	127	130	65	18	27	28
32	116	125	128				
33	114	125	125	66	17	25	26
34	113	125	123	67	16	23	24
35	111	121	121	68	15	21	22
				69	15	20	20
36	110	116	118	70	15	18	18
37	107	112	116				
38	104	105	113	71	15	17	17
39	100	100	110	72	14	16	15
40	95	97	106	73	14	15	14
				74	13	14	14
41	90	94	103	75	13	13	13
42	86	91	99				
43	82	90	96	76	12	12	13
44	78	87	93	77	11	11	12
45	75	83	89	78	10	10	11
				79	9	9	11
46	72	80	86	80	8	8	10
47	69	76	82				
48	65	74	78	81	7	7	9
49	61	70	74	82	6	7	9
50	57	67	70	83	6	6	8
				84	5	6	8

I remember being told by a most experienced and highly respected actuary, that the rate of mortality stated to prevail in Friendly Societies was almost the same as saying that the working classes were immortal; and I know there are many who still object to it, because it is contrary to preconceived notions. Let, however, the rate of mortality of the working classes IN OTHER CIRCUMSTANCES be what it may, it is certain that, AS MEMBERS OF FRIENDLY SOCIETIES, the general accuracy of this portion of the subject, in the three works before me, is so confirmed as to place it beyond all question. I think it would be difficult to point

out many observations, made by different parties under different circumstances, which have more strikingly agreed in their results. We have now a general mortality experienced in Friendly Societies, which may be relied upon, as each is corroborated from independent sources; and therefore the subdivisions of this part of the subject in each of the three works referred to are entitled to consideration, as they are each parts of a whole which is proved to be accurately ascertained.

I now propose to enter upon a consideration of some of the parts of the Report upon the *Sickness and Mortality of Friendly Societies* in which that work differs from previous ones, and where I cannot institute comparisons. I may however first remark, that there were no less than 3,787 returns, each of course representing a Society, and comprising in the aggregate a quinquennial account of 542,900 persons. They were sent from various localities in England and Wales, the kingdoms of Ireland and Scotland not being included in the arrangement. This however was not a great loss, as the small number of such Societies in the sister kingdoms induces the belief that they are there exotics, and not indigenous as in England, where there is scarcely a hamlet or village without its Benefit Club, and sometimes even rival ones—where every town has several, and every city many, of them. We may be surprised that the number of returns received was so small; but, as was to be expected, many Societies sent papers that were useless for the purposes intended, and others sent no account at all, the penalty for neglect being merely nominal. For this reason we are not to suppose that there are no more persons registered in Friendly Societies than those mentioned in the returns, there being, I think, good grounds for estimating the number at five or six times as many, or near three millions.

The returns contained an account of the amount of sickness experienced, and the number of deaths that had taken place, under the following heads; viz.—

1. Each individual's age when the account commenced.

2. Information as to whether he had been attacked by illness, or not, in any of the five years; and if attacked, for what length of time in each year.

3. A statement whether he had received any superannuation allowance during the five years; and if so, how long.

4. Information as to whether he had died, had been excluded, or had withdrawn himself from the Society, and the date of such event; and,

5. His occupation or employment.

Assuming that these particulars were obtained with the accuracy reasonably to be expected in a return which was quite a freewill offering of the parties making it, it is clear that they contained the elements from which nearly all problems of a practical character relating to the subject might be solved.

With regard to the results given in the Report, it is evident that any consideration which detracts from their value must arise from the character of the statements returned. There were, of course, many erroneous returns; but the errors were always, as far as could be discovered, those of ignorance, and not of intention, and to eliminate these was a task which was performed in the most careful manner that could be devised. To make "assurance doubly sure," this operation was repeated, occasioning what may appear to be a great loss of time; but it was considered advisable to spare no pains to secure accuracy as far as practicable. In fact, there cannot be too great scrutiny of the elementary matter in all matters of statistics, before commencing to operate upon it.

The collection of facts relating to each individual was contained on a slip of cardboard: and these facts were not divided, but remained together throughout the whole procedure; for, having passed the examination and been copied, they remained established, and were used over and over again, in viewing them in different combinations. It is not necessary, however, for me to enter into all the minutiæ of operations, as it is explained in the Report, and a repetition of it here would be tedious; suffice to say, that it was found to answer most completely for the purposes intended, and that it might be usefully adapted to many purposes of statistics.

Having thus sifted and separated the materials, Mr. Finlaison very truly remarks, that

"The most essential thing to be determined, and that for which the returns were most likely to prove of utility, was the quantum of sickness to which the members of Benefit Societies are liable at each age. The second great desideratum was the law of mortality to which the same class of persons is subject. But in setting out to collect this information, the knowledge that many influences are supposed to modify the results which are now the object of inquiry, and that circumstances of climate and occupation are believed by many to enter largely into the question, naturally had a general tendency to suggest the channels in which research should be carried on. It will appear to most minds that will give the subject attention, that among the primary conditions indispensable to the possession of bodily health must be ranged—food, warmth, and wholesome air. It is at the same time obvious, that many of the contributors to Friendly Societies are placed in very different degrees of command over these first requisites to

health and life. Those causes, therefore, which may be supposed to influence, more or less, a command over these first supports of existence, indicate of themselves the direction in which an investigation of this nature should be generally conducted. There are minor agencies, likewise, which are thought to deflect and impair the enjoyment of health and vigour, and which should not be passed by. It is desirable, therefore, to note these secondary causes as well, and to see, if possible, whether they also act in any material degree upon the constitutions of those who are subjected to their influence.

"The most obvious consideration of all would at first appear to be that of climate, which, through the medium of its temperature and quality of atmosphere, would seem to affect the vigour of the human frame in the most direct manner."

Commencing therefore with the question of climatic influence, he proceeds to map out the area of England and Wales into eight divisions—or, as he denominates them, "provinces"—each of which has a distinguishing feature, which may be well inferred from the names adopted to designate them, viz., the Northern province, the Manufacturing, the South-western, the South-eastern, the Eastern, the Midland, the Welsh, and the Metropolitan. The statements received from each of these "provinces" were collected, in order to discover whether any marked difference exists between the hardy Northumbrian and the Londoner, the Welshman and the manufacturing operative, &c., in the liability to disease; and in fact, to see what effect is produced by locality in conjunction with other causes. The results show that the locality in which a Friendly Society is established has very little to do with the average amount of demand that will be made for sick allowance, seeing that the average sickness for a whole year, to each person throughout England and Wales, differs in its most violent extremes but *one day and a half*.

	Average sickness per annum to each person, in days.
South-western province	11·01
Midland	10·65
Welsh	10·13
Manufacturing	10·06
Northern	10·02
Eastern	9·88
South-eastern	9·66
Metropolitan	9·45

Besides viewing sickness with reference to individuals, there is another light in which it may be placed, which is an important one in any inquiry into the subject. This, which I believe has been given for the first time in the Report, is the proportion of persons

attacked with sickness in one year. We find, that although there is so slight a difference in the amount of sickness when viewed in connection with every individual's proportional share of it, there are, in different parts of the country, very different degrees of probability of being the man who has to bear it.

Thus the proportion of sick persons out of each 100 persons liable to sickness, in the different provinces, is as follows:—

	Per cent.
Midland province	29·29
South-western	27·27
Eastern	26·41
South-eastern	25·96
Metropolitan	25·32
Manufacturing	22·34
Welsh	22·14
Northern	19·89

Showing a difference of full 32 per cent., or neary $\frac{1}{3}$, in favour of the Northman's chance of exemption from sickness.

These results, at first sight, seem to be contradictory to the statement just made, that the locality in which a Friendly Society is established has very little to do with the average amount of sickness experienced; but when we consult a column which shows the average length of time which every man who is attacked is confined by sickness, we perceive the remarkable fact, that exactly in the inverse ratio of these districts to *susceptibility* of attack is the *duration* of the sickness, thus on the whole producing an almost equal average amount of claims in each locality.

The average number of days' sickness per annum, to each person sick in each province, is as follows:—

	Days' sickness.
Northern province	50·38
Welsh	45·73
Manufacturing	45·02
South-western	39·65
Eastern	37·39
Metropolitan	37·34
South-eastern	37·22
Midland	36·37

The case may be stated in a more compendious form, perhaps; thus:—

Province.		No. per cent. taken sick.		Days of sickness to each sick person.
Northern	minimum	19·89	maximum	50·38
Welsh		22·14		45·73
Manufacturing		22·34		45·02
Midland	maximum	29·29	minimum	36·37

Showing, on the other hand, that the Northman, *when attacked*, suffers 28 per cent. longer affliction.

In concluding the observations upon sickness viewed in connection with locality, therefore, the result is that, in balancing the liability to sickness against its duration, it is a merciful consequence of their mutually compensating effects that the quantum of sickness to which each man throughout England is liable varies but little, let his abode be in what province it may.

We must not omit to notice, however, one observation in this part of the analysis, which shows how very important the means of obtaining a good supply of artificial heat are to the preservation of health. It is stated, that where the price of fuel is at its lowest, and where almost every cottage can command a glowing hearth, the lowest liability to the attack of sickness is found; while, on the contrary, in the counties where the difficulties in the way of the transport of coals are greatest, the liability is at its worst. Thus the northern coal fields show the minimum, and the midland seats of agriculture the maximum, as may be observed on reference to the figures on page 17. On this account it is very satisfactory to see the railways so effectually co-operating with the shipping in the more effectual distribution of this prime safeguard to health throughout the country; and we may also observe, that this is only one instance among many, in which railways produce an important effect upon the sanitary condition of a country.

The next aspect under which the returns were viewed was in connection with density of aggregation—or, in other words, in reference to whether the statements emanated from a rural, a town, or a city district. This arrangement was adopted in deference to preconceived and widely spread notions. The distinction between these various districts may have been more strongly marked in former times than it is at present, when town and country are more intermingled than ever, and sanatory measures are not treated as the visions of theorists, but are adopted with good results. It is observed in the Report on the Census, that of all the countries of Europe, England has the smallest demarcation between the population of rural and other districts. The facts, also, which came to light during the cholera visitation, have in some degree qualified the notion of the extraordinary salubrity of rural districts in comparison with other places. As the omission might have been considered a defect, the results were given; and we find that the average sickness per annum to each person is as given on page 20—showing, on the whole, a

decided superiority in favour of the city districts—which is contrary to usual expectation.

Thus failing to find any interesting distinction between the amount of sickness experienced in different localities, and also in different densities of aggregation, attention was directed to see what results were produced by the quality of labour expended by the classes under observation, and also by the circumstances attending it. The effect of different classes of labour upon the health has been already largely descanted on, but with such minute distinctions as detract from the value of the deductions. The plan adopted in the Report has at least the merit of originality and breadth of design. According to it, all classes of labour may be divided into four sections, viz.—

Light labour exercised in the open air.
" " exercised in shelter.
Heavy labour exercised in the open air.
" " exercised in shelter.

"There are certain influences which bear in a greater or less degree on the exercise of labour; but the principal agencies affecting the human constitution in such exercise are—the degree of force called out, the temperature in which that force is exerted, and the atmosphere wherein it is put into action. For instance, there is hard labour with exposure to weather, and there is hard labour subject to no such aggravation. There is a large class of labour requiring no great exercise of strength, which is carried on under shelter and in warmth, such as silk and cotton factory labour, while there are also light employments, as they may be termed, which are pursued in the open air. Again, there are many branches of industry which are followed in crowded workshops under circumstances highly unfavourable to the respiratory organs, while there are other callings which can be pursued at a man's own fireside—this last class being that which probably produces the greatest amount of deceptive claims.

"This general arrangement of the subject appears, on the whole, to have assembled in their respective classes the great bulk of industrial occupations in this country, with tolerable accuracy and relative fairness."

The results show that, on the whole, and especially in the case of heavy labour, exposure to the weather aggravates the amount of sickness.

"The remarkable mitigation of the sickness in the case of the light labour *with* exposure to weather, as compared with the other three classes, at once attracts notice, and indicates the most healthy condition of existence. But the result of exposure to weather is not to be conclusively inferred therefrom by any means, as it would appear in the case of heavy labour to be an ingredient materially aggravating the quantum of sickness attaching to this ruder class of occupation. The agency of more powerful causes than the influence of the elements begin to make themselves perceptible in this arrangement of the subject."

Age.	Average amount of Sickness per Annum to each Person, expressed in Days.			Average amount of Sickness per Annum to each Person Sick, expressed in Days.		
	City.	Town.	Rural.	City.	Town.	Rural.
18	7·6170	6·9026	6·7838	28·6400	27·4343	23·7641
19	7·4803	7·0273	6·8458	28·7879	27·6558	24·6780
20	6·8045	6·8950	6·8832	25·2558	27·2351	25·4196
21	6·5083	6·8345	6·9562	25·0533	27·2194	26·2788
22	6·3454	6·8539	6·9525	25·5284	27·6585	26·9647
23	6·1953	6·7844	6·9607	25·0982	27·8055	27·7068
24	5·8343	6·7840	6·9607	24·1463	28·4017	28·2194
25	5·9000	6·7811	6·9807	25·0414	28·8643	28·8560
26	6·1195	6·7746	6·9788	26·1202	29·3929	29·2121
27	6·2484	6·8390	7·0357	26·8868	30·2625	29·8394
28	6·4382	6·7995	7·1213	28·1548	30·5118	30·4672
29	6·4974	6·7576	7·1322	28·8651	30·5418	30·7295
30	6·4483	6·7458	7·1015	28·7735	30·9626	30·8337
31	6·4637	6·6916	7·0473	29·0368	30·9537	31·0025
32	6·3242	6·5547	7·0509	28·5763	30·9125	31·1110
33	6·2591	6·6868	7·0602	28·1780	31·3089	31·1354
34	6·4771	6·8477	7·1520	28·8502	31·9339	31·6841
35	6·7689	7·0706	7·2771	30·2309	32·3958	32·0435
36	6·9492	7·4296	7·5157	30·7487	33·5496	32·7802
37	7·3210	7·7325	7·6660	31·7158	34·5887	33·3169
38	7·4334	7·9323	7·8773	32·2577	35·3125	34·0048
39	7·7934	8·0201	8·0123	33·3814	35·6375	34·2312
40	8·0215	8·2174	8·2600	33·6409	36·6284	35·0061
41	8·2191	8·3574	8·5362	34·3144	36·7527	35·8766
42	8·6587	8·4027	8·6817	35·6194	37·2838	36·3261
43	9·1213	8·6079	8·7561	36·7644	37·8277	36·7053
44	9·3046	8·9019	9·0759	37·2154	38·7567	37·7987
45	9·5751	9·2531	9·3308	38·1352	39·4836	38·4808
46	9·8845	9·6536	9·4966	38·9659	40·8224	38·6250
47	10·1196	10·3167	9·8877	39·4774	42·2561	39·7732
48	10·4734	10·7897	10·2603	40·5774	43·2220	40·9762
49	10·9759	11·5371	10·7032	41·6320	45·0016	41·9947
50	11·6345	12·1226	11·0638	43·0322	46·4525	42·0553
51	12·0732	12·8588	11·4824	44·4726	47·8739	44·2921
52	12·4553	13·5272	11·8316	46·0043	49·3830	44·8553
53	13·0583	14·3539	12·1710	46·2859	51·1233	45·5701
54	13·3689	14·7879	12·5437	47·1096	52·2414	46·4499
55	13·7159	15·5269	12·9861	47·1067	53·5471	47·6128
56	14·0632	16·4876	13·5634	47·1668	55·3391	49·3661
57	14·4238	16·9611	14·3136	47·9177	56·0178	51·4436
58	15·2229	17·8730	15·3958	51·0369	56·9281	54·4180
59	16·2718	19·0847	16·6124	52·8581	58·8439	57·6737
60	17·2373	20·0673	18·1968	56·3412	60·6326	61·0253
61	19·1302	21·0959	19·7903	59·4077	63·3179	64·9559
62	20·5513	22·5085	21·6639	61·0857	66·3879	69·5617
63	21·6024	24·2391	23·4156	61·7026	70·7152	73·1276
64	23·0987	25·9519	25·6122	64·7011	74·1741	76·7819
65	24·6623	27·7974	27·7033	67·5363	76·4780	80·5265
66	26·0747	30·0664	30·4829	70·3426	80·0186	85·5011
67	27·8722	32·8324	33·0991	73·9506	84·4047	88·9019
68	29·8403	35·3634	36·4433	77·7879	88·9071	94·7371
69	32·3656	39·5769	40·3034	81·4741	96·7493	101·4379
70	34·5722	44·9672	44·8965	84·1968	106·2174	108·7371
71	36·2394	50·2123	49·7564	86·3399	111·9681	115·0776
72	39·1809	55·5946	55·9087	89·8979	119·4767	124·5084
73	43·7173	60·6206	62·0652	98·6442	126·6347	131·7403
74	44·2770	65·0862	66·2177	101·7273	128·9318	135·8664
75	43·9819	68·1116	70·4569	98·9592	133·2726	141·9663
76	45·3353	73·6694	75·0519	103·1974	141·4521	146·0587
77	47·2255	83·5938	79·9548	108·2250	156·7384	150·7565
78	45·3607	92·6358	85·2334	99·3400	172·3773	155·1686
79	48·7191	105·1417	92·7096	102·0235	187·2954	164·1529
80	54·5000	118·1040	97·4663	107·4203	209·4940	168·1637
81	61·8077	122·9214	102·4263	108·9492	216·5308	174·4794
82	52·8429	125·1736	105·7705	109·9750	213·3061	179·0625
83	57·3261	126·2881	106·1805	97·6667	209·3873	183·3640
84	45·2414	105·6279	102·4649	77·1765	193·2766	181·5696

Age.	Number of Persons Sick out of 100 Persons liable to Sickness.		Average Amount of Sickness per Annum.				Mortality per Cent.	
			Per Individual.		Per Sick Man.			
	Light Labour	Heavy Labour.	Light Labour.	Heavy Labour.	Light Labour.	Heavy Labour.	Light Labour.	Heavy Labour.
18	23·39	29·63	6·3731	7·0417	27·2528	23·7671	·72	·65
19	23·25	28·89	6·4853	7·0950	27·8996	24·5566	·76	·68
20	22·83	28·35	6·4179	7·0959	28·1156	25·0266	·76	·73
21	22·32	27·89	6·3068	7·1890	28·2538	25·7722	·84	·72
22	21·54	27·44	6·1660	7·2570	28·6327	26·4492	·80	·72
23	21·07	26·92	6·0929	7·2699	28·9127	27·0011	·77	·70
24	20·54	26·58	6·0194	7·2768	29·3093	27·3757	·74	·70
25	20·06	26·22	5·9684	7·3387	29·7480	27·9858	·76	·68
26	19·66	26·03	5·9411	7·3928	30·2135	28·3966	·73	·70
27	19·48	25·67	6·0687	7·4290	31·1477	28·9403	·76	·71
28	19·13	25·56	6·0970	7·5162	31·8769	29·4120	·77	·75
29	18·98	25·40	6·0592	7·5491	31·9212	29·7168	·80	·77
30	18·71	25·28	5·9796	7·5654	31·9671	29·9305	·81	·74
31	18·53	25·03	5·9414	7·5206	32·0654	30·0423	·86	·73
32	18·27	25·06	5·7749	7·5463	31·6058	30·1078	·86	·75
33	18·31	25·07	5·7408	7·6457	31·3579	30·4996	·88	·73
34	18·36	25·14	5·8917	7·7710	32·0945	30·9166	·87	·74
35	18·59	25·30	6·1306	7·9193	32·9831	31·3017	·90	·78
36	18·95	25·52	6·4702	8·1490	34·1358	31·9329	·95	·79
37	19·17	25·73	6·7305	8·3684	35·1137	32·5248	1·00	·80
38	19·27	25·91	6·9240	8·5782	35·9278	33·1034	1·08	·84
39	19·37	26·22	7·0492	8·7625	36·4013	33·4159	1·14	·88
40	19·48	26·43	7·1559	9·1025	36·7366	34·4348	1·15	·94
41	19·51	26·69	7·1838	9·4391	36·8319	35·3696	1·18	·97
42	19·55	26·84	7·3225	9·6769	37·2659	36·0497	1·20	1·02
43	19·78	26·97	7·5071	9·8413	37·9513	36·4913	1·20	1·03
44	20·02	27·13	7·6372	10·2870	38·1478	37·9202	1·24	1·08
45	20·28	27·48	7·9355	10·5762	39·1962	38·4934	1·31	1·12
46	20·47	27·86	8·2288	10·8111	40·2003	38·8090	1·31	1·20
47	20·99	28·22	8·7598	11·2118	41·7280	39·7281	1·37	1·25
48	21·49	28·60	9·2018	11·6872	42·8132	40·8637	1·42	1·31
49	22·08	28·86	9·8521	12·0573	44·6128	41·7756	1·51	1·34
50	22·65	29·05	10·3974	12·4801	45·9106	42·9551	1·59	1·42
51	23·30	29·26	11·0104	12·9477	47·2484	44·2444	1·66	1·47
52	23·76	29·69	11·5923	13·3124	48·7971	44·8408	1·77	1·51
53	24·29	30·20	12·2055	13·7391	50·2474	45·4945	1·88	1·59
54	24·55	30·63	12·6253	14·1729	51·4338	46·2751	1·96	1·85
55	24·99	31·18	13·1354	14·7445	52·5587	47·2910	2·13	1·92
56	25·35	31·79	13·7014	15·5511	54·0444	48·9160	2·34	2·14
57	25·60	32·34	13·8238	16·6521	54·0043	51·4859	2·43	2·29
58	26·06	33·11	14·3069	18·1720	54·8944	54·8859	2·63	2·40
59	26·79	33·96	15·1459	19·7776	56·5411	58·2381	2·75	2·30
60	27·41	34·87	16·1511	21·4153	58·9216	61·4158	2·79	2·43
61	27·98	35·60	17·2527	23·2393	61·6686	65·2798	2·85	2·50
62	28·78	36·33	18·8700	24·9854	65·5706	68·7815	3·09	2·61
63	29·49	37·16	20·5259	26·6735	69·6107	71·7739	3·09	2·85
64	30·35	38·35	22·5669	28·4801	74·3445	74·2664	3·56	3·19
65	31·24	39·69	24·6425	30·2534	78·8833	76·2332	3·86	3·50
66	32·34	40·87	27·2100	32·5361	84·1271	79·6001	4·12	3·75
67	33·57	42·40	29·7086	35·1748	88·4926	82·9678	4·60	4·16
68	35·04	43·35	32·9776	38·0270	94·1223	87·7161	5·13	4·48
69	36·06	44·53	36·6137	41·7366	101·5467	93·7371	5·31	4·90
70	37·96	45·63	40·8611	46·6527	107·6347	102·2489	5·94	5·04
71	40·22	47·37	44·6033	52·2168	110·8910	110·2323	6·43	5·34
72	42·05	48·88	48·9103	59·1383	116·3185	120·9925	6·66	5·67
73	43·08	51·53	52·7322	66·9898	122·4039	130·0064	7·14	6·09
74	44·56	53·28	54·9456	72·8668	123·2950	136·7509	7·55	6·67
75	45·10	54·47	56·8370	78·1977	126·0352	143·5594	7·55	7·58
76	45·85	56·37	60·3582	84·1791	131·6486	149·3308	8·76	8·35
77	46·77	58·01	66·1217	89·8373	141·3636	154·8660	9·04	9·40
78	49·10	58·61	72·8949	93·6064	148·4697	159·6996	9·77	10·79
79	51·65	59·35	82·4648	99·6599	159·6736	167·9096	10·00	11·62
80	53·22	60·56	90·9692	104·2469	170·9304	172·1398	12·15	13·38
81	54·55	61·45	100·3470	105·1627	183·9694	171·1471	12·09	14·63
82	55·69	61·74	105·9758	106·1074	190·2957	171·8479	13·37	16·04
83	54·87	61·47	104·3604	108·2783	190·1953	176·1542	15·24	16·11
84	50·45	51·70	93·2342	105·2808	184·8036	170·6276	17·54	17·30

When the four divisions or classes of labour are compared with respect to the average amount of *sickness experienced by each person sick*, the exposure to the weather appears to manifest no influence, the only principle perceptible being that which runs through the whole observation—viz., the inverse duration of the sickness to the frequency of attack.

The four classes of labour are then pursued through the rural, town, and city subdivisions; but, as before stated, these distinctions are in themselves sufficiently vague, and accordingly we find no principle of importance distinguishable in the results.

The conclusions to which we have now arrived are, that difference in respect of locality, density of aggregation, or shelter, goes for little so far as sickness is concerned. If, then, there be a difference in the amount of sickness undergone by one class as compared with another (which there certainly is), it is a consequence of the amount of physical force exerted. The effects of *light* and *heavy labour* upon sickness afford, when contrasted, some remarkable results. Thus there is an almost uniform difference of 7 per cent., in the proportion attacked by illness, in favour of *light labour*; and a continually increasing difference is to be observed in the other columns, between the sickness and mortality experienced by the two classes of labour.

In the mortality experienced by persons engaged in light labour, and by those engaged in heavy labour, we see (p. 21) that *the sickness per individual is in inverse ratio to the sickness per sick man*; and I believe that this relation will always be found to exist. Thus, the greater the sickness *per man*, the less the sickness *per sick man*; and *vice versâ*.

The average rate of exclusions and withdrawals in Friendly Societies had not been published before the issuing of the late Report. This we find to be very heavy. Up to about the middle of life, the probability of an individual leaving or being expelled from his Society is five or six times greater than the probability of his dying while a member; and taking the whole of life, the same probability is more than twice the probability of dying while a member. This, doubtless, has an influence in determining the rate of mortality. So long, however, as the working classes continue in their present social condition, and so many of them are migratory in their habits, the ratio of exclusions and withdrawals must always be high; but if, from any change of circumstances or habits, this rate should be permanently lowered, then we may expect a corresponding increase in the mortality.

It has always appeared to me, that the same reasoning which lessens the premium for a sickness assurance by the amount of the probability of living to receive it, also requires that the amount of the probability of remaining in the Society to receive it should be deducted; because a death, an exclusion, and a withdrawal, are equivalent in so far that each makes the receipt of the amount assured in sickness an impossibility, and each is alike the subject of calculation.

Age.	Exclusions, &c. per Cent.	Age.	Exclusions, &c. per Cent.
13	6·37	38	2·62
18	5·85	43	1·94
23	5·25	48	1·43
28	4·52	53	1·20
33	3·51	58	·99

We have seen that the probability of dying and the average amount of sickness increase with age, and as the vital power becomes exhausted; it therefore appears plausible that the average duration of sickness should bear a certain relation to the mortality, but what this relation is has never been ascertained. In order to attempt to discover this, let a_{20} be the number of persons of the age of 20 in a table of mortality, and a_{21} the number of persons of the age of 21; then the vital power in the 21st year is measured by the fraction $\frac{a_{21}}{a_{20}}$, and all events which happen in direct relation with the vital power must be proportional to the quotient of such fraction. Sickness, however, stands in reversed proportion to it; that is, it is proportional to the mortality. Now the probability of dying in the 21st year is $\frac{a_{20}-a_{21}}{a_{20}}$, since $a_{20}-a_{21}$ gives the number of deaths in the 21st year. If the average amount of sickness is proportional to the fraction, putting S_{20} for the sickness, the resulting equation is $S_{20}=C\frac{a_{20}-a_{21}}{a_{20}}$, where C is a constant: the same for all ages.

The ordinary tables of mortality give the reversed quotients, viz., $\frac{a_{20}}{a_{20}-a_{21}}$, &c., under the title of "specific intensity," or, "number out of which one will die." Let this column be represented by I_{20}, &c., then $I_{20}=\frac{a_{20}}{a_{20}-a_{21}}$; and since $S_{20}=C\frac{a_{20}-a_{21}}{a_{20}}$,

therefore $S_{20} = \frac{C}{I}$; and lastly, $I_{20}S_{20} = C$: that is, that the average number of days' sickness (S) which an individual experiences at a given age, multiplied into the "number out of which one will die" (I), should, if the reasoning be correct, give a product which should be the same for all ages = C.

The following calculation shows the result of the formula when applied to the "specific intensity" and average "sickness" given in the Report upon Friendly Societies:—

Age.	Specific Intensity.		Weeks of Sickness.		C
20	135	×	·98	=	132
25	141	×	·98	=	138
30	130	×	·99	=	129
35	121	×	1·02	=	123
40	97	×	1·17	=	113
45	83	×	1·33	=	110
50	67	×	1·64	=	110
55	49	×	1·99	=	98
60	38	×	2·68	=	102

The dissimilarity of the numbers in Column C is not surprising, since there are almost conclusive reasons for supposing that the mortality actually experienced in Friendly Societies is not that which would take place among the same individuals if they always remained under the observation. This would, of course, vary the quotient C.

Taking, however, the mortality shown by the Northampton Table, we obtain the following results:—

Age.	Specific Intensity.		Weeks of Sickness.		C	
20	71	×	·98	=	70	
25	63	×	·98	=	62	
30	58	×	·99	=	57	
35	53	×	1·02	=	54	
40	48	×	1·17	=	56	56 average.
45	42	×	1·33	=	56	
50	35	×	1·64	=	57	
55	30	×	1·99	=	60	
60	25	×	2·68	=	67	

These results lead to the conclusion that when we obtain the "*normal*" rate, the assumed equation IS = C will be found to be true, because the duration of sickness has a direct relation

to the vital power. Provided this view be confirmed, many important consequences will result, which it is unnecessary for me to point out. At any rate, we may hope that with the aid of new observations the relation between sickness and mortality will be more clearly exhibited than it hitherto has been by those whose attention has been devoted to the subject.

I trust that these remarks, while pointing out errors, will be sufficient to show the great difficulties with which the exposition of the sickness and mortality in Friendly Societies is surrounded, and that to attempt to lay down minute laws and refined distinctions, even at the present time, is only laying obstacles in the path of inquiry.

Report of the Proceedings at the Statistical Congress, held at Brussels, 19th to 22nd September, 1853. *By* Samuel Brown, F.S.S., *one of the Honorary Secretaries of the Institute of Actuaries.*

[Concluded from page 107, vol. iv.]

AFTER the inaugural address of M. Quetelet, the regulations for facilitating the business of the Congress were proposed and agreed to, and M. Ducpetiaux read a list of the papers and correspondence received.

The representatives of the Statistical Departments in the different Governments of Europe proceeded to give a summary view of the position occupied by statistics in their respective countries.

M. Bergsöe (of Denmark) explained, that 20 years ago Government statistics, or works containing statistical tables, were scarcely known in Denmark. An absolute Government gave no encouragement to the people to pry too closely into their proceedings. But when it was decided to form Provincial Councils, the want of statistical information began to be felt. The delegates of the people could not give advice without knowing the actual condition of affairs; and on the proposition of one of the most distinguished statesmen (M. Jonas Collin), the King named, in 1833, a Central Statistical Commission, with power to publish information respecting the condition of the country. This Commission was composed of the most distinguished men in the different branches of administration; but no grant of money was made to them, nor had they any official staff. Each member of the Commission worked separately in his own department, with such means as were

granted from time to time by special order of the King. Several of the members laboured with great energy and capacity, and in the years 1835 to 1849 eighteen large volumes were published by the Commission, relating to nearly all the important subjects of statistics.

In 1849 Denmark received a free constitution. The Commission was then suppressed, as not being in harmony with the new order of things, and a new Central Statistical Department created, consisting of seven *employés*, the head of the department having authority to sign for the different Ministers in statistical matters. At present M. Bergsöe held this post, together with his appointment as Professor of Political Economy in the University of Copenhagen; and under his direction, ten volumes of statistical information on various important subjects had been already published. The grant for the expenses of this department was 35,000 fr. (£1,400) per annum.

In Denmark the Government only has given attention to statistics. There are no societies, except that the Royal Society of Medicine at Copenhagen has a permanent committee for medical statistics. The Provincial Councils publish nothing; and no great assistance is derived from the communes or officials in provincial towns, but rather the contrary. It is to be hoped that, with the progress of civilization, men will begin to understand that it is by such inquiries the statesman must learn the real evils of society, and be enabled to devise the most suitable measures for their relief.

M. Dieterici (of Prussia) traced the origin of statistics in Prussia to the King Frederick William I., by whose order the provincial administrations and the Council of Ministers applied themselves to the question of what was the actual population of the country; but the questions proposed were answered rather by calculations than by a census. Frederick II. continued these labours, and was the real founder of the permanent statistics of the country. Since 1748, excepting during some years of war, the population of Prussia has been annually ascertained.

The Provincial Governments were ordered to number the inhabitants, distinguishing those in the town from the rural districts. Some of these documents contain remarks in the handwriting of Frederick II., and every year he required these population tables to be returned; so that it may be said that Prussia had already an official Department of Statistics. At this time these inquiries began to take rank as a science under the labours of Sussmilch, who, in

his *Gottliche Ordnung*, prepared the tables of births and deaths for 1,056 villages.

In 1805, under the auspices of the King Frederick III., a Central Statistical Department was organized, by the labours of M. Leopold Krug and the influence of the Minister de Stein, to collect the statistics of the provinces; but the war of 1806 interrupted the plan. After the peace of Tilsit, Prussia was reorganized, and M. Hoffmann, then Professor in the University of Kœnigsberg, was named Councillor in the Ministry of the Interior; and he made the first official report on the Statistical department of Berlin. By the care of M. Hoffmann, tables were compiled and some published, on the following subjects, at first every year; but afterwards the reports under the heads 1, 3, 4, and 5 were collected every three years, and only those of births, marriages, and deaths annually:— 1. General statistics; 2. Births, marriages, deaths; 3. Churches and schools; 4. Medical statistics; 5. Statistics of trades and manufacture. These tables, with some additions, are still arranged in the same form as under M. Hoffmann's management. By the advice of M. Alex. Humboldt, a meteorological section has been added to this department, organized in a manner worthy of the founder of this science; reports are received from 34 stations in Prussia, and scientifically arranged and published, under the skilful direction of M. Dove.

In 1834 the Zollverein was formed. One of the leading points in every treaty is that a general census should be taken every three years. It is made with great exactness, as on its results depends the appropriation of the customs' revenue amongst the different States of the Zollverein. Every inhabitant is taken according to his place of residence at the time, except persons stopping at hotels, who, as strangers, are not counted.

M. Dieterici then explained, that since he had had the honour of being named head of the department, he had published five volumes of statistics on the Zollverein. Three other volumes, to be followed by three more on general subjects, had been published, and also a journal devoted entirely to statistics, which appears once a fortnight, and in which a place is reserved for extracts relating to foreign countries.

M. De Hermann (of Bavaria) stated that in Bavaria statistical inquiries, which were conducted only by the Government, formed a department attached to the Ministry of Commerce and Public Works. The correspondence with the officials from whom statistical information is required passed through the ministerial depart-

ment. As to matters connected with the Ministry of Public Works, the orders to the provincial authorities emanate from this department; but those which concern any other branch of administration, from the Ministry of Commerce. The statistical returns so obtained are transferred to the Ministry of Commerce, arranged and prepared for publication by the Statistical Department. Any correspondence necessary with foreign countries is conducted through the Ministry for Foreign Affairs, employing the usual diplomatic agents.

M. De Hermann then explained that the questions did not originate from the Statistical Department; but if, in the Ministry of Commerce or any other branch of administration, special returns were required, the official charged with the inquiry put himself into communication with the head of the Bureau de la Statistique; and, having agreed together on the researches to be made, the Ministry of Commerce initiated the business to the other department, by whom the orders were issued and the returns obtained, and transmitted to the former.

M. LE BARON CZOERNIG (of Austria) said that a Statistical Department had been established in Austria in 1828, by the order of the Emperor Francis, with the special view of aiding the different branches of superior administration in the State. It was attached to the Supreme Court of Accounts, and all departments were called upon to remit to this department the information which had been demanded from them. From these returns tables were annually prepared, and lithographed for the use of the various branches of Government. No other use was made of them. M. le Baron de Kübeck, now President of the Council of the Empire, who was, in 1840, President of the Supreme Court of Accounts, proposed to form a special Statistical Department, and unite the scientific to the simply administrative part of statistics. Immediately afterwards, the Baron de Czoernig was appointed to preside over this department, and prepare the materials for publishing the official statistics of Austria. It was his aim to include all branches of political economy, agriculture, mining, manufactures, and commerce, as subjects of which the general knowledge was most required by the public. The returns so prepared were then for the first time published, with the addition of tables of population, public instruction, and judicial statistics.

In 1844 the entire publication of the statistical returns was authorized, including even what is purely administrative, so that a most complete and faithful *exposé* was made to the public of the

whole financial condition of Austria, of the resources and industry of her provinces, and the manner in which they could be made to contribute to the national wealth. For example: the tables showed that in 1841, in the administration of tobacco (a Government monopoly), 28 millions of cigars were manufactured; whilst the present production has increased to 800 millions, and would be much greater if the manufactories could produce as fast as the consumption increases.*

The total production of the manufactures of Austria appeared in 1845 to be of the value of 2,500 millions of francs (£100,000,000) —the same total value which was given by Schnitzler as the result of the cotemporary manufactures of France.

The Statistical Department in 1848 was attached to the Ministry of Commerce and Public Works, the branches of inquiry embraced therein forming the principal subject of statistical research.

Baron Czoernig then entered into a short statement of the mode in which the Statistical Department was carried on, so as to accomplish the purpose for which it was formed. All the information and returns which the different branches of administration may have required and procured for special purposes, and which would have been laid aside amongst the archives when the temporary object is achieved, are transferred to the Statistical Department, where these returns, frequently of the highest importance, become the subject of further labours.

If the branches of administration, before collecting the information they require, confide to the Statistical Department the care of preparing the forms and queries, the results become so much the more valuable for future use. In this manner a sure check is established, when the facts obtained and arranged under different points of view agree in their final results. The critical examination of statistical returns must be used with more severity in cases where they are obtained in a great degree from private individuals; as, when the question is respecting the manufactures or interior commerce of the country. The statistical inquiries into manufactures have been attended with the most satisfactory results, whenever private individuals have been questioned as to the force of machinery or power of production, the raw or partly prepared material consumed, as well as the quantity which enters into foreign trade, which are generally known to the larger establishments in

* May we not infer that this is a strong argument against State monopoly? since it is reasonable to suppose that, in such a case, competition would lead to an increase in the number of manufactories, so that the wants of the public would be supplied as they arose. (S. B.)

direct connection with the manufacturers. By an acquaintance with the technical part of the different processes and their combined operation, we may arrive, even without a direct and detailed valuation of industrial products, at an approximation very near the reality, and amply sufficient for all purposes of statistics.

Baron Czoernig concluded by enumerating the periodical and other publications under the direction of the statistical administration of Austria. These are—1. The official *Statistical Tables*, published annually, of which those for the two years 1847 and 1848 would appear in a few weeks. 2. The *Statistical Communications* have been published, for the last four years, monthly or quarterly. They contain the reports of the Austrian Consuls, especially those in the East, on the commerce of their countries, other subjects omitted in the great work, and the *resumé* of the most recent information. 3. The *Tables of Foreign Trade—Import, Export, and Transit*—are published annually. The volume for 1852 is in the press. 4. The *Austria*, a journal of political economy, founded by the order of the Minister of Commerce: and, finally, 5. The great *Ethnographical Map*, which has been ten years in course of preparation, and the text of which is not yet finished, completes the list of Government publications.

M. Legoyt (of France) explained that in France statistical labours are divided amongst the different departments of Government, each one publishing the documents relating to its own branch of business. Some Ministries have, however, special Statistical Departments attached. The Minister of Justice publishes the criminal statistics, which are confided to the care of M. Arondeau, one of the most eminent statisticians of France. The Finance Ministry possesses also a department for custom-house statistics.

In 1834 a Department of General Statistics was established in connection with the Ministry of Commerce, and placed under the direction of the eminent M. Moreau de Jonnès. Since that date there have been published by him about thirteen volumes on different subjects of political economy—population, administration, finance, commerce, agriculture, manufactures, &c.

In 1852 M. Legoyt succeeded M. Moreau de Jonnès on his retirement, and soon perceived, from the correspondence of the Prefects, that too much was required from the provincial authorities, who had neither time, talent, nor knowledge sufficient for many of the researches they were called upon to make. He proposed therefore to associate private individuals with the agents of

the Government; and by a decree of 1st January, 1853, a new system was established. By this decree, in every canton a Statistical Committee has been formed, the members being named by the Prefect. It is subdivided into communal subcommittees. As an experiment, a form of questions has been prepared for the present statistics of agriculture, and, to a limited extent, of manufacturing industry. Each subcommittee is required to fill up the form submitted to them and send it to the cantonal committee, where it is discussed; and this committee may consult with those members of the subcommittee who have filled up the form, in order to resolve any doubts or difficulties which may suggest themselves. The separate forms for the communes being agreed to, the cantonal form is prepared, and, when adopted, referred to the Subprefect, who submits it to a double verification—first by the Central Commission of the arrondissement, and secondly, as far as regards agriculture, to the Chamber of Agriculture. These verifications being finished, the various cantonal forms are sent to the Prefect, who submits them to a final examination, recapitulates the results, and forwards them to the Minister of the Interior.

The system is still only at its commencement, and whether it will work satisfactorily not yet ascertained.

The General Statistical Department corresponds, through the Minister, with the Prefects, to procure the documents which result from the simple performance of their administrative functions. Thus the Prefects transmit to the Commission documents respecting population, the management of the *enfans trouvés*, mendicity, &c.

Mr. William Farr (of England), previous to describing the organization of statistics in England, requested permission to state that he had been deputed to assist at the Congress by the Registrar General, with the approbation of the British Government, and had been charged to make a report on the proceedings of the Congress. This fact, and the communication which he had to make on the part of English statisticians, were proofs of the desire of England to aid in accomplishing the important objects set forth in the programme—to communicate the results of her experience, to gain new information from the distinguished talent here assembled, and to consider with the utmost attention any plan that might be proposed to produce uniformity and the means of comparison between the statistical results of all countries.

The fact of uniting all the statisticians of Europe, engaged as they were in observing and analyzing social phenomena, is in itself a benefit for us and for science—a benefit which we owe to

the enlightened character of the Belgian Government, so honourably represented by the President, who had so long cultivated the field of statistics, and whose reputation and labours were, he ventured to say, fully appreciated by all the statisticians of Europe.

Mr. Farr then proceeded to describe the system adopted in England for procuring statistical information, and analyzing the facts and consequences resulting therefrom. He enumerated the various departments of Government having the power of making statistical inquiries through their local agents and officials—the army and navy, the Board of Customs and Taxes, the Post Office, the Poor Law Board, and the Registrar of Population. All departments publish information in different forms, which is more or less statistical. The Chancellor of the Exchequer, in making an annual statement of the financial position of the country, lays before the House of Commons a general account of the revenue and expenditure of the country. The two Houses of Parliament have the right of calling for information on all branches of the public service, and of naming committees for special investigations, who take the opinions not merely of public functionaries, but of men of science and any others versed in the particular subject of inquiry; and the evidence is printed in *Blue Books*, frequently containing facts of the highest interest and importance.

The statistics of customs and other branches of commerce have been long preserved, and, since the time of Mr. Porter, have been annually published by the Board of Trade. At present the publication is under the direction of Mr. Fonblanque. Mr. Valpy, who is preparing the last volume, has explained the publications of the Board of Trade in a note to the President of the Congress.

Criminal statistics are furnished to the Home Minister from the criminal and police courts, and are analyzed and discussed by Mr. Samuel Redgrave, in a letter to the President.

The statistics of the army are prepared by Colonel Tulloch and Dr. Balfour, and those of the navy by Sir William Burnett and Dr. Bryson.

Other statistical publications emanate from the Poor Law Board.

The General Register Office, under the direction of the Home Minister, is charged with the execution of the statute for the registration of the births, deaths, and marriages; and the Registrar General, in accordance therewith, lays the returns before Parliament.

Mr. Farr then went into a description of the mode of taking

the census, and the results of the last census, which would form a very interesting statement, if space permitted.*

Mr. Farr then explained that, besides the official departments, there were other means in England of following out statistical researches, as it was not the custom for the public to leave subjects of such general interest entirely in the hands of the Government.

The Statistical Society of London, of which H. R. H. Prince Albert is the illustrious patron, and which was represented at the Congress by Viscount Ebrington, has made many important researches; and its members have furnished many papers of the highest interest, both for discussion at its Sessional Meetings and for publication in their *Journal.*

Amongst the private individuals who have contributed to the diffusion of statistical knowledge, are the late Mr. Porter, and Mr M^c^Culloch — the former distinguished for the clearness of his writings, and the latter no less celebrated for his efforts to analyze and discuss statistical subjects, than for his brilliant theories on political economy.

The PRESIDENT stated that there were two representatives from Saxony present, M. Weinlig and M. Engel, and called upon one of those gentlemen to favour the meeting with information as to the progress of the question in their country.

M. WEINLIG mentioned that in the kingdom of Saxony M. Engel was the head of a Department for Statistics attached to the office of the Ministry of the Interior, which was also the Ministry of Commerce, of which he was himself Director. In two years they had published three quarto volumes, containing the census, the movement of the population, and the population divided into professions and occupations. M. Engel had just completed the publication of another volume, containing a concise *resumé* of the statistics of the whole country.

M. RAMON DE LA SAGRA (of Spain) expressed his embarrassment in following speakers who had made reports for countries so rich in the results of statistical labours; but his Government had deputed him with a view of laying the foundation of an improved system in Spain. Coming late into the field, they had the advantage of being able to select and adopt the best without having to destroy the past. Notwithstanding the length of time that statistical labours had been known in Spain, dating even from the fifteenth century, they were still unorganized as a science.

* I would beg also to refer the readers of this report to the very clear and concise summary in Mr. Cheshire's pamphlet, *The Results of the Census of Great Britain in* 1851; *with a description of the Machinery and Processes employed to obtain the Returns.*

Physical and physiological statistics promised well for the future. A remarkable map, still in course of correction, with the recent discoveries, had nearly been completed, of Spain considered in the point of view of its geography, geology, mineralogy, and agriculture. The records of the climate had not been much attended to, though a monthly meteorological report was made from the observatories of San Fernando and Madrid.

As to the statistics of population, they had no census of the population at the present date, but the Minister of the Interior was actively engaged in the preparations for one. The records of births, deaths, and marriages were carefully preserved in the Church registers; but those particulars not connected with religious rites were still wanting, such as the number of the stillborn, the conditions of the parents, &c. The provisions of municipal law, however, recently promulgated, which resemble the French, will, with the transmission of the Church registers, which the Mayors send to the Prefects to be forwarded to the Ministry of the Interior, enable a report on the movement of the population of Spain to be published shortly.

As to medical statistics, the only information extant is found in some of the medical journals, which notice the mortality, epidemics, &c.

The statistics of mining may be found in the *Journal of the School of Mining*, and in other private publications at Madrid. The report of M. Caveda, the Director of the Department of Commerce and Industry, made to the Government respecting the Great Exhibition of 1850, affords a history of production in the principal branches of industry in the Peninsula.

In official publications on commercial statistics, an interval of more than thirty years since the beginning of this century has occurred, and those preceding are not published with regularity nor on any uniform method. Recently, the publication of tables of foreign commerce has been resumed, under the title of *Balanzas*. Since the institution of a constitutional *régime*, the Ministers are obliged to present to the Chambers a budget, and the summary of the revenues of the State is also published.

With regard to moral and criminal statistics, some information may be obtained as to schools and the number of scholars frequenting them, from the *Bulletin of Public Instruction;* and as to the state of crime in the south, some curious details in a document published about ten years ago, on criminal statistics, in the Balearic Isles; but he would more especially refer to an important

work by a private individual, M. Pascual Madoz, Member of the Chamber of Deputies, entitled *Geographical and Statistical Dictionary of Spain*, in 16 large vols., 8vo., in which the author, by his agents and *employés*, has taken every pains to make his work of the highest value, using all the means which in general Governvernments only can command.

M. Ramon de la Sagra then entered into a statement of the much more complete statistics collected in the Spanish colonies, especially Cuba. Tables prepared officially by the Government of the colony, on population, commerce, exports and imports, revenues and expenses, and many other similar topics, have been published, commencing with the year 1817 to the present time, from which he had been enabled to complete the two first volumes of *The History of the Island of Cuba*, which he had presented to the Congress.

M. Mittermaier (of the Grand Duchy of Baden) desired to prove that his country was not behind in these inquiries. The Grand Duchy of Baden was the first of the German States which published an annual report of criminal statistics, in which for 15 years they have followed the model of the French reports. Since then the statistics of civil justice have been added. Each department publishes its statistical labours, and last year a valuable report on the taxes, commerce, and riches of the Grand Duchy appeared from the Ministry of Finance. Private journals and a medical journal also contain contributions. A Statistical Department has recently been established in connection with the Ministry of the Interior, at Carlsruhe, and which, though not yet completely organized, has issued to the communes a series of 45 questions on the important subject of the moral statistics of the country.

M. le Chevalier Bertini (of Sardinia) thought that the organization of statistics in Sardinia was too well known to many present for him to occupy much of their time. It accorded very nearly with that of Belgium. With the exception of her geographical configuration, Sardinia has much analogy with Belgium, by her constitutional system and the different branches of her administration. She looked upon Belgium as her elder sister; and the sovereigns of both countries, types of constitutional kings, were alike honoured and beloved by their subjects.

A Statistical Department was instituted by Charles Albert at the commencement of his reign, with which 37 Juntas or Provincial Commissions, each composed of six members, are in correspondence. These Juntas collect information in each commune from the local authorities, the curés, and the principal inhabitants.

The Central Commission publishes the results of these labours under the title of *Informazione Statistiche*, of which four volumes have already appeared. The one of last year contains medical statistics. Besides these volumes, two reports have been prepared on the administration of civil justice, of which the part relating to criminal statistics will appear shortly, and that relating to education, primary and secondary, has already been published.

The President of the Ministerial Council, the Ministers of Finance, of Public Works, and the head of the Post Department, publish monthly accounts in the *Official Gazette* of the produce of the indirect taxes, the State railroads, the postage, &c. The annual budgets presented to the Chambers by the Ministers, as well as the reports of the Intendants General to their respective Councils of Administration, contain many important statistical details.

Amongst other works may be mentioned the general calendar published every year by the Minister of the Interior, and a statistical annual which appeared in 1852 and is rich in varied information on the population, industry, commerce, navigation—railroads finished, in course of construction, or projected, &c.

M. DE BAUMHAUER (of the Netherlands) stated that the Dutch Government had established in 1826 a Department of Statistics, by the officials of which two volumes on general statistics had been published, and that the principal object of their attention had been the population, collecting the documents relating to births, deaths, and marriages since the year 1804, and in some communes from an earlier date. They also arranged the census of 1829. Their functions ceased in 1836; but a third volume, completing the other two, appeared in 1836.

Since 1826, M. Lobatto, Professor at Delft, has published by order of the King an annual relating principally to population and the statistics of prisons.

In 1840 a census was taken, being entrusted to the Governors of provinces, and the results were published in a small volume.

On the revision of the fundamental law in 1848, two Statistical Departments were established—one in connection with the Ministry of the Interior, and the other, of which M. de Baumhauer is the head, with that of Finance. This department has published, since 1815, reports on the statistics of education and of benevolent institutions. Three volumes of reports, the first of which contains the years 1847, 1848, and 1849, have been published respecting criminal statistics, by the Ministry of Justice.

Reports respecting navigation and foreign commerce have been published regularly by the Ministry of Finance for the years 1846 and subsequently, of which the seventh volume is in course of preparation; besides detailed reports on the finances, which are laid annually before the legislative chambers.

Since 1849, the Ministry of the Interior has been actively engaged in the third census (taken, with the exception of agricultural and industrial statistics, in the same manner as in Belgium), the results of which are just published. They comprise benevolent institutions and schools. A volume is devoted to each of the eleven provinces, and the twelfth (of 1,000 pages large folio) contains the recapitulation for the whole kingdom. Since then, the reports made by permanent deputations to the Provincial Councils have been deemed of greater importance, and an attempt has been made, by adopting an uniform plan, to improve these valuable records, which form the basis of the statistics of the country.

The Ministry of Justice has published regularly, since 1847, a statement on judicial, criminal, and civil statistics, which has appeared for five successive years; and the Inspectors of Hospitals for the Insane, which are under the control of Government, also publish annually detailed and interesting accounts.

The Colonial Ministry gives annual reports of the statistics of the Indian colonies, both eastern and western.

M. Sick (of Wurtemberg) explained that the kingdom of Wurtemberg possessed a Bureau (founded in 1820, under the orders of the Minister of Finance) for Statistics and Topography, of which M. Hardegen was the chief. Under the last head, the *Topographical Atlas* of the kingdom has just been completed, on a scale of $\frac{1}{50000}$, and a *General Map* on a scale of $\frac{1}{400000}$.

The *Description of the Kingdom*, by arrondissements of administration, called *Oberamter*, is in course of publication; 33 volumes have appeared, edited by four of the nine members who constitute the Bureau. There is also published annually the movement of the population (births, deaths, changes of domicile), a report on the prices of corn, wool, and statistics of fires, with a summary every ten years. Every three years, also, an enumeration is made of domestic animals. The produce of vines appears annually; but the complete statistics of agriculture are still deficient. Instruments for meteorological observations have been established in five telegraph offices, and the results will be shortly printed in the *Würtembergische Jahrbucher*.

M. DE MEYER DE KNONAU (of Switzerland) considered that, though there was no Government Department for Statistics in Switzerland, yet much progress had been made since 1830. For half of the 25 Cantons, there were reports more or less complete; but the most noticeable were those of the Cantons of Basle, of St. Gall, and Zurich. The private philanthropic and industrial societies numbered amongst them many individuals who had distinguished themselves in these pursuits. He would particularly mention M. Etienne Franscini, author of the *Statistics of Switzerland*, written in Italian, since translated into German, and about to be translated into French. M. Gonzenbach, also, a distinguished member of the Government, had published interesting works on the manufactures of Switzerland. There is also a minute topography of all the Cantons, of which M. de Knonau was himself the author.

The PRESIDENT returned thanks to the various speakers for their very valuable communications.

M. GARNIER (of France) inquired if no communication had been received from Russia.

The PRESIDENT replied that Russia had no representative in the Congress. The reason was indicated in the reply they had received, and which was not without interest in a statistical point of view.

The different sections having been constituted, for three classes of subjects, the meeting adjourned at half-past two o'clock for the following day.

The speeches and explanations offered by the delegates from the different Governments of Europe to the Statistical Congress have been given somewhat at length, because they serve to show the progress and present position of statistics in their different countries. They also enable those who take an interest in these subjects to form an idea of the publications in foreign languages which are published by Government authority, and to which therefore they may have recourse with confidence. But the space allotted in this *Magazine* will not allow me to give more than a very brief summary of the further proceedings of the Congress, and of the interesting discussions which occupied the following three days.

The subjects were referred to three different Sections for discussion. The first comprised Population, Survey and Valuation of Land, Emigration, &c.—M. Quetelet, President; the second, Agricultural and Commercial Statistics—President, M. Horace Say, of

the Paris Chamber of Commerce; and the third was devoted to the Statistics of the Social Condition of the People, Pauperism, Education, Crime, &c.—M. Visschers being the President.

The various subjects had been arranged in a programme, the heads of which could during the mornings be discussed in the different Sections at the same time; and reporters having been selected from each Section to state the alterations proposed, and what was finally carried, with the reasons for the changes, the resolutions were in the afternoon sittings submitted to the consideration of the whole Congress united, and confirmed or altered by them. Time was thus saved; whilst the duty charged upon the reporters from the Sections gave rise to many lucid and eloquent speeches, comprising a *resumé* of all the arguments *pro* and *con.*, on the subjects debated in their respective Sections.

The summary of the conclusions of the First Section was—

1. That for the organization of statistics and the adoption of an uniform system of inquiry and publication, without omission or repetition of details, it is necessary that the officials engaged in preparing the tables and statements for the public should use the same divisions and the same terms to denote subjects of the same kind.

2. That every Government should establish a Central Statistical Commission, or some similar office, composed of the heads of the various departments, and others, who by their knowledge and experience may combine the practical working with the science of statistics.

3. That as accuracy in minute details can be best ensured on the spot, local Commissions should be established in the provinces, to be in communication with the Central Commission; the advantage of which, besides the individual knowledge of local peculiarities, which might escape the notice of strangers, will be felt in overcoming, by their personal interest, the prejudices and opposition with which Government inquiries are sometimes received.

4. That delegates from the Central Commissions of different countries should periodically assemble in an international Congress, whose duty it would be to reduce the documents for publication to a uniform and the most simple plan, and at the cheapest rate of charge to the public.

The second question submitted to this Section was the general census of a population—the information to be required, and mode of collecting it.

In Belgium, as in England, the census is taken on a given day,

and every individual is entered on the lists for the place where he passed the night. As to the date, there is scarcely any fixed arrangement. In France and England it has been in the months of April and June; in Norway, in November; in Denmark, in February; in Belgium, in October; and in the German States, in December. The periods have been equally varied; in England, the United States, Sardinia and Norway, and in the Netherlands, the period is decennial; in the German States, triennial; and in other countries, as in Belgium, at indeterminate periods. The resolutions agreed to were—

1. That the census of a population should comprise the name of every individual actually in the country *(population de fait)* at the date of enumeration; but that information should also be required as to individuals legally domiciled, though absent at the time *(population de droit)*.

2. That the interval of taking the census should not exceed ten years, and that it should be in the month of December.

3. A special return for each family.

4. Special agents to be employed to deliver and collect the lists; and either to see that they are properly filled up, or to enter the necessary particulars themselves, as obtained by them from the inhabitants.

5. The returns to include—full name, age, place of birth, language, religion, civil condition, profession, residence (distinguishing temporary from permanent), children receiving public or private instruction, houses by stories, with the number of rooms to each family, gardens, &c.; to distinguish also individuals suffering under such infirmities as affect the labour of the country, as, the blind, the deaf and dumb, the insane in public and private establishments.

The registers of population should comprise the facts relating to each family; and the following important inquiries were recommended to ascertain the movement of the population;—The births, legitimate or illegitimate, according to sex and the ages of the parents; number of twins and stillborn; the deaths, marriages, and divorces, all classified according to profession, by ages and in months, and the causes of death to be certified by medical men; the ages of persons contracting marriage to be stated, with their civil condition or profession. The causes of death and the varying nomenclature of diseases formed a very important subject of discussion, and the necessity of adopting some general nomenclature applicable to all countries was reserved as a point worthy of consideration at the next Congress.

Two other subjects of great interest were also discussed in the First Section; the former as to the survey and valuation of land, and the second as to emigration. The radical reform in the method of survey, adopted by France in 1808, has formed the basis of all the modern improvements. It was recommended that the general survey of a country should be on a scale of 1-2,500, but varying, according to the minute subdivision required, to 1-1,250, or even 1-500, and the general index maps to be 1-10,000 or 1-20,000; that the maps should comprise the name of every proprietor, and the nature of the soil and cultivation of every property; that from a careful comparison of prices of produce and rents of farms for several years, the annual value should be affixed against each property, according to the quantity under various kinds of cultivation; and, lastly, that the changes in limits, in mode of cultivation, in ownership, and in value, should be noted, so that the survey might be continued as a permanent record of the property and produce of the country.

The question of emigration has lately become of the most vital importance, especially in this country; and the information to be sought for must be considered in reference to the influence of emigration on the fortunes of the mother country, and the effects it may produce. It should comprise the name, place and date of birth, sex, age, religion, profession or occupation, place of embarkation and destination, and the motives of emigration, if they can be ascertained, together with the capital which the emigrant takes with him, and the average cost of the passage to different places; number and tonnage of emigrant ships; deaths on the passage, with their causes; and the number sick, with the nature of the diseases, on their arrival. The resolutions originally proposed led to much discussion, and were modified so as to avoid as far as possible questions which might be considered too inquisitorial.

To the Second Section was referred the very important question of agricultural statistics, and the report thereon was brought up by M. Cogels on the last day of the Congress. The time at which such statistics should be collected is fixed in all countries as it were by nature, being the period when the labours of the agricultural year are finished, and when those for the next season commence; and although various countries may differ from each other as to the nature of their products, and consequently as to the period of the year when the required information can be obtained, yet it was considered by the Congress—

That in order to compare the agricultural products of different

countries, the facts should be collected simultaneously, either for the same agricultural year or for a period which may be considered as expressing a general average; and that, unless unavoidable difficulties intervened, or unless the intervals were too long, the inquiries should be conducted at the same time as the census of the population.

That special agents, capable of verifying and correcting the facts stated, should be employed to collect the returns, which should be left with each occupier long enough to enable him to fill in all the particulars ready for the Inspector's visit; and that in every district the returns should be revised by some local Committee, and reduced into some general form with which they should be furnished, so as to allow of being compared with others.

That the particulars required should be such as to enable a general knowledge to be formed of the capability of a country in its agricultural productions at any given time—its soil, the weather and other natural phenomena, the power and instruments for cultivating the ground, the means of supplying the exhaustion or improving the nature of the soil, agricultural stock, rotations of crops, the laws of production, and the relations of agriculture to society, or the regulations under which its operations are carried on. These inquiries, however, though desirable, were thought by the members too minute to be prescribed, and the report finally adopted was in more general terms.

This Section also considered the questions of industrial and commercial statistics, which, after discussion, were somewhat modified from the original propositions, and divided into four branches: (1) Agricultural, (2) Mining, (3) Manufacturing, (4) Commercial statistics. The report of the discussion on the last subject was made by M. Partoes, of Belgium. In the subdivisions of commercial statistics, it was proposed to distinguish the imports and exports by land, rivers, canals, and by sea—the last showing also the quantities brought by national or foreign ships. The tables should comprise the total quantities in weights, measure, or number, and the valuation with and without the duties imposed. Under the head of maritime service it was proposed to require the number and tonnage of ships both home and foreign, with the places of their destination and their ports of departure, as well as the statistics of the mercantile service and the coasting trade, the number and tonnage of the vessels at present existing, with those that are constructed and purchased in the year, or lost and sold; the number of sailors, also, to be enumerated. The Section also recommended,

that previous to the ensuing meeting of the Congress a report should be prepared by the Commission Centrale de la Statistique de Belgique on the commercial tables which have either been published or collected in different countries, pointing out the discrepancies in their form and the deficiencies in the information which they exhibit.

The report of the discussions of this Section on manufacturing statistics was eloquently made by M. Horace Say, on the third day of the Congress. His Majesty the King of the Belgians, accompanied by their Royal Highnesses the Duc de Brabant and the Comte de Flandres, and attended by his Ministers and a numerous suite, honoured the Congress with his presence on this occasion. At one o'clock, the approach of the King being announced, M. Quetelet, the President, with M. Piercot, Minister of the Interior, M. Liedts, Minister of Finance, and M. Chas. Faider, Minister of Justice, with the Vice Presidents and other officers of the Congress, went to receive His Majesty at the foot of the grand staircase, and many of the foreign members of the Congress had the honour of being introduced. The King and the Princes having taken the seats prepared for them near the President's table, M. Quetelet addressed His Majesty in the following terms:—

"Sire,—En venant assister aux travaux de ce Congrès, votre Majesté donne une nouvelle preuve qu'elle s'associe de cœur à tous les efforts que tentent les peuples pour resserrer entre eux les liens d'amitié et de bienveillance qui doivent les unir, et pour developper, dans le domaine des sciences politiques, l'esprit d'ordre et les lumières qui concourent si puissamment à leur bonheur."

His Majesty having graciously received this address, and ordered the business to proceed, M. Horace Say entered into an eloquent discourse on the connection of political economy with statistics, and on the light which these inquiries would throw on the important question of the condition and prospects of the industrial classes. It was proposed to ascertain the number of workpeople—men, women, and children—under sixteen or fourteen years of age, employed, and in the last case the number engaged as apprentices, and the conditions under which they are bound; the number of workpeople receiving average wages, and those obtaining less or more than the average. The same observations are to be made relative to the mining population, which formed another subject for a report. Independent of these inquiries, the number of establishments, the motive power (steam, hydraulic, &c.), and in mines the quantities extracted, &c., should form part of the returns.

To the Third Section were referred three questions of great importance: Question 8, the economical condition of the labouring classes—the actual cost of subsistence, divided into necessary expenses of living, the cost of religious, moral, and intellectual education, and the cost of such articles as may be considered luxuries. Question 9, the statistics of pauperism—number of the poor of different sexes and ages, charitable and other institutions, causes of pauperism, cost of maintenance of the poor, institutions designed to avert or diminish pauperism, &c. Question 10, statistics of education—public and private schools, number of scholars, nature of instruction, &c.; and Question 11, comprising crime and the means of its suppression—number and nature of offences committed, penalties inflicted, or the number acquitted, &c.: all being distinguished, as far as possible, by sexes and ages. The reporters and principal speakers on these important questions were, besides the President, MM. Visschers, Horace Say, Legoyt, le Comte Cieszkowski, &c., Ramon de la Sagra, and Ducpetiaux.

Before the conclusion of the Congress, a resolution was passed expressing a hope that the postage reforms would be extended to international systems of postage, and a similar expression of opinion was given as to the assimilation of the commercial laws of different countries.

It would be unjust to conclude even this brief report without a grateful tribute to the generous attentions and hospitality which all the members of the Congress received from the Government of Belgium and the distinguished men of science residing at Brussels. Independent of all the public establishments, and objects of art and interest, being gratuitously shown by the production of the tickets of the members, dinners and soirées were given by the Ministers, and by M. Quetelet, the President of the Statistical Commission, thus enabling the number of eminent men assembled from all parts of Europe to form those social connections, and to impart to each other that knowledge and experience, which may prove of the utmost value hereafter in extending and simplifying the study of statistics. His Majesty the King of the Belgians took the liveliest interest in the proceedings of the Congress, and, besides honouring the meeting by his presence at the discussions, invited a large body of the members to a grand banquet at the Palace, at which the Royal Family, the Ministers, Foreign Ambassadors, and other distinguished guests, were present.

The variety and interesting character of the subjects discussed at the Congress could barely be indicated in so brief a summary as the

foregoing. Much still remains to be accomplished in future meetings; but there cannot be a doubt that the union of so many eloquent writers and scientific men, who have profoundly studied the great social questions of the age, cannot but result in improving the means of collecting and classifying the facts which bear upon them. Governments will thus be enabled to legislate for the true welfare of the people, and philosophers and moralists may correct many theories which, however ingenious, can never be practically useful unless based on that truth which forms the subject of all statistical inquiries.

Suggestions in regard to the Regulation of the Rate of Interest on Landed Securities. By Wm. Thos. Thomson, *Manager of the Standard Life Assurance Company and of the Colonial Life Assurance Company.*

THE above is the title to a pamphlet just published by Mr. Thomson. The subject is one of such importance, that we should have been glad had our space permitted to give it *in extenso;* as it is, we place before our readers the more prominent parts of it. Mr. Thomson says—

"It has been the practice in Scotland for a considerable period to fix the rate of interest on landed securities periodically, with reference to the views expressed on the subject by, or more correctly the decision of, a meeting held in Edinburgh three months before each money-term of Whitsunday and Martinmas.*

"The parties assembled to discuss the question have not represented by any fixed arrangement, or by appointment, the various moneyed interests of Edinburgh, or of Scotland; but their decisions have always been so much in accordance with the general opinion, and have been so satisfactory, that the constitution of the assembly has not till recently been called into question.

"I have no information to enable me to explain the origin of these meetings, but it is evident that the practice of appealing to their decisions has arisen from a desire on the part of lenders and borrowers to have some mode of fixing the rise and fall of interest according to the value of money, by reference to the opinions of those who were considered best capable of judging, without leaving the adjustment to be matter of bargain between parties as the market rose and fell.

"The Widows' Fund of the Ministers of the Church of Scotland and of the Professors of the Scottish Universities, established in 1744, was for a considerable period the only moneyed body having considerable sums to lend

* "Changes in the rate of interest were made on three occasions—in 1828, 1829, and 1831—at the intermediate term of Lammas; but I do not think it probable that changes at Lammas or Candlemas will again be made."

on heritable security in Scotland. The Widows' Fund of the Writers to the Signet, established in 1808, afterwards became a large lending body; and the trustees of these two institutions, or, more correctly, the Trustees of the Writers to the Signet's Widows' Fund (who are invariably leading law agents), and the law agent of the Ministers' Widows' Fund, constituted for a considerable period the meeting, from which the decision as to the rate of interest emanated. About twenty years ago, the Scottish Widows' Fund and Life Assurance Society, established in 1815, were called to take a part in the proceedings, in consequence of their having become large lenders on land, and they are still represented. Since that date, the constitution of the periodical meeting has, I believe, remained unchanged.—At the same time, it is right to state that I understand it has been the custom for some time to consult, before each meeting, some of the leading law agents who were not privileged to attend; also to communicate with the banks; and I know it has been the practice lately to ask the views of gentlemen connected with the management of the Assurance Companies, although they have not, with the exception before mentioned, been invited to take part in the proceedings of the meeting."

Mr. Thomson then expresses an opinion that a revision of the constitution of this assembly is desirable, considering the great magnitude of the interests not represented in it, and continues—

"I think the chief object to be kept in view is—that an opportunity be given for the expression of opinion by the different moneyed bodies throughout the country who lend on landed security, and that the interests of private individuals who lend or borrow be at the same time fully attended to, by enabling their law agents to express their opinions and take part in the proceedings.

"To accomplish this, I think a meeting should be held periodically, at which the representatives of the Widows' Funds already referred to—the Society of Writers to the Signet, and such other Law Societies as may be considered eligible, also the Managers of the Scottish Assurance Offices, should be permitted to attend and express their views: that such meeting should come to no decision on the question under discussion, but that a committee of twelve persons, chosen so as to represent the different interests, should be appointed, and, after giving due weight to the views and opinions expressed at the meeting, decide what the rate of interest should be for the half year to be entered on.

"It would require some consideration to regulate the admission of bodies to the deliberative assembly; but I would suggest a wide basis, the meeting being held for the expression of opinion merely. The members of the committee instructed to decide the question might be chosen from the bodies already represented, from the Society of Writers to the Signet, and from the Assurance Companies—so many from each, to be elected by the bodies themselves.

"An Association so constituted would, I think there is every reason to anticipate, receive the entire support of the country, and their decision be invariably followed, tending greatly to the interests of borrowers and lenders by the fairness of the principle and the confidence which would ensue.

"Let me now call attention to English and Irish practice, to show the superiority and importance of our arrangements.

"In England the rate of interest is entirely regulated by the principle of supply and demand. Is there a better principle? I may be asked; and theoretically I answer, No: but what is the fact? A landed proprietor has borrowed a large sum at 4 per cent.; how can he get a reduction of rate? Only by a threat to pay off, and a certainty of procuring the money elsewhere, if he gives notice. But it is a matter of no small difficulty to get money in such circumstances, for the interest of the large money lenders being to keep up the rate, they will not enter into competition for new securities on lower terms. There is no communication between agents representing lender and borrower, except by sending to each other proposals for loan; there is no generally expressed opinion as to the rate of interest to appeal to; and although new transactions, when the security is very ample, may occasionally be got at a lower rate than the average, they have a strong tendency to get up to a higher rate, and are not easily brought down again. An easy and inexpensive mode of assigning securities in both countries would tend to let the value of money regulate the rate on juster principles; but without communication, and an inclination on the part of lenders to let the terms be regulated by justice, and not by mere force of necessity, the position of matters, although improved, would not gain what we aim at—*fairness.*

"I have not ventured to attribute the higher rate of interest which obtains in England to what, in Scotland, is considered the defective state of the law of England as to titles to property, from the want of a general system of registration of deeds, as I am not prepared to go deeply into the subject; but I have no doubt that it is an element, taken in connection with the difficulties and expenses attending the transfer of securities, which tends to create a difference in the rate of interest in the two countries.

"As a further proof of what I state, take the rate of interest in England for the last five years. While money has been plentiful at 3½ per cent., and even at a lower rate, in Scotland, 4 per cent. has been the rule, and 3½ per cent. the exception, in England; and although securities were bought occasionally, if I may so express it, on reduced terms for a limited period, there was no fall on old transactions to any extent."

Mr. Thomson quotes the practice of the Equitable Assurance Society, under a resolution of the court of the 9th December, 1830, and observes—

"The principle embodied in this resolution is consistent and fair, if 3 per cent. Consols and the value of money for investment on mortgage of land are influenced by the same causes and in the same degree; but although I am prepared to admit that they are influenced in the main by the same causes, I cannot allow that these causes influence them in the same degree. By reference to the diagram which I have prepared,* showing the fluctuations of consols, interest on mortgages, and bank interest, since 1789, it will be observed that in Scotland the movements of interest on landed securities follow the movements in consols and the bank movements, steadily, at a certain distance; and in any well regulated system, which I

* We regret that we have not room for this very illustrative statement. It is, however, we believe, in the hands of many of our readers.—Ed. *A. M.*

consider the Scotch one to have been, the changes must always follow that rule: but violent fluctuations in 3 per cent. Consols may be brought about by temporary causes, and banking finance may be affected by the same causes; and it is not judicious, I think, to adopt any regulation by which an important fluctuation at a particular time of the year should depress or raise the value of money in the way adopted by the Equitable. I am informed that the working of the Equitable system, however, has not been quite successful, and that new loans are not regulated by the scale of 1830.

"In making inquiry as to English practice, a friend largely interested in money dealings writes to me thus from London:—'The rate of interest on mortgages here is a matter of entire uncertainty. The number of capitalists of one sort and another is so great, that any coherence amongst them, or unity of action, seems quite out of the question; and accordingly, a person seeking for a loan on such a security would find lenders at any time, at very different rates of interest.' I see the difficulty, I confess, of making an arrangement in such a world as London, similar to that in Scotland, combining all interests; but I do not see that it would be impossible to bring together the larger moneyed bodies for the purpose of assimilation of practice and unity of action. Some would probably hold aloof: but if the practice of appealing to the decision of certain large bodies were once introduced, an important step would be gained towards the object in view; and if once a standard were recognized, others would be forced to follow.

"In IRELAND the practice is similar to England; but as capital is scarcer, a higher rate of interest obtains, and the per centage fixed at the outset of the transaction—which is, I may say, invariably higher than the average in England or Scotland—does not vary with the money market, but regulates the transaction during its whole course. Such a state of matters can only arise from a small supply of capital or want of proper communication among lenders and borrowers, or from both, and must operate prejudicially.

"In fixing the rate of interest periodically, the parties who are entrusted with the consideration of the subject should have in view the following:—

"1. The proceedings of the banks with reference to the rate of discount and of deposit interest.

"2. The fluctuations of the public funds.

"3. The prospects of the country generally, politically and commercially.

"It will no doubt be remarked, that I have not inserted as a point of consideration the amount of money waiting for investment; but I have advisedly left out that point, for it is not one which should influence the question *primarily*. If it can be ascertained distinctly what is the amount so waiting, it is right that it should be kept in view; but the impression as to the amount must not be taken from the advertising columns of a newspaper. There is another point, however, which would require to be ascertained before we could admit as a real element of consideration the money waiting investment, viz., the amount already lent; for it must be remembered that the benefit to be derived from the decisions of the periodical meetings is far more important with reference to existing securities than to new transactions. Because a few thousands, or a few ten thousands, wait investment, it is no fitting argument for keeping down the interest on millions; and if

the money market contains few new lenders, it is no reason why interest should not be lowered.

"A lender in the market may say—I have various sums waiting investment; why should not a borrower deal with me at a lower rate of interest, for a certain limited period, than that which has been fixed by the periodical meeting, if we can come to an agreement? and I can only answer, You are quite entitled to do so; but that is what I have mentioned before as the PURCHASE OF A SECURITY, and must not be referred to as a criterion of the value of money on such securities generally.

"It follows from what I have now urged, and will, I hope, be admitted, that it is very desirable to ascertain how much money is lent on landed securities in Scotland; and as the information could be obtained from the registers at an expense trifling as compared with the object in view, I would humbly suggest that steps should immediately be taken to procure the necessary information.

* * * * * * *

"I would only further ask attention to the question of usury as regards landed securities, being strongly of opinion that the restriction is useless, detrimental to the smaller class of borrowers at all times, and detrimental to all in times of scarcity of money.* If interest rises to 4½ per cent., of which there is every probability at present, or it may be to 5 per cent. on first-class landed securities, what are borrowers on postponed securities, house property, leasehold property, &c., to do? They will not be able to raise money at all in the ordinary way. And if money becomes of greater value, even first-class transactions will be disturbed, and money will be called up, but may not be obtained, as new loans will be transacted with difficulty. During such temporary pressure in the money market, the whole machinery comes to a dead lock, much distress ensues, and much discredit is created, by the attempt to control that which should be left to natural laws. Under such a state of matters the difficulty of raising money leads to annuity transactions and other skilfully devised ways of evading the Act of Parliament, under which the borrower is sure to pay an increased rate for the accommodation. So it was in the last war—so it was in the railway times—and so it will be again. The suspension of the usury laws as regards personal securities has worked well since 1837; and I am confident it would be no less successful if landed securities were untrammelled, and the ordinary laws of supply and demand left to regulate the rate.

"In conclusion, let me urge the importance of this subject generally, not only with reference to the interests of lenders and borrowers, but more particularly with reference to the general prosperity of the country. When interest is raised, it has the effect of checking the prosperity of the country, as high rates of discount in mercantile matters and high rates of interest on landed securities cause diminished adventure and expenditure; a decrease in that portion of the revenue which arises from all commodities which pass under the departments of customs or excise ensues, and consequently the less rapid circulation of money tends to a diminished basis of taxation. It is sufficient for my purpose at present to suggest this as a matter of serious import, without going more fully into the consideration of those influences which bear upon the wealth of nations; but it will convey to the mind more

* "While these sheets are going through the press, the Chancellor of the Exchequer has introduced a bill to repeal the Usury Laws entirely.—30*th June*, 1854."

distinctly what I wish to represent, if the following results of the revenue of the country for thirty years are studied with reference to the rise and fall of interest :—

Year.	Surplus of Income.	Deficiency of Income.	Year.	Surplus of Income.	Deficiency of Income.
	£	£		£	£
1822	4,744,518		1838	....	345,226
1823	4,300,747		1839	....	1,512,792
1824	3,888,172		1840	....	1,593,971
1825	3,049,156		1841	....	2,101,869
1826	....	645,920	1842	....	3,979,539
1827	....	826,675	1843	1,443,304	
1828	3,246,994		1844	3,356,105	
1829	1,711,550		1845	3,817,642	
1830	2,913,673		1846	2,846,308	
1831	....	698,858	1847	....	2,956,683
1832	614,759		1848	...	796,419
1833	1,513,083		1849	2,098,126	
1834	1,608,155		1850	2,578,806	
1835	1,620,941		1851	2,726,396	
1836	2,130,092		1852	2,417,559	
1837	....	655,760			

"The years 1825, 1837, and 1847, are well marked periods in the history of speculation, and the events which occurred at these periods are too well known to require more than a simple reference; but it is to the years which succeeded that I would more particularly direct attention, as showing invariably a deficient revenue when interest rose. I do not mean it to be inferred as my opinion that the changes in the revenue have been brought about by changes in the rate of interest alone; but keeping also prominently in view the financial operations which have affected the revenue, I am humbly of opinion that the value of money has been and is a regulating principle, bringing about results of surplus or deficiency, primarily to a great extent, and secondarily to a still greater. How deeply then is the country and the Government of the country interested in the question generally! Of course interest will rise and fall in all departments of finance, irrespective of the wisest management of public affairs on the part of those entrusted with it; but it is well that those whose opinions are sought to regulate these matters, should know the vast responsibility which rests upon them in influencing the decisions by which the money market is governed, whether in a limited or extended sphere."

There is much suggestive matter in these observations; and at a future day we shall hope to see a recurrence to them. Our readers, we are sure, will not fail to appreciate any remarks or investigations calculated to throw light on the important subjects here referred to.—Ed. *A. M.*

On the Amount of Reserve made by Life Assurance Companies.

THE peculiar nature of the business of life assurance must ever make it a matter of some difficulty for the public to comprehend the statements of account put forward by Companies engaged in it. The difficulty is further enhanced by the variety of modes in which such statements are usually made up. Thus, some simply contain a "debit," which is stated to be the net amount of the liability under the Company's contracts. Others have a debit, on the one hand, of the value of the Company's obligations under such contracts; and a "credit," on the other, of the value of the obligations by which the parties dealing with them are bound. A great variety of modes, too, are adopted in arriving at the value of these debits and credits, each of which must be understood, and the one selected known, before an accurate estimate can be formed of the financial position of any given concern. Hence, as we have hinted, the question of the greater or less ability of any Company to meet its engagements is one which those only who are fully conversant with the theories of the subject can definitively solve. Nevertheless, there are some broad indications which, in the absence of more refined ones, may be usefully resorted to, as in a certain degree measuring the capabilities of these Associations; and which, considering the vast importance of the question, it may be well to point out. Thus we may infer that the liability of Companies under their contracts will generally be in direct proportion to the amount assured and the duration of the assurances; and hence the "reserves" made by them should exhibit some such proportions. It is true that the average duration of the assurances in one Company may be different from that of those in another, even though the age of each Company may be the same. But this is not, we believe, commonly the case. With a few exceptions, the incidents and fluctuations in the business of life assurance are common to all, and any particular average in one case is not likely to differ materially from the corresponding one in another. It is however to be borne in mind, that the rates of premium charged by the several Companies differ a good deal, and that some allowance is to be made on this score; since it is evident, that of two Associations similarly situated in other respects, and making the like amount of reserve, the one charging the higher rates will have the advantage.

Premising these few observations, we annex a short table, exhibiting in certain instances the amount reserved per cent. after a

given number of years. At a future day we hope to be able to give the like particulars as regards all the existing Offices, with such explanatory notes as we may be favoured with. At present the statements issued by many do not supply the necessary information; and of those which do, many are not, while we are writing, accessible. It will be observed that great irregularity exists in the proportions, but that extreme discrepancies are apparent notwithstanding; were the number of quotations increased, we believe the extent of these discrepancies would become better defined, and that the statement would be found useful, as calling timely attention not only to insufficient reserves, but to excessive ones also—the Scylla and Charybdis between which it behoves all engaged in the conduct of these undertakings so carefully to steer.—Ed. *A. M.*

Office.	Years in operation.	Amount assured.	Estimated Liability under Assurances.	Amount Reserved per cent.
		£.	£.	
Alfred	12	601,190	83,223	13·8
Victoria	14	815,171	100,844	12·3
Scottish Provident..	15	2,071,125	206,079	9·9
Metropolitan	19	2,753,657	38,671	1·4
Mutual	20	1,017,372	96,918	9·5
Scottish Equitable ..	22	3,892,031	602,857	15·5
National.....	23	1,000,000	138,000	13·8
Crown	28	1,940,530	503,185	25·9
Palladium	28	1,280,098	289,933	22·6
Economic	30	5,336,000	1,034,381	19·3
Albion	45	1,443,505	341,176	23·6
Eagle	45	2,723,513	529,616	19·4
London Life	47	6,006,061	715,857	11·9

NOTES AND QUERIES.

Demonstration of Formulæ.—The following demonstration of the expression $\frac{1-\rho A}{1+\rho}$ is perhaps worth recording:—

Since ρ is the annual interest of £1, this last is the present value of a perpetuity of ρ pounds, and $1-\rho A$ is therefore the value of a perpetual reversionary annuity of ρ pounds, to be entered upon at the death of A. But such reversionary annuity is then clearly equal to $1+\rho$. Hence, as $1+\rho : 1-\rho A :: 1 : \frac{1-\rho A}{1+\rho}$, the present value of £1, to be received at the end of the year in which A dies.—Ed. *A. M.*

THE following methods of finding the amount and present value of annuities increasing or decreasing by a constant quantity, are by Mr. EDWIN H. GALSWORTHY, of the Medical and Invalid Assurance Company:—

I. *To find the Amount of an Annuity increasing or decreasing by a Constant Quantity.*—Let the annuity payable at the end of the first year $=a$, and suppose this annuity to increase yearly by any constant quantity, c; then what will the annuity amount to at the end of n years, at i per £ per annum compound interest?

Now the first payment will be out at interest for $(n-1)$ years,
" second " " " $(n-2)$ "
" third " " " $(n-3)$ "
&c. &c. &c.
nth or last " " 0

Hence, then,

a which is payable at the end of the 1st year, will amount to $a(1+i)^{n-1}$
$a+c$ " " " 2nd " $(a+c).(1+i)^{n-2}$
$a+2c$ " " " 3rd " $(a+2c).(1+i)^{n-3}$
&c. &c. &c.
$a+(n-3)c$ " " $(n-2)$nd " $\{a+(n-3)c\}.(1+i)^2$
$a+(n-2)c$ " " " $(n-1)$th " $\{a+(n-2)c\}.(1+i)$
$a+(n-1)c$ " " " nth " $a+(n-1)c$

And the total amount of the increasing annuity S, will be the sum of the last column, or $S=a(1+i)^{n-1}+a(1+i)^{n-2}+\ldots\ldots+a \quad . \quad (\alpha)$
$+c(1+i)^{n-2}+2c(1+i)^{n-3}+\ldots\ldots+c(n-1)$

Now line (α) represents the amount of an ordinary annuity of £a say,$=$

$$A=a.\frac{x^n-1}{x-1}, \text{ where } x \text{ is put for } (1+i) \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

Then we have

$$S=A+cx^{n-2}\left\{1+\frac{2}{x}+\frac{3}{x^2}+\frac{4}{x^3}+\ldots.\frac{n-2}{x^{n-3}}+\frac{n-1}{x^{n-2}}\right\}.$$

Let $\frac{1}{x}=z$; then the series written within the brackets is $1+2z+3z^2+\ldots\ldots(n-1)z^{n-2}$, which is a series to $(n-1)$ terms, the sum being*

$$=\frac{1-nz^{n-1}+(n-1)z^n}{(1-z)^2}.$$

But we put z for $\frac{1}{x}$; therefore,

* This series, or a similar one, is summed in almost every Algebra (*see* Wood's, 12 Edit., Art. ccxcv., &c.), and in the following manner:—

Let $1+2z+3z^2+\ldots\ldots(n-1)z^{n-2}=S$
$\therefore z+2z^2+\ldots\ldots(n-2)z^{n-2}+(n-1)z^{n-1}=z.S$
$\therefore S-z.S=1+z+z^2+\ldots\ldots z^{n-2}-(n-1)z^{n-}$
or, $S(1-z)=\frac{1-z^{n-1}}{1-z}-(n-1)z^{n-1}$
$\therefore S=\frac{1-z^{n-1}}{(1-z)^2}-\frac{(n-1)z^{n-1}}{(1-z)}=\frac{1-nz^{n-1}+(n-1)z^n}{(1-z)^2}.$

$$S=A+cx^{n-2}\left\{\frac{1-\frac{n}{x^{n-1}}+\frac{n-1}{x^n}}{\left(1-\frac{1}{x}\right)^2}\right\}=A+c.\left\{\frac{x^{n-2}-\frac{n}{x}+\frac{n-1}{x^2}}{\left(1-\frac{1}{x}\right)^2}\right\}$$

$$=A+c\left\{\frac{x^n-nx+n-1}{(x-1)^2}\right\}=A+\frac{c}{x-1}\left\{\frac{x^n-1}{x-1}\right\}-\frac{nc}{(x-1)^2}.(x-1)$$

and by $(\beta)=A+\frac{c}{x-1}.\frac{A}{a}-\frac{nc}{x-1}=A\left\{1+\frac{c}{(x-1)a}\right\}-\frac{nc}{x-1}$.

But x was put for $(1+i)$;

$$\therefore S=A+\frac{Ac}{ia}-\frac{nc}{i}, \text{ and finally}=A+\frac{c}{i}\left\{\frac{A}{a}-n\right\}.$$

When the constant $c=1$ and a also $=1$, we have $S=A+\frac{A-n}{i}$. If the annuity be a decreasing one, write $-c$ for c, and we get the amount $S=A-\frac{c}{i}\left\{\frac{A}{a}-n\right\}$; and when, as before, $c=1$ and $a=1$, then $S=A-\frac{A-n}{i}$.

Example.—Find the amount of an annuity of £1 increasing £1 annually, in 5 years, at 5 per cent.

Here $c=1$ and $a=1$, hence S, or the amount $=A+\frac{A-5}{\cdot05}$.

(Jones, vol. i.) A= 5·525631

$$\cdot05\overline{)5\cdot525631}$$

$$10\cdot5126=\frac{A-5}{\cdot05}$$

$$5\cdot5256=A$$

$$16\cdot0382=\text{Amount required.}$$

Proof.—In order to prove the formula, the above result should evidently equal that produced by adding together the amounts of the several payments when accumulated, for the terms during which each would respectively be invested; thus:—

£1	would be invested	4	years,	and would amount to	1·2155
£2	„	„	3	„	„	1·1576 × 2 = 2·3152
£3	„	„	2	„	„	1·1025 × 3 = 3·3075
£4	„	„	1	„	„	1·0500 × 4 = 4·2000
£5	„	„	0	„	„	5·0000
					Sum =	16·0382 as before.

II. *To find the Present Value of an Annuity increasing or decreasing by a Constant Quantity.*—Let the annuity payable at the end of the first year $=a$, and suppose the yearly increase to equal any constant quantity c;

then what is the present value of the annuity payable for n years at i per £ per annum compound interest?

Now at the end of the 1st year a is payable,

" " 2nd " $a+c$ "
" " 3rd " $a+2c$ "
&c. &c. &c.
" " nth " $a+(n-1)c$;

and $1+i$ being the amount of £1 in one year, we obtain the total present value of the increasing annuity, which call P, thus:—

$$P=\frac{a}{(1+i)}+\frac{a+c}{(1+i)^2}+\frac{a+2c}{(1+i)^3}+\ldots\ldots\frac{a+(n-1)c}{(1+i)^n}$$

$$\therefore P=\left\{\frac{a}{(1+i)}+\frac{a}{(1+i)^2}+\frac{a}{(1+i)^3}+\ldots\ldots\frac{a}{(1+i)}\right. \quad . \quad . \quad (\alpha)$$

$$\left.+\frac{c}{(1+i)^2}+\frac{2c}{(1+i)^3}+\ldots\ldots\frac{(n-1)c}{(1+i)^n}\right\} \quad . \quad . \quad (\beta)$$

The series (α) represents the present value of an ordinary annuity of $£a=a.\frac{1-(1+i)^{-n}}{i}=a.p$, say; then in the series (β) let $\frac{1}{1+i}=x$, and it becomes $=cx^2\{1+2x+3x^2+\ldots.(n-1)x^{n-2}\}$; and summing the series within the brackets (as in the preceding investigation of the amounts), we get

$$\text{Series }(\beta)=cx^2.\frac{1-nx^{n-1}+(n-1)x^n}{(1-x)^2}=c.\frac{x^2-nx^{n+1}+(n-1)x^{n+2}}{(1-x)^2} \quad ..(\gamma)$$

Now p (from above)$=\frac{1-(1+i)^{-n}}{i}=\frac{1-x^n}{i}$; $\therefore x^n=1-ip$, $\therefore x^{n+1}$ $=x-ipx$, and $x^{n+2}=x^2-ipx^2$; and hence, by substituting these values, we have

$$(\gamma)=c.\frac{x^2-nx+nipx+(n-1)x^2-(n-1)ipx^2}{(1-x)^2}$$

$$=c.\frac{(n+ip-nip)x^2-(n-nip)x}{(1-x)^2}.$$

Here restore the value of x, and the above value becomes

$$c.\frac{(n+ip-nip)(1+i)^{-2}-(n-nip)(1+i)^{-1}}{1-2(1+i)^{-1}+(1+i)^{-2}}.$$

Multiplying numerator and denominator by $(1+i)^2$,

$$=c.\frac{(n+ip-nip)-(n-nip)(1+i)}{(1+i)^2-2(1+i)+1}=c.\frac{ip-ni+ni^2p}{i^2}=c.\frac{p(ni+1)-n}{i};$$

and hence $(\alpha)+(\beta)$, or the total value of $P=ap+c.\frac{p(ni+1)-n}{i}$;

where $a=1$ and $c=1$, $P=p+\frac{p(ni+1)-n}{i}$.

If the annuity be a decreasing one, then the expression for the present value will evidently be represented by the above, after changing the connecting sign + to the sign —.

Example.—Find the present value of an annuity of £1 for 5 years, increasing £1 annually, at 5 per cent.

Here $a=1$, $c=1$, $n=5$, $i=\cdot05$, $P=p+\frac{p(ni+1)-n}{i}$.

(Jones, vol. i.) $p=$ 4·329477
1·082369

5·411846 $=p(ni+1)=p(1\cdot25)$

$n=$ 5·

$i=\cdot05\,)\cdot411846$

8·23692

$p=$ 4·32947

12·56639 = Total present value of the increasing annuity.

To check this result, we find the present value of the whole annuity by summing the present values of each distinct payment; and we have—

Years hence.		Present Value of £1.		Amount of Payment.		Present Value.
1	=	·952381	×	1	=	·95238
2	=	·907029	×	2	=	1·81406
3	=	·863838	×	3	=	2·59151
4	=	·822702	×	4	=	3·29081
5	=	·783526	×	5	=	3·91763
Total present value, as before =						12·56639

which proves the correctness of the formula obtained.

FOREIGN INTELLIGENCE.

DENMARK.—*Statistics of the Royal Octroied Fire Insurance Company, at Copenhagen (Kyl Octroierede Assurance Company for Varer og Effecter).*—This Company, established on the 11th May, 1778, and remodelled on the 24th May, 1843, has a privilege for insuring goods, furniture, and all moveable property, at Copenhagen, so that neither the formation of another Danish Society nor the agency of a foreign Insurance Company is permitted; nevertheless, some English and German Societies have agents at Copenhagen, and do a large amount of business. A fine of 1,000 rix dollars* (£111 sterling) which must be paid by the insured to the Royal Fire Insurance Company, in case he is found to have insured with another Office, has only the effect that this fine is very often insured with the other property; and no inquiry being allowed about the insurance, except in case of fire, the fine can only then be levied.

The original capital amounted to £66,666 sterling, in 500 shares of £56, wholly paid up; but in 1843 it was provided that the reserved fund accumulated at that period, of £4,444, should be added to the original

* 9 Rix dollars = £1.

capital, thus raising it to £111,111, and the value of each share to £92. 10*s*. Since that time, a new reserved fund of £22,920 has been accumulated.

The interest paid to the shareholders is 4 per cent., which originally was paid only on £66,666; but since 1843, on £111,111.

A review of the Society's business is annexed.

Year.	BUSINESS EXPRESSED IN POUNDS STERLING.					PROPORTIONAL REVIEW.				
	Sums insured.	Amount of Premiums.	Claims Paid.	Dividend, including Interest.	Dividend, including Interest, each Share.	Average Premium per Cent. on Sums insured.	Claims per Cent. on Sums insured.	Claims per Cent. on Premiums.	Dividend, including Interest per Cent. on Sums insured.	Dividend, including Interest per Cent. on Premiums.
	£	£	£	£	£					
1815	1,694,444	9,006	301	5,333	4 4/5	0·53	00·018	3·3	0·31	59
1816	2,000,000	13,096	1,348	9,333	7 1/5	0·66	0·067	10·2	0·47	71
1817	1,925,925	14,459	281	10,666	8 4/5	0·74	0·014	1·9	0·55	74
1818	1,814,814	12,933	1,696	9,333	7 1/5	0·71	0·093	13·1	0·52	72
1819	1,222,222	7,995	4,773	9,333	7 1/5	0·65	0·391	59·7	0·76	116
1820	1,111,111	7,149	248	9,333	7 1/5	0·64	0·022	3·4	0·84	131
1821	944,444	5,974	32	8,933	7 1/5	0·63	0·003	0·5	0·95	150
1822	888,888	5,448	0	8,266	6 4/5	0·61	0·000	0·0	0·93	152
1823	833,333	5,269	0	8,266	6 4/5	0·63	0·000	0·0	0·99	157
1824	750,000	5,208	12	9,333	7 1/5	0·69	0·001	0·2	1·24	179
1825	861,111	5,029	172	8,933	7 1/5	0·58	0·020	3·5	1·04	179
1826	1,000,000	5,798	567	9,066	7 1/5	0·58	0·057	9·7	0·91	156
1827	1,000,000	5,734	527	8,933	7 1/5	0·57	0·053	9·2	0·89	156
1828	1,055,555	5,944	658	8,800	7 1/5	0·56	0·062	1·1	0·83	148
1829	1,055,555	6,005	165	8,800	7 1/5	0·57	0·016	2·8	0·83	147
1830	1,111,111	5,718	310	8,800	7 1/5	0·51	0·028	5·5	0·79	154
1831	1,148,148	5,374	918	8,133	6 1/5	0·47	0·083	17·1	0·70	151
1832	1,222,222	5,506	23	8,800	7 1/5	0·45	0·002	0·4	0·72	160
1833	1,305,555	5,860	361	9,066	7 2/5	0·45	0·028	6·2	0·69	155
1834	1,277,777	5,681	123	9,066	7 2/5	0·44	0·009	2·1	0·71	160
1835	1,333,333	5,876	378	9,066	7 2/5	0·44	0·028	6·4	0·68	155
1836	1,355,555	6,128	956	9,066	7 2/5	0·45	0·071	15·6	0·67	148
1837	1,622,222	7,316	3,456	9,066	7 2/5	0·45	0·213	47·2	0·56	124
1838	1,555,555	7,148	271	9,066	7 2/5	0·46	0·017	3·8	0·58	127
1839	1,740,740	7,361	4,391	9,066	7 2/5	0·42	0·252	59·7	0·52	123
1840	1,638,888	6,975	1,144	9,066	7 2/5	0·43	0·070	16·4	0·55	130
1841	1,666,666	6,645	306	9,066	7 2/5	0·40	0·018	4·6	0·54	136
1842	2,148,148	6,774	126	10,400	8 1/5	0·32	0·016	1·2	0·48	154
1843	2,250,000	7,191	1,414	6,000	5–	0·32	0·063	19·7	0·26	83
1844	2,444,444	8,231	16,874	8,000	6 1/5	0·34	0·690	205·0	0·33	97
1845	2,666,666	8,709	1,707	8,533	7 1/5	0·33	0·064	19·6	0·32	98
1846	2,833,333	9,261	4,738	8,000	6 2/5	0·32	0·167	51·2	0·28	86
1847	3,000,000	9,899	2,981	8,533	7 1/5	0·33	0·099	30·1	0·28	86
1848	2,972,222	9,863	1,628	8,000	6 2/5	0·33	0·055	16·5	0·27	81
1849	3,027,777	9,926	7,714	5,333	4 4/5	0·33	0·255	77·7	0·17	44
1850	3,259,259	10,535	7,188	5,866	4 2/5	0·32	0·221	68·2	0·18	56
1851	3,333,333	11,101	6,991	8,000	6 1/5	0·33	0·210	63·0	0·24	72
1852	3,555,555	11,775	3,161	9,066	7 2/5	0·33	0·089	26·7	0·25	77
Total.	66,625,911	293,900	77,939	325,719	271 1/5	0·44	0·117	26·6	0·49	111

The expenses of management have been £1,111 to £1,333 per year; thus varying from 10 to 25 per cent. of the amount of premiums—a rather high proportion.

Examining this table, it is obvious that the average premium, which

rose in 1817 to 0·74 per cent., has fallen since then every year. Since the year 1825, the competition has regularly increased, and there seems to be a remarkable connection between average premium and sum insured. As long as the premiums of this Company have remained higher than those of foreign Companies acting in Copenhagen, the sum insured remains small, and diminishes from year to year. Even a reduction of premiums, which in their reduced state still exceed those of other Companies, does not cause a change. But when in 1825 the average premium is reduced from 0·69 to 0·58 per cent., the sum insured begins to rise, and regularly rises while the average premium falls; and in 1842, when a *considerable* reduction of the average premium took place, the sum insured increased more than in any other year.

The claims paid differ from 0·0 to 0·69 per cent. of sums insured, and only once exceed the amount of premiums in 1844, the year of their maximum.

The dividends paid are very considerable, and exceed the amount of premiums by nearly £33,333. Thus the interest taken would have been sufficient not only for all claims and expenditure of management, but also for the accumulation of a reserved fund of £100,000, which gives an average income of £5,555 per year.

The interest to the shareholders is fixed at 4 per cent., giving till 1843 £2⅔ per share, and since then, £4⅔ per share. Since 1815, this amounts to £91⅓ per share. The shareholders have received £271⅓, which produces an actual profit of £180; or, if we consider the augmentation of interest in 1843 as a profit, and the increased value of shares now really amounting to £111, each share of £66⅔ has gained since 1815, besides the interest of 4 per cent., £242⅓, or 363 per cent.—making £6⅓, or 9½ per cent., each year.

GERMANY.—*Gotha Mutual Life Assurance Bank.*—Extract from the report for 1853.

I.—*Assurances.*

Proposals:—	Persons.	Amount.
Proposals made in the course of 1853	1,804	£368,128*
Standing over from 1852	44	11,186
	1,848	£379,314
The accepted of these proposals were—		
Of persons who were not yet assured	1,324	£268,471
Of persons who were already assured	233	39,229
	1,557	£307,700
Refused or reduced by the Society	235	55,871
Withdrawn, not completed, or reduced by the parties	18	7,786
Standing over for consideration at the end of the year	38	7,957
	1,848	£379,314
Amount of assurances:—		
At the beginning of the year	17,715	£4,004,057
New assurances during the year	1,324	307,700
In force during the year	19,039	4,311,757

7 Thalers = £1.

	Persons.	Amount.
Brought forward	19,039	£4,311,757
Cancelled—		
By death.	399 £92,614	
By surrender, running off, &c.	213 59,829	
	612	152,443
In force at the end of the year	18,427	4,159,314

The clear increase was therefore 712 persons, with £155,257.

Classes of assurances :—

Whole life—	Persons.	Sums assured.	Yearly premium.
Payable at death, or on attaining the 90th year of age	18,023	£4,063,842	£142,225
Payable at death, or at any younger age than the 90th year . .	150	34,743	2,048
	18,173	4,098,585	144,273
Survivorships	51	11,000	339
For terms of years	203	49,729	989
	18,427	£4,159,314	£145,601

According to the sex of assured—	Persons.	Sums assured.
Males	17,256	£3,928,400
Females	1,171	230,914
	18,427	£4,159,314

According to the ages of the assured and the amount of assurances :—

Age.	Amount of Assurances.										Total.	
	£284—£142½.	£157—£285.	£300—£428.	£443—£570.	£586—£714.	£728—£857.	£871—£1,000.	£1,014—£1,143.	£1,157—£1,286.	£1,300—£1,429.		
	Number of Persons.										Persons.	Sums Assured.
												£.
19—20	7	..	..	..	..	..	..	..	..	..	7	629
21—25	70	11	1	1	1	1	..	..	..	..	85	11,471
26—30	442	107	16	12	13	7	..	..	..	2	599	101,686
31—35	1,118	300	102	55	49	19	2	1	2	22	1,670	348,014
36—40	1,546	438	149	82	68	30	4	10	2	28	2,357	507,457
41—45	1,878	594	217	87	86	39	8	19	1	33	2,962	652,171
46—50	1,944	684	207	136	76	57	6	20	1	33	3,164	709,228
51—55	1,690	659	220	138	84	49	16	32	2	29	2,919	698,571
56—60	1,225	480	169	92	54	34	6	16	5	23	2,104	491,729
61—65	763	281	99	85	54	37	6	12	4	16	1,357	348,072
66—70	421	161	60	46	21	20	8	12	..	2	751	178,843
71—75	179	73	19	11	10	9	2	7	1	..	311	75,900
76—84	92	27	10	10	2	..	..	..	..	..	141	26,543
Total . .	11,375 persons, £1,169,800.	3,815 persons, £938,357.	1,269 persons, £513,057.	765 persons, £420,500.	518 persons, £365,929.	302 persons, £257,571.	58 persons, £57,043.	129 persons, £146,357.	18 persons, £22,200.	188 persons, £268,500.	18,427	4,159,314

The average age of the assured was, at the beginning of the year, 48 years 2 months; and at the end of the year, 48 years 3 months. The average amount of the sum assured on each life was at the beginning of the year £226, and at the end £225. 14*s*. 3*d*.

Deaths:—

Age.	Assured in the course of 1853.	Should die, according to the Tables used.	Died.	Plus.	Minus.	Mortality per cent.
15—25	100	0·65	..	..	0·65	..
26—30	636	5·24	5	..	0·24	0·79
31—35	1,702	16·97	11	..	5·97	0·65
36—40	2,417	27·24	20	..	7·24	0·83
41—45	3,018	36·92	35	..	1·92	1·16
46—50	3,245	49·59	50	0·41	..	1·54
51—55	3,012	59·60	55	..	4·60	1·83
56—60	2,179	55·59	60	4·41	..	2·75
61—65	1,423	46·67	63	16·33	..	4·43
66—70	809	36·34	54	17·66	..	6·67
71—75	338	24·25	27	2·75	..	7·99
76—84	160	23·01	19	..	4·01	11·87
Total..	19,039	382·07	399	41·56	24·63	

Of the 399 persons, on whose life the total sum of £92,614 was assured, only 389 (for £90,229) were entitled to full payment. Six assured (for £1,671), who had died by suicide, left to their representatives no claim to the sum assured, but only to a return out of reserved fund. This return was, in four cases of suicide which were "*non compos mentis*," the full value of the policies; in the other two cases of intentional suicide, the usual purchase price. Two others (assured for £571), who had shortened their lives by intoxication, left only a similar claim. A third case of the same sort (£100) is still under examination. In another case (the first of the kind since the establishment of the Society), the claim (£43) was forfeited because the assured died by capital punishment, for having murdered his wife. The amount paid to his representatives was only the usual purchase price of the policy.

The proportion of deaths was 361 males and 38 females. In the course of 1853, 17,813 males and 1,226 females being assured, the rate of mortality of the former was 2·03 per cent., of the latter 3·10 per cent. The mean age of the males and of the females was about the same; the latter showed, as usual, a much higher mortality than the former.

An accurate table in the appendix of the report shows, in the case of every death, the age of the assured at the date of the assurance, the age at his death, and the cause of the death. There died 60 by apoplexy, 54 by consumption (phthisis), 46 by nervous fever, 31 by dropsy, 22 by marasmus senilis, 15 by cholera Asiatica, 6 by suicide, &c.

II.—*Cash account for the year* 1853.

Dr.	£.
Invested capital at the end of 1852	986,619
Amount of new premiums for 1853	10,375
„ renewal premiums for 1853	136,424
„ premiums paid in advance	6
„ commuted premiums and extra premiums	1,036
Interest on loans	39,062
Dividends unclaimed	220
Profit from agio on money, existing bonds, bills of exchange, &c.	164
House rent, deducting all charges	94
Extra receipts for deposits, &c.	530
Total	£1,174,530

Cr.	£.
Paid 1 claim for the year 1849	86
„ 1 „ „ 1851	186
„ 53 claims „ 1852	14,528
„ 325 „ „ 1853	75,228
„ 3 claims fallen due in lifetime	1,214
Dividends to the assured for the year 1847 (residue)	603
Dividends to the assured for the year 1848	28,610
For policies purchased	2,760
Agents' commission	3,268
Expenses of management	4,129
Extraordinary expenses	139
Invested capital at the end of 1853	1,043,779
Total	£1,174,530

III.—*Balance-sheet for the year* 1853.

Dr.		£.
68 claims unsettled		15,814
Unclaimed dividends for year 1848		499
Deposits received in cash		4,861
Reserve, or the value of all existing policies at the 31st Dec., 1853, without any regard to the premiums or parts of premiums paid since this date		776,905
Gross amount of premiums in advance for the time after the 31st December, 1853 (*Prämien übertrag*)		69,804
Surplus (guarantee fund)—		
Surplus of 1849-52	£136,786	
Surplus of 1853	39,110	
		175,896
Total		£1,043,779

Cr.	£.
Cash in hand	8,593
Mortgages	961,244
Advances on policies	35,594
Interest due	9,876
Value of the Society's house	4,787
Balances due by agents	13,268
Balances at bankers	10,417
Total	£1,043,779

In order to show that the above-mentioned reserve covers the risk, the following extract from the tables of reserve, computed on the list of mortality at 3 per cent., is published.

Age of entrance.	Reserve for a whole Life Policy of 100 after Years.													
	5.	10.	15.	20.	25.	30.	35.	40.	45.	50.	55.	60.	65.	70.
21	4,972	10,019	15,413	21,320	28,212	35,559	43,135	51,117	59,721	68,881	77,729	84,677	90,051	100,000
26	5,311	10,988	17,204	24,457	32,187	40,160	48,560	57,614	67,253	76,563	83,875	89,530	100,000	
31	5,995	12,560	20,220	28,384	36,804	45,675	55,237	65,417	75,248	82,971	88,942	100,000		
36	6,984	15,133	23,816	32,774	42,211	52,382	63,211	73,669	81,885	88,236	100,000			
41	8,762	18,096	27,727	37,873	48,807	60,449	71,692	80,525	87,353	100,000				
46	10,230	20,786	31,907	43,890	56,651	68,972	78,554	86,137	100,000					
51	11,760	24,149	37,497	51,711	65,435	76,222	84,557	100,000						
56	14,041	29,167	45,276	60,828	73,053	82,498	100,000							
61	17,596	36,336	54,427	68,650	79,637	100,000								

It is by this table that the reserve made in the annual balance-sheets, as well as the purchase price of surrendered policies, and the advances on policies, have been computed ever since the establishment of the Society, and will continue to be computed, until the experience of the Society requires an alteration of the table of mortality in use.

The guarantee fund comprises, according to the former balance-sheets—

	£.
The residue of the surplus of 1849	16,557
The surplus of 1850	30,389
„ 1851	46,819
„ 1852	43,021
„ 1853	39,110
Total . .	£175,896

In the course of 1854 the surplus of 1849-50 is divided among the assured who have paid premiums in 1849. It consists of—

	£.
The residue of surplus of 1849	16,557
Part of the surplus of 1850, corresponding to the premiums paid in 1849, which are due in 1850 (£59,167)	14,505
Total . .	£31,062

The premiums for whole life assurances and survivorships paid in 1849 being £124,250, the surplus of £31,062 gives a dividend of 25 per cent. This dividend is made in the course of 1854—on all existing policies, by way of reduction of premiums; and on all policies cancelled by death or surrender, by payment in cash on bonds, which are issued at the extinction of the policies.

The investments on loans and mortgages are—

£.	
882,789	first mortgages on land, of at least double the value.
26,996	„ „ house property, of at least double the value, and insured against fire.
20,665	in bonds of public Loan Societies, based on land securities.
14,815	in bonds of public corporations, whose management is under the control of the Government.
15,979	on mortgage of stocks.
£961,244	. Total.

Hamburg.—Janus Life and Annuity Insurance Company. *Balance sheet for the year* 1853.

I.—*Life Insurance.*

Proposals made in 1853, 814		£118,258*	
„ accepted, 661		87,831	
Cancelled, died, surrendered, &c., in 1853		31,589	
Increase in 1853		56,242	
Policies in force 31st December, 1853—	Persons.		
On single lives	3,238	£418,679	
„ joint lives	132	13,076	
	3,370	£431,755	
Premiums		£14,920	
Interest		842	
Brought forward from 1852		15,181	
			£30,943
39 claims, exclusive of £600 reassured		5,049	
Dividend for the year 1850		523	
For reassurance and policies purchased		589	
Commission		814	
Expenses of management		1,830	
Stamps, and loss by exchange		109	
			8,914
Balance carried forward			£22,029

II.—*Annuities.*

	Persons.			
Accepted in 1853	12		£103	
Died	2		28	
	10		75	
Which have been deferred	1		6	
	11		£81	
Deferred annuities	10		£148	
Cancelled, died, &c.	7		102	
	3		£46	
In force, 31st December, 1853—				
Annuities	62		£648	
Deferred annuities	51		729	
Persons	113		£1,377	
Receipt of purchase-money			£1,086	
„ premiums			255	
„ interest			193	
Balance of 1852			5,633	
				£7,167
Annuities paid			595	
Dividend of 1850			27	
Agents' commission			51	
Expenses of management			155	
Loss by exchange			9	
Policies purchased			35	
				872
Balance to be brought over				£6,295

The dividend granted to the insured for 1851 is 10 per cent. of the net premiums, and to the shareholders £1. 17*s.* 6*d.* per share.

* 13⅓ Banco marcs = £1.

Balance-sheet of the Hamburg Fire Assurance Company of 1843 *for the Year* 1853.

		£.	
Premiums—			
For the assurances in force, 31 Dec., 1852, viz.	£1,729,091	7,567*	
New assurances	2,247,351	9,581	
Deducted for brokerage, commission, &c. £510	3,976,442	17,148	
Expired before 31 Dec., 1853	2,285,566		
In force 1 January, 1854	1,690,876		
Premiums for this amount carried forward		7,793	
Net premiums		£9,355	
Interest	£4,297		
To shareholders	3,000		
Surplus of interest		1,297	
			£10,652
For claims paid		1,348	
Expenses of management		798	
Expenses of printing, &c.		102	
Commission of dividend, gratuities, &c.		304	
			2,552
Net profit			£8,100
To be divided, 50 per cent.			£4,050
To be reserved		4,050	
Reserved fund, 31 Dec., 1852		33,600	
		37,650	
Capital 2,000 shares, £37. 10s., wholly paid up		75,000	
		£112,650	

Premiums reserved for current risks, £7,793

Assicurazioni Generali in Trieste.—Extract from the Report and Balance-sheet for the Year 1853.—The claims paid in the year 1853 amount to £131,300* sterling, published by the papers. The business of that year left nevertheless a net profit of £21,723, after augmenting the fund of reserved premiums by £23,024. From the net profit of £21,723, £6,369 have been added to the fund of reserved profits, and £15,354 are to be divided among the shareholders.

The total amount of the reserved fund now amounts to £459,433, viz., £403,589 for reserved premiums, £55,844 for reserved profits. The reserved premiums include £134,822 for the liability to pay £15,711 annuities; and £30,882 for the liability to pay for deferred life assurances, viz., sums, £75,937—annuities, £581. These deferred assurances yield a yearly income in premiums of £1,604.

The profit realized in this year is consequent on the great number of risks assured, of their distribution in different branches, and of the amount secured by frequent reassurances, but not on the increase of the premiums. The divisible profit of £9,369 of the Balance-sheet (A) scarcely exceeds 4 per cent. of the premiums received in this branch. A reduction of the premium of $\frac{1}{8}$ per mille only at an average per year would even have caused a loss; for the assurances effected in 1853, although only stated to

* 13⅓ Banco marcs = £1. † 12 Florins = £1.

be £36,000,000, would exceed the sum of £66,000,000, if the sums assured for more than one year were comprised in each year separately, as is done by other Societies.

In the course of 22 years from the commencement of the Society the claims have been £1,276,900; and the amount paid in 1853 exceeds that of six of the earlier years.

With respect to life assurance, those circumstances fatal to the development of this branch in the Austrian empire enumerated in our report of last year still exist; but it is to be hoped that the settlement of financial matters, the cessation of the dearth, and the consequent improvement in the moral and social condition of the population, will induce the public to avail themselves of life assurance on a larger scale than before, as it is in England, France, and the northern parts of Germany.

In the Tontine branch the sums deposited in the first three months of 1854 exceed those of the whole year 1853; the status was, on the 1st July—Tontine ending 31 December, 1862—1,183 subscriptions, in 2,606 shares, for the sum of £28,403, of which £7,547 is paid up. Tontine ending 31 December, 1870—524 subscriptions, in 1,612 shares, for the sum of £13,869, of which £2,025 is paid up.

The directors express a desire for the increase of assurances of this description, and their wish to popularize them in Austria.

(*A*) *Balance-sheet for* 1853 (*exclusive of Life Assurance*).

	£.	£.
Brought from 1852—		
For premiums of fire insurance	146,295	
„ claims unsettled	7,500	
„ reassurance of current risks	12,375	
Premiums in 1853 for sums assured (£36,083,900)	211,123	
Interest for this branch (£17,030)	4,782	
		382,075
For 4,576 claims paid	104,656	
Brokerage, agents' commission, Ristorno premium of reassurance	68,997	
Expenses of management	7,000	
Interest to shareholders	2,000	
To be carried forward—		
For premiums of fire insurance	161,433	
Claims unsettled	8,333	
Reassurance of current risks	13,333	
		365,752
Profit		£16,323
To the directors and auditors, 18 per cent.		2,939
Net profit		£13,384
To be reserved, 30 per cent.		4,015
To be divided		£9,369

Reserved Fund of Profits.

	£.
Balance 31 December, 1852	33,058
Brought from 1853	4,015
Interest	645
Balance, 31 December, 1853	£37,718

Balance-sheet of Life Assurances in 1853—*Payable at Death.*

	£.	
Brought forward for liabilities	62,434	
Interest	2,497	
Assurance premiums of sums, £527,657; annuities, £10,400	29,049	
		93,980
Claims, commission, brokerage, expenses	£19,798	
Carried forward for liabilities	70,197	
		89,995
Profit		£3,985
To the directors and auditors, 18 per cent.		717
Net profit		£3,268
To be reserved		980
To be divided		£2,288

Fund of Reserved Profits.

Balance, 31 December, 1852	10,067
Brought forward, 1853	980
Balance, 31 December, 1853	£11,047

Balance-sheet of Annuities terminated in 1853.

	£.	£.
Class 1. For £1,280 in favour of persons who died 1852-3, payments were made, including interest	16,281	
„ 2. For £157 in favour of persons who died 1852-3, payments were made, including interest	1,761	
„ 3. For £354 in favour of persons living 31 December, 1853, payments were made, including interest	3,209	
		21,251
Annuities paid to the members of Class 1	11,145	
„ „ „ 2	1,908	
„ „ „ 3	3,563	
Reserved fund for annuities to be paid to the members of Class 3, according to the tables	2,206	
		18,824
Net profit		2,429
To be reserved		729
To be divided		£1,700

Fund of Reserved Profits.

	£.
Balance, 31 December, 1852	5,704
Brought forward, 1853	729
Balance, 31 December, 1853	£6,433

Jamaica.—*Extract from the Report of the Directors of the Jamaica Mutual Life Assurance Society, to 30th April,* 1854.—The directors have to submit to the members of the Society their report on the business of the Institution for the year ending on the 4th January last, as also for the further period to the 30th April, with its exact position at each of such dates; it having been considered more convenient, and more acceptable to the members, to take in, by adjournment of the usual annual general meeting in the month of March, the further period required to complete the first ten years' operations of the Society, which commenced its career on the 1st May, 1844; and the more so, as their report in respect to the appropriation of the cash bonus, recently declared, could have been only of a

very incomplete character had the supplemental account been made up, as on former occasions, only to the 25th January preceding.

In respect to such bonus—amounting, after deduction of certain policies discontinued in the interim, to £8,312. 4*s*. 3*d*. as a payment in hand—it will be in the recollection of the members, that a free option was given to each participant between the three different forms of a money payment, an equivalent reduction of the future premiums, and a reversionary bonus; in the event of the first, an order for a moiety embodying interest at 3 per cent. being given on the Island Treasury, and payable on the 25th July next. And the directors have now to report that the sum of £8,312. 4*s*. 3*d*. in question, under the option given, has been discharged as follows:—

	£.	*s*.	*d*.
By cash payments (as above stated) amounting to . . .	5,284	12	6
By reduction of future premiums	363	3	11
By additions to policies, and supplemental policies, amounting to £4,711. 18*s*. 3*d*.; their present value	2,664	8	10
	£8,312	4	3

while the sum of £17,717. 9*s*. 1*d*., previously standing on the books of the Society as "gross surplus on the 4th January, 1852," has been disposed of in the following manner:—

	£.	*s*.	*d*.
Values of policies, 4th January, 1852	5,219	2	3
Guarantee fund	4,186	2	7
Dividend surplus	8,312	4	3
	£17,717	9	1

the last item having been closed in the books of the Society; and, under the modification of the new premium scale (diminished below, and increased above, the age of 45), a further sum of £402. 5*s*. 2*d*., arising on the excess of payments since the 4th January, 1852, has also been returned to the members, with the exception of £38. 12*s*. 11*d*., to be available to them in reduction of premiums falling due in July next.

The entire settlement has been effected, it will be seen, with but little inroad on the vested capital, under the eventual mode of liquidation; while it is satisfactory to state, that the premiums and semi-annual interest due in July, amounting to £3,000, will prevent any further encroachment on the fund in the Island Treasury, in reference to the obligations of the Society presently becoming due.

In the business of the Institution between the meeting in September last and the promulgation of the Government Actuary's report, but little addition was made to the assurances of the Society, although during such half year the directors have only one further death to notice, involving a call of £500, to be provided for in July; but subsequently to the receipt of the report in question, the directors have to announce an important accession to the business of the Society, by the acceptance of 22 proposals, duly carried out, involving assurances to the extent of £10,450; while 4 proposals, amounting to £1,250, have been declined; and 5, involving assurances for £2,600, are under consideration—the existing assurances of the Society, including the recent additions to policies and free policies granted, amounting, on the 30th April last, to £100,282. 1*s*. 9*d*., and the income of the

Society, on premiums alone, to £5,300 per annum, while only one term policy for £500 has been discontinued; and as regards the mortality over the last six months, the directors have only to notice one death, involving a call of £379, and falling due in October next.

Statement of Accounts (*No.* 1) *on 4th January*, 1854.

Dr.	£.	s.	d.
Outstanding claims, due or not due	1,574	11	4
Receipts towards contingent reversionary annuity	218	16	0
Gross surplus, 1853	46	4	0
Assurance premiums, after deducting those appertaining to the ensuing year	4,698	7	0
Interest on investments	1,528	1	11
Assurance premiums, 1854	275	8	2
Interest account	14	11	3
Gross surplus arising on the year 1852	4,383	17	2
Gross surplus arising between the 1st May, 1844, and 4th January, 1852, and which formed the sum for investigation, valuation, and subsequent reference of the whole to the Government Actuary	17,717	9	1
	£30,457	5	11

Cr.	£.	s.	d.
Island Treasury, due as of the 25th July last	25,500	0	0
Interest thereon, since accrued	676	18	5
Drafts on Island Treasury bearing interest	458	9	11
Loans upon policies ditto	501	13	5
Colonial Bank	95	1	0
Office furniture	86	7	0
Policies emerged	2,300	0	0
Grant to the secretary by general meeting	315	0	0
Law expenses on the year	11	8	0
Current expenses on the year, viz.—			
Rent — £20 0 0			
Office servant — 8 12 0			
Secretary and accountant — 275 0 0			
Medical fees and commission — 20 16 0			
Advertising and printing — 54 16 0			
Stationery, stamps, and petty disbursements — 31 0 2			
Colonial Bank—Bank commission — 20 0 0			
430 4 2			
Fees to directors — 67 4 0			
„ auditors — 15 0 0			
	512	8	2
	£30,457	5	11

Statement of Accounts (*No.* 2) *on 4th January*, 1854.

Dr.	£.	s.	d.
Outstanding claims (of which £500 due in July next)	1,619	13	8
Receipts on contingent reversionary annuity	218	16	0
Premium account, 1854	275	8	2
Interest ditto, 1854	14	11	3
Gross surplus arising on the year 1852	4,226	7	2
Ditto ditto, 1853	3,246	4	5
Gross surplus to the 4th January, 1852, appropriated as follows—say £17,717. 9*s.* 1*d.* To value of policies for the whole term of life, as of the 4th January, 1852, amounting to £80,150;			
Say, *then* present value of assurances — £42,735 14 6			
Ditto ditto of future premiums thereon, after throwing off 5 per cent. towards future expenses — 37,516 12 3			
Value of policies as the difference	5,219	2	3
Carried forward	14,820	2	11

	£.	s.	d.
Brought forward	14,820	2	11
To assurance guarantee fund for one third of £12,498. 6*s.* 10*d.*, over and above the value of the policies £4,166 2 3			
Add on policies since abandoned 20 0 4			
	4,186	2	7
To dividend surplus for distribution among members having paid not less than one full year's premium, as of the 4th January, 1852	8,312	4	3
	£27,318	9	9

Cr.	£.	s.	d.
Island Treasury	25,500	0	0
Interest accrued	676	18	5
Drafts on Island Treasury	458	9	11
Loans upon policies	501	13	5
Colonial Bank	95	1	0
Office furniture	86	7	0
	£27,318	9	9

Contracts—say, assurances pending 4th January, 1854, 167 in number, on 147 lives, amounting to £87,000, £1,800 of which for terms; and a contract for a contingent reversionary annuity of £200.

Continuation Account, to the 30th April, 1854.

Dr.	£.	s.	d.
Outstanding claims, of which £500 due in July, and £379. 16*s.* 5*d.* in October next	1,038	2	0
Receipt towards contingent reversionary annuity	248	16	0
Cheques remaining to be granted to close the dividend surplus of £8,312. 4*s.* 3*d.*, as of 4th January, 1852, or unpresented	305	7	9
Bills on Island Treasury, payable 25th July next, being the amount granted in such form in part of the preceding	2,047	4	7
Assurance premiums, 1854 to date	3,147	14	5
Interest on investments	485	2	10
Assurance premiums paid in 1852 and 1853—say £402. 5*s.* 2*d.*, less returned £363. 12*s.* 3*d.*, leaving returnable in July	38	12	11
Fund for the discharge of future premiums, in whole or in part, in the case of single premiums, or diminution of previous annual premiums	2,690	17	6
Gross surplus arising on the first two years (1852 and 1853) of the triennium ending on the 4th January next	7,070	6	5
Value of policies as of the 4th January, 1852 (to merge in the *then* present value of the Society's contracts, as of the 4th January, 1855)	5,219	2	3
Guarantee fund in respect of the preceding—say	4,186	2	7
	£26,477	9	3

Cr.	£.	s.	d.
Island Treasury	24,500	0	0
Three months' interest accrued to 25th April	382	10	0
Drafts upon same	415	14	0
Loans upon policies, bearing interest	292	9	6
Colonial Bank, at call	134	2	6
Outstanding premium	10	10	2
Purchase of policies	58	5	8
Office furniture	86	7	0
Charge upon premiums, see *per contra*	379	16	5
Directors' fees	21	10	6
Current expenses	196	3	6
	£26,477	9	3

Contracts of assurance pending 30th April, 1854, 202 in number, on 158 lives, amounting to £100,282. 1*s.* 9*d.*; and further, a contract for a contingent reversionary annuity of £200.

Mortality Experience—1*st May*, 1844, *to* 4*th January*, 1854 (*in round numbers*, 10 *years*).

Years.	Lives.	Deaths.	Assurances.	Calls.
			£.	£.
1844	50	1	30,959	500
1845	79	0	50,750	„
1846	124	1	80,800	1,000
1847	141	1	86,500	1,000
1848	134	2	71,250	1,500
1849	117	1	70,700	500
1850	126	7	78,010	5,100
1851	137	2	85,460	1,100
1852	151	1	89,160	1,000
1853	154	4	91,060	2,300
	1,213[a]	20[b]	£734,640[c]	£14,000[d]
	Average Mortality, 1·65 per cent., or 1 in 61.		Average Calls, 1·91 per cent.	

(a) Average age, about 40 years.
(b) Attributable to Asiatic cholera, 7—in 1850-51; or seven twentieths of the whole.
(c) Average risk *per capita*, as computed, £606.
(d) Attributable to Asiatic cholera, £5,100; or more than five fourteenths of the whole.
Average aggregate annual calls, £1,400; average annual calls *per capita*, as experienced, £700.

The deaths occurring in 1,213 observations as above—the lives averaging the age of 40—should have been

According to the table of the Society (1,213 × ·04) 48
„ „ Northampton Table (× ·02) 24
Against deaths actually experienced..... 20

But the average calls *per capita, as experienced*, having been £700, in lieu of £606 as computed, we have, on the one hand, 24 calls by the *Northampton* Table, *as computed at* £606.....£14,544
And, on the other, 20 calls averaging £700 *as experienced* 14,000

Leaving, on *such* a comparison, a difference of 4 lives in favour of the Society, and of money value £544

The calls in the preceding table appear in each year in which the deaths respectively occurred, without reference to the maturity of the money payments. The lives, with little variation, have been treated as having been on risk from the *commencement* of the several years, although some entered only *during* such years respectively; but the difference arising on a more rigid adjustment would be unimportant.

As strongly contrasted with the very limited mortality experienced by the Society over the first decade of its career, under the age of 50, its experience so far, at the more advanced ages, evinces the utter inadequacy of the corresponding premiums as diminished by Mr. Woolhouse in 1849, and only recently augmented. It should also be observed that, as in the mother country (as noticed by Mr. Milne), the larger average risks have attached to the higher ages, although these last have been numerically few: thus, on the 30th April, 1854, only 9 lives appear on the register above the age of 55, but averaging £756, and only 4 above that of 60, but averaging £825; the average risk over the entire number of 158 lives then under assurance

being £685, and their combined ages presenting an average or mean age of 41·70. The minimum and the maximum risk existing on any one life are respectively £100 and £1,324, any recent enhancement included; the highest risk undertaken, according to the regulations of the Society, being £1,600.

The ages at death, and the disorders, of the 20 deceased members, were as follow:—

Diseases.	Deaths.	Ages.
Dysentery	2	27, 34
Gout and other causes	1	50
Decline	1	38
Organic disease in the intestines	1	48
Suffocation following on the rupture of a blood vessel	2	55, 37
Apoplexy	1	45
Asiatic cholera	7	38, 41, 43, 48, 51, 58, 59
Fever of the country	2	37, 53
Disease of the heart	1	44
Paralysis	1	62
After the amputation of a limb, consequent on a fall from a gig	1	47
Total	20	

And here the question naturally presents itself—What would probably have been the mortality before us but for the *exceptional* feature of Asiatic cholera, unknown in the island of Jamaica until the autumn of 1850; admitting, as applied to the seven decrements it produced, that disease, to a certain extent, might have developed itself in some other form, and possibly also *in other subjects*, if not in the corresponding year, in the two or three years (1851 to 1853) which immediately succeeded? To such a question we should be disposed to answer—from a personal knowledge of the individuals themselves, and with due regard to other circumstances which would legitimately enter into the consideration—that *four* out of the seven decrements so occasioned might reasonably be deducted on such a score from the 20 deaths experienced on the whole; leaving attributable to ordinary causes of death, *on such an assumption*, 16 in $(1213+\overline{4\times3})\,1225$; or 1·31 per cent., against 1·30 per cent., which, as of the age of 40, the Carlisle Table would have presented.

While the results here given, considered under any aspect of the case, furnish no reliable criterion for the government of the future, looking at the limited time and numbers involved, and evidence merely the important difference between acclimated and unacclimated lives, and the large value of selection in the infancy of a Society and in a community in which health and longevity so greatly depend on the habits and occupation, temperament and disposition, of the individual; it must, nevertheless, be conceded, that no such results, however considered, could have been anticipated ten years ago, as those which were to characterize the experience of a Society in the island of Jamaica over the first ten years of its career.

With respect to the treatment here of Asiatic cholera, as exceptional, and as a feature *per se*—as irreferable to any law of mortality thus far known, as setting all anticipatory computation at defiance, and as a disease

altogether *sui generis*—in no country can it be treated otherwise. Over the last three years (and embracing one death from cholera), it will be seen that the mortality has been $\frac{7}{442} = 1{\cdot}58$ per cent.; over the last six months the mortality having been 1 in 159.

CORRESPONDENCE.

SUGGESTIONS FOR A BETTER MEANS OF MAKING PROVISION FOR THE WIVES AND FAMILIES OF PERSONS ENGAGED IN THE BUSINESS OF LIFE ASSURANCE.

To the Editor of the Assurance Magazine.

Sir,—As the accredited organ of the assurance interest, I feel sure that you will open the pages of your *Magazine*, and also lend the aid of your own pen, to call the attention of the directors of Assurance Companies to a question of no mean importance to those in their employment. I refer to the question of allowing to those engaged in the business of life assurance the same advantage as is enjoyed, I believe, by persons engaged in all kinds of trades—I mean, that of obtaining the article dealt in by their employers at the cost price.

My attention has been more particularly called to this subject lately (though it has always been one the equity of which I have never questioned, and one which I have thought a good deal about) by reading the last report of the National Provincial Bank of England, the *Twenty-first Annual Report*, and from which I extract the following :—

"It will be in the recollection of the proprietors that, on the occasion of their last annual meeting, it was intimated that a strong desire had been expressed among the officers of the Company for the formation of a Provident Society for their mutual benefit—embracing also, if possible, the principle of a guarantee fund. The precise nature of the scheme it was then left for the directors to consider and determine. At the outset of their deliberations the directors found that the guarantee scheme was objected to by a large number of the officers, and it was therefore abandoned.

"The other objects aimed at are—*First*, a provision for the widows and children of officers at their death; and, *secondly*, a superannuation allowance for the officers themselves.

"To effect the first and most pressing of these objects, the directors recommended to the proprietors that the officers and clerks should be assisted in effecting insurances upon their lives to the following extent, by the Bank paying for them one half of the yearly premium—viz. :

"1st. Managers of branches and agencies to insure for not less than £500 and not more than £1,000.

"2nd. Junior officers and clerks to take out policies for sums not under £250 and not exceeding £500.

"With respect to the question as to a superannuation allowance, the directors intend to postpone the consideration of it, in the hope that at a future period they will be able to deal with it in a more satisfactory manner than they could do at the present moment.

"In recommending the above measure for the adoption of the proprietors, the directors feel satisfied that they are consulting the welfare of the establishment, and that any expense which it will entail will be more than compensated by attaching to the Bank experienced and well tried officers, and by the increased devotion to its interests on their part which such consideration for their personal comfort cannot fail to inspire.

"The guarantee project having, as already stated, been abandoned, the £5,000 voted by the proprietors last year for the benefit of the officers and clerks of the establishment, as a mark of appreciation of their past zealous and faithful services, the directors, acting upon a suggestion thrown out at the last meeting, propose to divide amongst them as a bonus, of which they hope the meeting will approve."

It must be conceded, that the payment of 50 per cent. of the premiums for life assurance, within the limits prescribed, and of course under certain necessary regulations, is a great boon to those in the employment of the Bank, and an act of the greatest liberality on their part.

The General Post Office, too, I am happy to see, have recently agreed to allow certain funds arising from void money orders and from unclaimed property found in dead letters to be applied to a purpose similar to the above—to the extent, however, of the payment of 20 per cent. only of the premiums for life assurance, within certain limits, and similarly, of course, under certain regulations.

I extract the following from a circular issued by command of the Postmaster General to all the officers of the Post Office, and bearing date June, 1854.

"In order to encourage the officers of the Department to make provision for their families at their death by insuring their own lives, the Lords of the Treasury have consented to give assistance in the payment of the premiums on such insurances, and have appropriated to this object a fund arising from void money orders and from unclaimed property found in dead letters.

"This fund now amounts to about £1,600 per annum, exclusive of an accumulated sum of about £15,000, which also, by their Lordships' direction, will, with the future interest thereon, be applied to the same purpose. . .

. . . . "It is intended, if the number of officers who may effect insurances be not too great, that one fifth part of each officer's premium shall be paid by the Department; provided, however, that no officer shall have aid in his payments for more than £300 of insurance. . . . . .

. . . . "In all circumstances the Postmaster General will have full power, if at any time he think fit, to withhold the pecuniary assistance contemplated, either in part or altogether. . . . . .

. . . . "The plan will extend to every Post Office officer in the United Kingdom.

"An existing life insurance in either of the Offices enumerated will entitle the insured to the payment, by the Department, of a portion of his premium equally with a new insurance. . . . . .

. . . . "The Department's contributions will not extend to premiums greater than annual premiums. Thus if an officer engages to make a single payment only, or a limited number of payments, the contribution from the Department will apply only to such sum as he would have had to pay if his premium had been continuous and annual.

"On quitting the service, an officer will of course cease to receive

assistance in the payment of his premiums, and to partake of any other advantage of the plan."

It is not my intention to enter upon any criticism of the regulations under which the respective advantages above referred to are to be enjoyed. There are one or two points of detail (I am now referring more particularly to the Bank) the objections to which would readily strike any of your readers that are versed in life assurance business; but I doubt not that those slight imperfections have only to be pointed out to be corrected. As, however, some alterations in the regulations are undoubtedly necessary, I shall refrain from quoting them; but I forward for your own private inspection a copy of the regulations issued by the Bank, with a few marginal comments upon them.

The plan of public Companies assisting those in their employment in so desirable an object as life assurance is so highly to be commended, that I am astonished to find that no such boon is at all general; nor have I been able to ascertain, upon inquiry, that any such advantage is held out to those employed even in Assurance Companies, where it would be reasonable to suppose such a scheme should have originated, with the single honourable exception of the Albion Life Office, which allows, if I am rightly informed, a reduction of 33 per cent. upon the premiums paid by those in the Company's employment. Some Assurance Companies, I believe, object to allow even the usual commission of 5 per cent. upon the premiums to those in their service.

Now, Sir, it seems to me (and I should feel proud to find that you were of the same opinion) that it is a most reasonable thing for those employed in an Assurance Company to obtain their assurances at the net premium, at least. To grant such an advantage would only be to place those engaged in Assurance Companies upon a similar footing with those employed in all other descriptions of business.

I feel sure that the attention of the directors of Life Assurance Companies, which are almost invariably conducted in a spirit of the highest liberality, only requires to be drawn to this question to induce them at once to agree to some considerable reduction of the usual premiums on assurances upon the lives of their *employés*.

The benefits conferred by a scheme of this kind would be greater than almost any other that could be devised. It would tend to encourage habits of forethought and prudence in their servants, which would redound in many ways to the advantage of the Company, besides the relief it would afford of freedom from claims being made upon their funds, in consequence of the death of any of their servants, by those dependent upon them for support.

It would, of course, be quite out of place in this *Magazine* to enter upon any arguments in favour of life assurance. There can be, however, no doubt that the practice is not nearly so general as it should be, not only among the public at large, but even among those who ought to be the first to set the example, the servants and agents of Life Assurance Companies. Strange to say, this is one of those curious anomalies one sometimes meets with; and as that close observer of human nature, Charles Dickens, has pointed out that proctors, as a class, frequently die intestate, so those engaged in life assurance business not unfrequently fail to avail themselves of its advantages.

It has, I understand, been proposed to establish an Office for the

express purpose of assuring the lives of those engaged in the business. This, it appears to me, is not only perfectly unnecessary, but indeed quite undesirable. A person engaged in a Life Office can effect his policy in his own Office with the least possible trouble to himself and inconvenience to his employers, in the first place; and in the second, many cases may arise in which an Assurance Company might be disposed to assure the life of one of their servants which another Company might decline altogether. I am now supposing the existence of a slight defect in the life proposed, which might be really unimportant; and the Company in whose daily service he was engaged, and whose officers were in the daily habit of seeing him, would be, it is quite clear, much better judges of the case than any other Company could possibly be.

I am not now entering upon the discussion of the question as to whether the Companies should go so far as to assure the lives of *all* their *employés*, whether first class lives or not, though I think that this is a question far from unimportant, and one in favour of which a good deal might be said.

Should this matter be taken up at all, it will then be quite time to inquire how far it might be desirable and proper to admit such lives to the incalculable benefits of life assurance—(benefits far greater, in their unhappy case, than in any other). Of course, a person in radically bad health would not be fit for his duty, and his case would come under another category; and though, of course, organic disease might be latent when he obtained his appointment, or might supervene thereafter, this would probably not be a case of so frequent occurrence as to make it necessary to legislate particularly for it. I should have quite sufficient confidence in the board of directors of a Company of repute, to trust to their taking a fair and liberal view of any case that might come specially before them.

Now it seems to me that no complicated arrangements for carrying out such a scheme as I have advocated are at all necessary. Let the directors of Assurance Companies merely agree to allow as a commission, to those in their service, a deduction from their premiums of assurance of—say, 25 per cent. (which would probably, upon an average, reduce the tabular premium of the several Companies to the net premium), as long as the life assured remains in their service, to revert to the usual commission of 5 per cent. should he, from any cause whatever, leave the service of the Company, and the thing is done.

If the life assured retire from the Office from ill health, or, after length of service, upon a pension, it would of course be discretionary with the directors to continue to him the previous allowance, or to deal specially with the case according to the estimation in which his services were held, and in proportion probably to his pecuniary means.

If such a boon as this were conceded, I think—

1st, That the assurance of the lives of all the Company's servants, whether married or single, should be made compulsory, for sums proportionate to their respective salaries.

2nd, That the power of deducting from the salaries the total amounts of the annual premiums, by equal quarterly instalments, less the commission agreed to be allowed, should be reserved to the directors.

3rd, That no assignment or dealing with the policies should be allowed without the permission of the directors, to be specially obtained in each case: to prevent which, I would place a special indorsement upon every policy issued to the Company's servants.

4th, That as, upon the institution of any plan of this nature, many of the more prudent of the Company's *employés* would probably be already assured, and some of them with other Offices than their own (and it would be more likely to be so in the case of prudent and intelligent men, who would in many cases have passed from an inferior position in one Office to a superior appointment in another), a similar allowance of 25 per cent., or whatever amount might be agreed upon, on the premiums actually payable, should be allowed under similar regulations, provided of course the Company so making the allowance should express themselves satisfied as to the respectability and solvency of the Company with whom the existing assurance had been effected.

I do not pretend to have entered into all the details necessary to perfect a plan of this nature, in the first place because this is hardly the time for doing more than what the lawyers call *ventilating* the case; and, secondly, because the necessary precautions to be taken and steps to be adopted would necessarily occur to every habitual reader of your *Magazine*—a magazine, Sir, if I may be allowed to express an opinion, which has already done much to promote the *theory* of life assurance and its cognate subjects, and which I trust will now assist in the good cause of advocating the *practice* of life assurance, as applied to those engaged in daily assisting to carry its blessings into the families of thousands.

You will see that there are many points that I have not touched upon; not because I have been unmindful of them, but because I am fearful of intruding too much upon your space—as for instance, whether similar allowances should not be made, as respects the insurance from fire, to those engaged in Fire as well as Life Companies. I have not proposed an allowance to the extent of 50 per cent., as agreed to be made by the National Provincial Bank of England. Of course, such Assurance Companies as were in a position to make such an allowance might do so; but I feel that many Assurance Companies are hardly in so prosperous a condition as the Bank in question, and that this accordingly could hardly be expected.

A scheme of this kind should be made as universal as possible, for many reasons: one I will name, which is, that if the same plan be agreed to by all the respectable Companies, a person leaving one Company and entering into the service of another would still obtain an equal amount of allowance, and would know what he had to expect when he effected his assurance in the first instance. I feel sure that no boon that could be granted by the directors of Companies to those in their employ would be of more real advantage than the one proposed. It could be granted at a very small sacrifice of profits, and I should hope that the advocacy of your able pen would bring about its almost universal fulfilment.

I have the honour to be, &c.,

WM. PORTER.

Alliance Life Office, 1st August, 1854.

NOTE.—The subject referred to in this communication is of such obvious interest and importance to the profession, and Mr. Porter's observations upon it are so much to the purpose, that, for the present at least, we feel it quite unnecessary to do more than submit his letter to our readers, not doubting that it will attract the notice and secure the consideration it so well deserves.—ED. *A. M.*

ON THE INTEREST IN ASSURANCES MADE BY ONE PERSON ON THE LIFE OF ANOTHER.

To the Editor of the Assurance Magazine.

SIR—I have just received the last Number of the *Magazine*, which contains much interesting matter, and on which I have great pleasure in complimenting you.

My reason for writing you is because an article there, that is dated from the Eagle Office, does not meet my ideas of what is desirable, and I should be glad to learn that Mr. Bailey is not expressing the sentiments of "*the Eagle*" in his letter.

I believe that the Act 14 Geo. III., cap. 48, has been and is a valuable protection to life assurance, and has a material effect on the whole of that class of business against which the Act was intended to provide. It is, indeed, very rarely that any case occurs in which the Offices would ever think of pleading a want of interest as a reason for not paying, but the rarity of such cases arises from the wholesome influence of the Act; and, when necessary, an Office can fall back upon the want of interest with a facility and a confidence that it could not have, if it had to prove fraud or swindling.

I thought it was a point which was universally admitted to be an advantage which we possessed, in this country, above the sister isle, that we had a comparative immunity from gambling insurances. The proof of this is the fact that we have so very seldom occasion to agitate the question of interest, or to object, on any ground, to the right of the insurer to claim the contents of the policy.

Mr. Bailey says, that the interest in assurances on the "life of another" must be "*pecuniary*," and that no other will suffice; but the case he refers to as an example of an illegal insurance is strictly within the meaning of that term. A man who has *a mother, sister*, or other relative dependent upon his exertions, and for whom he is desirous of making some provision at his death, can have no difficulty in satisfying any Office of the validity of their interest, and a policy taken in their names upon his life would be quite unobjectionable. The real ground of objection that might be started in such a case, and which is the same with that of *a wife* insuring her husband, arises from a totally different cause—viz., the fear that *the interest* (admitted to exist) may prove too great a temptation to parties who have the power of hastening the contingency upon which the payment depends, or who may not exert themselves in warding off that contingency.

The case of insuring a debtor's life may be a bad speculation, but it is quite a fair mode of securing payment of a bad debt. I may not choose to lose a large amount of my capital, and may prefer a certain outlay each year which I can easily afford, and by which the capital will at a future period be replaced.

Again: in cases where policies are sold, the law, as it stands, is very useful; for the party purchasing is well aware that, if anything fraudulent or incorrect be mixed up with the policy, the Office can, through the question of interest, stand upon strong grounds in refusing to pay either in whole or in part. It is only the invariable practice of the Offices to pay, because cases of an exceptionable character are hardly ever met with; and this, I believe, arises solely from the salutary influence of the law as it at present stands.

I confess myself in total ignorance of the question of interest having ever diverted the attention of boards of directors from the more important points of health and habits; but I believe that the high mortality which has prevailed among Irish assurances has been in part attributable to the state of the law, and consequently to the ideas of the Irish people being very different from ours on that subject.

I trust the views of Mr. Bailey are not at all general, and not likely to have any influence upon the intentions of Mr. Wilson or the legislature, in the Bill about to be brought before Parliament.

I am Sir,

Your most obedient servant,

Edinburgh, 7th July, 1854. VERUS.

REPORTS OF ASSURANCE COMPANIES.

London Mutual Life and Guarantee Society.—Third Annual Meeting, held 6th July, 1853.—The report stated that the amount of business completed since the commencement of the Society to the 30th June last has been as follows:—

	Policies issued.	Sum assured.	Annual Income.
		£.	£. s. d.
Total.....	2,357	391,941	12,546 8 2
Of which there were issued in the year 1852.....	553	92,010	2,938 2 3
During the first six months of the present year, 1853..	323	60,225	1,940 6 0
Being at the rate of	646	120,450	3,880 12 0

for the entire year, and showing an increase at the rate of upwards of 32 per cent. from the business of the present year over that of the year 1852. During the past year five deaths occurred, the claims on which amounted to £650, all which have been paid, except the last, for £150, which is not yet mature. The average age of the assured for the whole term of life is nearly 37½ years; average amount assured on each life, £268. 11*s.*; average premium per cent., £3. 0*s.* 9*d.* The amount advanced on loan to the members, on approved security, during 1852, was £1,970; £3,208 has been lent during the past six months; and the total amount loaned since the commencement has been £10,189.

Statement of Receipts and Disbursements, from 31st December, 1851, *to 31st December*, 1852.

Receipts.	£.	s.	d.
To balance of last account	879	7	3
Assurance premium fund	7,880	0	8
Premiums due, but unpaid as per last account	99	12	4
Immediate annuity premiums	739	2	2
Interest on investments, India bonds, loans, &c.	274	19	5
Sundries	1	0	7
Carried forward	£9,874	2	5

	£.	s.	d.
Brought forward	9,874	2	5
Repayment on account of loans	1,621	7	6
Deposit account, cash from depositors	624	0	0
Temporary capital	5,000	0	0
Amount at credit of sundries	331	4	10
	£17,450	14	9

Disbursements.

	£.	s.	d.	£.	s.	d.
Charges of management—						
Office expenses	28	18	11			
Rent and insurance	180	13	6			
Salaries	729	3	6			
Advertisements	312	2	7			
Printing, stationery, and books	412	4	5			
Postage and carriage of parcels	170	9	4			
Directors' fees, 1851	546	0	0			
Auditors' fees, 1851	31	10	0			
Receipt stamps	5	12	0			
Petty and incidental	22	12	8			
				2,439	6	11
Agency charges—						
Salaries and travelling expenses of superintendents of agencies	721	8	10			
Commission	447	3	7			
Incidentals	147	15	5			
				1,316	7	10
Expenses—Establishing committees of reference in Yorkshire, Lancashire, and the midland counties, and to extend business				117	1	9
Medical fees and medical officers' salaries				457	4	3
Policy stamps, not being charged to the assured				212	15	0
Interest—						
On temporary capital, 25 Dec., 1851, to 9 Aug., 1852	£414	2	2			
Deposit accounts	16	11	10			
				430	14	0
Law charges—						
Salary of solicitors	100	0	0			
Sundries	23	8	1			
				123	8	1
Fees, Joint Stock Registration Office				18	9	8
Office furniture (tin box)				2	11	4
				5,117	18	10
Temporary capital fund paid off				5,000	0	0
Claims	300	0	0			
Policies surrendered	3	14	0			
				303	14	0
Reassurance				98	3	9
Annuities paid				5	0	0
Costs in Pepper *v.* Chambers, and repayable by Mr. Chambers				165	5	5
Interest on temporary capital fund, due 25th December, 1851				339	15	10
Income tax, short paid, October, 1851				0	11	1
Deposits returned				808	8	3
Half credit premiums, and premiums due but unpaid on 31st Dec., 1852				111	6	8
Investments—						
India bonds	£1,038	0	0			
Loans	1,970	0	0			
Cash at bankers'	2,265	15	7			
In agents' hands	203	12	6			
Petty cash	7	6	0			
Policy stamps	15	17	6			
Balance				5,500	11	7
				£17,450	14	9

Balance-sheet, 31st December, 1852.

Assets.	£.	s.	d.
To Rogers, Olding, & Co., balance in their hands	2,265	15	7
Petty cash in hand	7	6	0
Policy stamps in hand	15	17	6
Loans, balance due from borrowing members	3,426	16	10
India bonds, valued at cost	2,080	15	0
Office furniture and fittings	546	9	0
Agents, balance due from	203	12	6
Sundry assured, for unpaid premiums, 1852	74	18	6
Credit premiums	35	3	2
T. Pepper, costs in Pepper *v.* Chambers	165	5	5
Suspense account	4,862	10	1
	£13,684	9	7

Liabilities.	£.	s.	d.
By temporary capital	5,000	0	0
Deposit account, amount due to depositors	42	16	0
Unclaimed interest	1	14	0
Interest on deposit account unpaid	0	19	5
Income tax on shareholders' interest	33	6	1
Amount at credit of sundries	331	4	10
Directors' fees for attendances, 1852	636	6	0
Balance	7,638	3	3
	£13,684	9	7

The chairman then explained that the directors had turned their attention with much anxiety to the expenditure of the Society, with a view to its curtailment, having at the same time a due regard to the safety of the assured and the efficiency of their operations. The first matter, and one which had long perplexed them, was the large amount of interest paid on the temporary capital fund. The amount of the capital was £50,000. Five thousand pounds only of that sum had been paid up, and on that the interest was 5 per cent. on £250 per annum; on the remaining £45,000 not called for, 1 per cent. was allowed, or £450 per annum, making a total of £700 per annum, or 14 per cent. on the sum really advanced. With the advice and opinion of Mr. Neison, the directors had advanced, to pay off the paid-up capital, £5,000 at 5 per cent., with an engagement not to recall it until the reserved funds of the Society to meet claims amount to £20,000. The business has increased in the following proportions, viz.—

	No. of Policies issued.	Sums Assured.	Annual Income therefrom.		
In the first six months of—		£.	£.	s.	d.
1851	284	40,644	1,228	15	4
1852	310	48,360	1,513	19	0
1853	323	60,225	1,940	6	0

A saving will be found under the following heads:—

Interest temporary capital	£450	
Salaries, about	300	
Printing and advertising	150	
Travelling agent, or agency superintendent	450	
Directors' fees, less this year than last	200	
Carried forward	——	£1,550

Brought forward	£1,550
Alteration in stamp duties, say	150
	1,700
Deduct from this expected outlay in establishing committees of reference, agencies, &c. &c., say	300
Decrease of expenses, 1853, over 1851 and 1852, about	£1,400

During the twelve months which have elapsed since the last annual meeting, there might have been expected at least ten deaths, and a loss therefrom of more than £2,600. But instead of ten, only five assurers have died during all the year, and the claims have only amounted to £650. There have been, since the commencement of the Society, proposals to assure £609,956; of these there have been declined, withdrawn, or not completed, with the exception of those still under consideration, £218,015. There have been 2,357 policies issued, for £391,921, giving an annual income of £12,546. After deducting lapsed policies, the present net income from life assurance premiums is about £10,000 per annum.

A resolution was passed, to the effect that "any director who shall at any time, at the request of the board, give his attendance upon the business of the Office, during any day other than the ordinary board day, shall receive the same fee as that fixed to be given for attendance at board meetings"; such fee to be received by the chairman for his extra attendance during the past six months.

Metropolitan Life Assurance Society.—Report of Receipts and Payments for the Year ending 4th April, 1854, with a Statement of the Assets of the Society on that day.

	£.	s.	d.
Cash and policy stamps on hand on the 4th April, 1853 . . .	5,365	3	9
Receipts.			
Members' new assurances (effected since 5th April, 1853) . . .	5,990	6	11
Miscellaneous new assurances . . same	214	10	11
Members' renewal premiums, after allowing the abatement on the premiums of members of five years' standing	54,897	3	0
Renewal premiums on miscellaneous assurances	3,854	14	9
Extra premiums	966	17	4
Fines	112	8	1
Discount on policy stamps	1	16	0
Dividend on New 3½ per Cent. stock	650	0	0
„ on 3 per cent. Consols	10	11	10
„ on Annuities for terms of years	4,270	0	0
„ on Bank stock	520	0	0
Interest on City of London bonds	175	0	0
„ on India bonds	251	10	0
„ on Exchequer bills	92	0	7
„ on loans	19,690	19	11
Loans repaid	29,909	0	0
Sale of £23,800 India bonds	23,910	12	0
„ £10,000 New 3½ per Cent.	10,250	0	0
Repayment of £5,000 Exchequer bills	5,000	0	0
Over-receipt on mortgage, to be accounted for	132	0	0
	£166,264	15	1

Payments.	£.	s.	d.	£.	s.	d.
Ground rent and taxes	287	14	5			
Advertising	58	11	4			
Printing and stationery	157	8	10			
Carried forward				503	14	7

	£.	s.	d.	£.	s.	d.
Brought forward . . .	503	14	7			
Receipt stamps	24	8	7			
Policy stamps	45	9	0			
Postage	54	0	9			
Office salaries	1,578	1	6			
Fees to medical examiners	339	5	0			
„ directors	1,200	0	0			
„ auditors	31	10	0			
Fire insurance	8	11	0			
Office expenses and messenger	136	2	8			
Fittings and repairs	13	15	0			
Law charges	203	16	4			
	4,133	12	5			
Less—Rent received and income tax allowed . .	201	0	2			
				3,932	12	3
Claims on members' assurances				30,398	0	0
„ on non-members' assurances				100	0	0
Purchase of policies surrendered				2,071	12	10
Property and income tax				730	15	7
On mortgage account				3	3	0
Purchase of £109,000 Railway and other debentures . . .				108,691	2	9
Advanced on mortgage				9,500	0	0
„ on policies effected with the Society				8,888	0	0
Cash and policy stamps in hand on 4th April, 1854 . . .				1,949	8	8
				£166,264	15	1

The Assets of the Society on the 4th April, 1854.

£2,000 annuities for terms of years ending 1859.
2,270 do. do. do. 1860.

	£.	s.	d.
£4,270	20,653	11	7
£15,000 New 3½ per Cent. stock	14,618	15	0
£6,500 Bank stock	13,978	2	6
£5,000 City of London bonds	5,000	0	0
Advanced on mortgage	286,655	0	0
„ Railway and other debentures	215,774	9	0
„ for purchase of life annuities	2,173	10	0
„ on policies of the Society	46,828	7	0
Invested in the purchase of reversionary interests	10,328	2	0
House in Princes Street	4,593	12	4
Furniture and fixtures	363	14	6
Cash and stamps	1,949	8	8
Due on investments	8,600	18	7
	£631,517	11	2

A General Statement of the Affairs of the Metropolitan Life Assurance Society, estimated on the 5th April, 1854, to justify an abatement of 50 per cent. and also an extra abatement of 1 per cent. for the Year ending 4th April, 1855, to members of five years' standing.

	£.	s.	d.
The present value of £2,654,142 assured by policies effected on the lives of members to 5th January, 1854	1,201,946	2	0
The present value of £99,515 assured on the lives of non-members .	60,400	16	0
Reserve for value of assurances for short terms, contingent and extra risks	6,171	18	0
Claims allowed but not yet paid	10,450	0	0
	£1,278,968	16	0

The present value of £88,376. 13*s.* 9*d.*, being the whole amount of annual premiums on policies of members effected on or before 5th of January, 1854 . . . £1,154,062 0 0

	£.	s.	d.	£.	s.	d.
Brought forward . . .				1,154,062	0	0
50 per cent. abatement thereof £577,031 0 0						
Deduct—						
The proportion of this abatement to be paid in full on policies of less than 5 years' standing . . .	28,367	4	9			
				548,663	15	3
						605,398 4 9
The present value of £3,588. 6*s.* 4*d.*, being the annual premium on policies of non-members						41,247 8 10
House and furniture in Princes Street						4,957 6 10
Due on investments						8,600 18 7
Cash and stamps in hand						1,949 8 8
						662,153 7 8
Balance						616,815 8 4
						£1,278,968 16 0

The income of the Society is as follows:—

	£.	s.	d.	£.	s.	d.
From £4,270 terminable annuities, after allowing the annual sum required to replace the capital .	840	2	8			
„ £15,000 3½ per Cent. stock	455	1	8			
„ £6,500 Bank stock	520	0	0			
„ £5,000 City of London bonds	169	18	0			
„ £215,774. 9*s.* invested in Railway and other debentures	8,861	9	9			
„ £286,655 invested on mortgage	12,725	1	5			
„ Life annuities purchased for the sum of £2,173. 10*s.*, after deducting the premium for assuring the return of the principal	96	7	11			
„ £46,828 7*s.* 0*d.* advanced on policies effected with this Society	2,262	9	2			
Invested in the purchase of reversions £10,328. 2*s.* 0*d.*	516	8	0			
						26,446 18 7
One year's interest on the above balance of £616,815. 8*s.* 4*d.*, at 4 per cent.						24,672 12 4
						1,774 6 8
One per cent. additional abatement on £63,943. 19*s.* 3*d.* premiums on members' assurances of five years' standing for the current year .						639 8 10
Surplus income						£1,134 17 5

Metropolitan Life Assurance Society.—Nineteenth Annual Meeting, held 10th of May, 1854.—The summary of the experience of the Society, to the 5th of April last, is as follows:—

The number of policies issued has been 3,677, of which 747 have ceased, leaving 2,930 existing policies, the annual premiums receivable on which amount to £94,000. 16*s.* 1*d.* The premiums received on the above 747 policies are as follows:—

			£.	s.	d.
On 261 policies, constituting the claims after mentioned, amount to			82,090	15	0
On 154	„	purchased (deducting the purchase money) .	15,272	19	11
On 332	„	lapsed and forfeited, and risk expired . .	22,25	4	3
747			£119,614	19	2

The aggregate of the claims on the Society by death is £275,270. 15*s.* 5*d.*, and of that sum £30,948 has arisen during the past year. The total amount of abatement of premiums made to its members has been £223,012. 3*s.* 5*d.* The accumulated capital to the 5th April last was £631,517. 11*s.* 2*d.*

Minerva Life Assurance Company.—Sixteenth Annual General Meeting of Proprietors, 23rd June, 1853.—The report states that during the year 1852 the new policies issued were 324, assuring the sum of £182,957. 18*s.*, and the new premiums received thereon amounted to £7,462. 16*s.* 11*d.*, or adding the second half year's premiums on policies effected by the half yearly scale, representing in new premiums the sum of £8,148. 13*s.* 3*d.* This is the largest amount of business hitherto transacted in any one year. Twenty deaths were announced during the year, the loss being £12,450, assured by 24 policies; 33·62 deaths might have been expected amongst the 1,634 lives at risk during the year, so the actual mortality still continues favourable. The whole number of policies discontinued during the year, from non-payment of premiums, effluxion of time, surrender, death, and cancellation, was 114, assuring £75,095. 12*s.* 6*d.*; leaving existing on 31st December last 1,793 policies, assuring £1,207,705. 11*s.* 8*d.*, on 1,546 lives. The average on each policy was £673. 11*s.* 4*d.*; the average on each life was £781. 3*s.* 7*d.*

	£.	*s.*	*d.*
The income of the life assurance fund was	52,386	12	1
Of the proprietors' fund	1,432	13	4
Total	£53,819	5	5

	£.	*s.*	*d.*
The accumulated amount of the life fund was	214,972	14	8
Proprietors' ditto	33,994	1	11
Total	£248,966	16	7

The expenses of management for the year were only £3,753. 16*s.* 11*d.*, being less than 7 per cent. of the gross income. A dividend was declared on the paid-up capital for the past year at the rate of 5 per cent., clear of income tax, payable on the 5th July next. This dividend is paid entirely out of the interest and accumulations of the proprietors' fund.

National Life Assurance Society.—Report of the Receipts and Payments for the year ending 31st December, 1852.

			£.	*s.*	*d.*
Balance in hand 1st January, 1852			2,088	17	5
Sale of £11,600 Long Annuities			79,750	0	0
Ditto £3,700 Annuities for terms of years			24,512	10	0
Ditto £27,900 New 3½ per Cents.			28,314	2	6
Ditto £10,000 Consols			10,000	0	0
Premiums on assurances renewed	£32,340	3 4			
Ditto new business	1,947	2 2			
Extra premiums, and fines for revival of policies	484	17 9			
Additional premium paid on Policy 1,395	1	17 5			
			34,774	0	6
Interest on mortgages and dividends on stock	£8,312	11 0			
Profit on temporary investment in 3½ per Cents.	377	12 6			
			8,690	3	6
Portion of dividend on annuities replaced to capital			6,709	0	10
Loans on policies repaid			2,095	0	0
Received for policy stamps			129	10	0
Commission on reinsurances			6	9	7
Remitted from Trinidad, in excess			0	13	0
Ditto Manchester, ditto			0	5	4
			£197,070	12	10

	£.	s.	d.
Purchase of £15,900 New 3¼ per Cents.	15,979	10	0
Ditto £104,132. 6s. 9d. Consols	104,262	10	0
Ditto £5,000 Canada Government bonds	5,787	10	0
Advanced on mortgage	34,000	0	0
Ditto policies of the Society	2,818	18	4
Ditto policies, with collateral security	2,850	0	0
In discharge of claims on policies:—			
Arrears, 1851, £4,600; casualties, 1852, £7,500	12,100	0	0
Purchase of policies	1,190	13	7
Agencies	221	19	4
Fees to directors, auditors, and medical officers	941	10	0
Office salaries	1,136	5	0
Annuity to late secretary (two quarters)	150	0	0
Purchase of furniture	40	0	0
Postages, printing, advertising, stationery, &c.	421	12	6
Law charges	34	14	0
Amount allowed the assured in reduction of premiums	9,983	19	6
Rates and taxes	152	16	11
Income and property tax	490	13	0
Advanced for policy stamps	254	0	0
Reinsurance	169	6	8
Portion of premiums returned, Policy 857	9	6	10
Balance in hand, 31st December, 1852 £3,650 16 5			
Interest due but not yet paid 424 10 9			
	4,075	7	2
	£197,070	12	10

Statement of Property belonging to the Society, and Income arising thereon.

	Property. £.	s.	d.	Income. £.	s.	d.
New 3¼ per Cent. Stock	56,100	0	0	1,823	5	0
Consols	94,132	6	9	2,823	19	5
6 per cent. Canada Government Bonds	5,000	0	0	300	0	0
Loans on mortgage and on policies	128,048	18	4	4,848	17	11
				9,796	2	4
Annual dividends and interest on mortgages						
Cash at bankers' and petty cash	3,650	16	5			
Policy stamps on hand	241	7	6			
Freehold premises in King William Street	7,500	0	0			
Amount of annual premiums				34,264	17	11
				£44,061	0	3

A General Statement of the Affairs of the National Life Assurance Society, estimated on the 31st December, 1852, as up to 25th March, 1853.

	£.	s.	d.
Present value of £994,214 assured on the lives of members	514,823	14	7
Present value of non-members' policies	5,137	8	10
Claims admitted but not yet paid	8,250	0	0
Reserve for additional claims to 25th March, 1853	15,000	0	0
Reserve for salaries, fees, and outstanding accounts to ditto	1,292	2	1
	£544,503	5	6

Capital:—	£.	s.	d.	£.	s.	d.
New 3¼ per Cents., £56,100	54,978	0	0			
Consols, £94,132. 6s. 9d.	89,425	14	5			
6 per cent. Canada bonds, £5,000	5,000	0	0			
Loans on mortgage and policies	128,048	18	4			
Premises in King William Street	7,200	0	0			
Policy stamps on hand	241	7	6			
Balance at bankers, and petty cash	3,650	16	5			
				288,544	16	8

			£.	s.	d.
Brought forward			288,544	16	8
Various sums payable to the Society before 25th March, 1853			3,360	0	0
Present value of £5,988. 16*s.* 9*d.*, annual premiums which members not yet entitled to reduction will pay in full			14,965	7	11
Present value of £31,955. 18*s.* 4*d.*, total amount of annual premiums on members' assurances	£398,244 10 0				
Less value of amount to be paid in full, as above	14,965 7 11				
Present value of amount subject to reduction	£383,279 2 1				
38 per cent. reduction thereon	145,646 1 2				
			237,633	0	11
			£544,503	5	6

Palladium Life Assurance Society.—Capital and Assurance Account, 1850.

Dr.	£.	s.	d.
Paid-up capital, with additions, making the capitalized value of each share £2. 4*s.* 2*d.*	66,285	10	5
Reserved fund	7,569	4	11
Revision for outstanding risk on assurances, subject to periodical revaluation	260,288	8	4
Dividends due	1,095	17	4
	£335,239	1	0

Cr.	£.	s.	d.
Mortgage investments	244,905	15	2
Government securities	34,543	10	0
Reversions and annuity investments	16,045	10	3
Advances on policies and bonds	14,599	0	0
Bank stock, Palladium stock, and India transfer loan	13,455	13	10
Estimated value of house and fixtures	4,601	5	0
Amount at debit of agents	3,449	13	8
Policy stamps in hand	5	10	0
Cash at London and Westminster Bank	3,633	3	1
	£335,239	1	0

Transactions of the year.

Receipts.

		£.	s.	d.
Jan. 1.	Balance of assets from previous year	312,161	9	0
Dec. 31.	Premiums on new policies	5,046	17	2
	Ditto on renewals	34,403	11	7
	Commissions on reassurances	117	19	4
	Interest	16,160	0	5
	Proceeds of reassurances cancelled	1,528	10	8
	Profit on sale of Three per Cent. Reduced Bank Stock	132	6	4
	Transfer fines	5	15	0
		£369,556	9	6

	£.	s.	d.
Amount assured 31st December	1,187,835	13	11
Ditto, previous year	1,132,171	5	2
Increase in year	£55,664	8	9

Payments.

	£.	s.	d.
Claims paid in the course of the year	21,962	14	10
Returns on surrender of policies	885	14	2
Premiums on reassurances	3,164	16	9
Charges, including agencies and medical reports	3,882	4	11
Commission	1,459	10	10
Proprietors' dividends	2,962	7	0
Balance (assets) invested in Government stock, mortgages, and other securities	335,239	1	0
	£369,556	9	6

	£.	s.	d.
Balance of assets at close of year	335,239	1	0
Ditto, previous year	312,161	9	0
Increase in year	£23,077	12	0

Capital and Assurance Account, December 31, 1851.

Dr.	£.	s.	d.
Paid-up capital and additions	66,288	0	5
Reserved fund	7,569	4	11
Provision for outstanding risk on assurances	274,501	7	10
Dividends due	1,263	17	4
	£349,622	10	6

Cr.	£.	s.	d.
Mortgage investments	239,213	10	2
Government securities	33,029	11	1
Reversions and annuity investments	14,821	17	4
Advances on policies and bonds	26,232	4	1
Palladium stock, and India transfer loan	13,455	13	10
Estimated value of house and fixtures	4,601	5	0
Amount at debit of agents	3,558	14	7
Policy stamps in hand	26	0	0
Cash at London and Westminster Bank	14,683	14	5
	£349,622	10	6

Transactions of the year.

	Receipts.	£.	s.	d.
Jan. 1.	Balance of assets from previous year	335,239	1	0
Dec. 31.	Premiums	39,936	14	10
	Commission on reassurances	186	0	11
	Interest	15,652	12	9
	Transfer fines	2	10	0
		£391,016	19	6

	Payments.	£.	s.	d.
Dec. 31.	Claims paid in the course of the year	27,595	8	2
	Returns on surrender of policies	2,563	0	11
	Premiums on reassurances	2,965	0	10
	Charges, including agencies and medical reports	4,031	18	3
	Commission	1,409	9	10
	Proprietors' dividends	2,829	16	0
	Balance (assets) invested in Government stock, mortgages, and other securities, as above	349,622	10	6
		£391,016	19	6

Number of new policies issued in the year, 165. Amount assured, £139,637.
New premiums, £4,062. 2*s.* 7*d.*

	£.	s.	d.
Amount assured 31st December, 1851	1,230,141	0	7
Ditto ditto 1850	1,187,835	13	11
Increase at the close of 1851	£42,305	6	8

	£.	s.	d.
Balance of assets, 1851	349,622	10	6
Ditto, 1850	335,239	1	0
Increase in 1851	£14,383	9	6

Palladium Life Assurance Society.—Report of the Directors to the Twenty-eighth Annual Meeting of Proprietors and Policyholders, 12th May, 1853.—The close of the fourth septennial period having arrived, the directors laid before this meeting a report not only of the ordi-

nary annual business of the Society, but of the result of their proceedings during the last seven years, and presented the following comparison of the septennial period that has just expired, with the three which preceded it:—

	1831.	1838.	1845.	1852.	Increase. 1838.	Increase. 1845.	
Number of policies in force on the 31st December	379	674	798	1,378	295	124	580
Sum assured	£470,502	£712,169	£768,609	£1,280,098	£241,667	£56,440	£511,489
Assets, exclusive of accruing interest	109,003	185,634	272,159	373,625	76,631	86,525	101,466
Reserve	1,341	4,261	7,569	12,377	2,920	3,308	4,808
Proprietors' fund	56,062	60,168	66,185	74,951	4,106	6,017	8,766
Provision for policies, exclusive of claims ascertained	46,741	116,525	190,265	277,556	69,784	73,740	87,291
Divisible Surplus	13,412	24,365	33,073	48,085	10,953	8,708	15,012
viz., to Reserve	1,341	2,436	3,307	4,808	1,095	871	1,501
„ Proprietors	2,414	4,385	5,953	8,655	1,971	1,568	2,702
„ Policyholders	9,656	17,543	23,812	34,621	7,887	6,269	10,809

The result of the recent operations of the Society is, that the directors are able to declare a divisible surplus of £48,085. 4*s*. 6*d*. The surplus will appear the more considerable, when it is stated that it is the result of calculations, not made as heretofore according to the Northampton Tables, which would give a larger amount by nearly £8,000, but by the Experience Tables, which the directors, after careful and mature consideration, saw reason to adopt, with a reservation of £6,000 a year for future bonus and expenses. When it is further considered that in these calculations the interest of money has been taken at the very low rate of 3 per cent., instead of 4 per cent. which many persons would not deem excessive, the proprietors and policyholders will feel that, though the bonus far exceeds that of any previous septennial period, the provision against outstanding risk is considerably greater.

The deed provides that one tenth of the surplus shall be set aside to form a protecting fund. This leaves £43,276. 14*s*. for the proprietors and policyholders; and of this sum one fifth, or £8,655. 6*s*. 10*d*., is allotted to the former, and four fifths, or £34,621. 7*s*. 2*d*., to the latter. The proportion allotted to the proprietors is ordered by the deed to be added to the capital stock of the Society, or the so-called proprietors' fund, which is now increased from its original amount of £59,956 to £74,951, or upwards of 25 per cent. Each proprietor is accordingly held by the deed to have subscribed, beyond his first deposit of £2, the additional sum of 10*s*. per share; and the Office value of the shares is thus raised to £2. 10*s*., being an increase of 25 per cent. on the original subscription. Under these circumstances the directors feel justified in declaring a dividend, free from income tax, at the rate of 2*s*. 6*d*. per share, being an increase of 25 per cent. upon the dividend paid during the last twenty-one years, and 50 per cent. on that paid in the first septennial period. As this dividend is paid out of interest receivable on the funds set apart for the protection of

the proprietors, in contradistinction from those appropriated to meet claims on policies, no loss is thereby entailed on the assurance fund.

Capital and Assurance Account, December 31, 1852.

Dr.	£.	s.	d.
Paid-up capital and additions	74,951	7	3
Reserve fund	12,377	15	5
Provision for outstanding risk on assurances	283,451	0	7
Dividends due	2,845	15	4
	£373,625	18	7

Cr.	£.	s.	d.
Mortgage investments	245,595	8	3
Government securities	31,637	13	4
Annuity investments	15,793	11	3
Advances on policies and bonds	38,179	3	4
India transfer loan, and Palladium stock	13,455	13	10
Estimated value of house and fixtures	4,530	0	0
Amount at debit of agents	1,782	1	5
Premiums otherwise outstanding	1,987	5	9
Cash at London and Westminster Bank, on deposit, and in current account	20,665	1	5
	£373,625	18	7

Transactions of the year.

	Receipts.	£.	s.	d.
Jan. 1.	Balance of assets from previous year	349,622	10	6
Dec. 31.	Premiums	42,997	14	11
	Commission on reassurances	184	7	5
	Interest	14,932	2	9
	Transfer fines	8	0	0
	Profit on valuation of assets	1,154	8	7
	Proceeds of reassurance cancelled	14	16	0
		£408,914	0	2

	Payments.	£.	s.	d.
Dec. 31.	Claims paid in the course of the year	20,399	14	11
	Returns on surrender of policies	1,756	14	8
	Premiums on reassurances	3,399	13	4
	Charges, including agencies and medical reports	5,221	1	0
	Commission	1,596	1	8
	Proprietors' dividends	2,914	16	0
	Balance (assets) invested as above	373,625	18	7
		£408,914	0	2

Number of new policies issued in the year, 167. Amount assured, £136,528. 16*s.* 6*d.*
New premiums, £4,995. 13*s.* 4*d.*

	£.	s.	d.
Amount assured 31st December, 1852	1,280,198	19	7
Ditto ditto 1851	1,230,141	0	7
Increase at the close of 1852, exclusive of bonus to be added after valuation	£50,057	19	0

	£.	s.	d.
Balance of assets, 1852	373,625	18	7
Ditto 1851	349,622	10	6
Increase in 1852	£24,003	8	1

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

Fifth Ordinary Meeting, Session 1853-54.—*Monday, 27th March*, 1854.

JOHN FINLAISON, Esq., President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

Mr. David Chisholm, upon the recommendation of the Council, was elected a Fellow under the provisions of Rule 6 of the Constitution and Laws.

Mr. G. T. Oldfield, duly nominated at the last ordinary meeting, was elected an Associate of the Institute.

Mr. S. Brown read a paper "On a method of classifying life policies, so as to afford a ready means of forming a table of mortality from the experience of the Office."

Sixth Ordinary Meeting, Session 1853-54.—*Monday, 24th April*, 1854.

JOHN FINLAISON, Esq., President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

A paper by Mr. Holmes Ivory was read, "On the value of deferred and reversionary annuities, payable half yearly and quarterly."

Seventh Ordinary Meeting, Session 1853-54.—*Monday, 29th May*, 1854.

CHARLES JELLICOE, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

Eighth Ordinary Meeting, Session 1853-54.—*Monday, 26th June*, 1854.

JOHN FINLAISON, Esq., President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

Mr. J. J. P. Anderson, duly nominated at the last ordinary meeting, was elected an Associate of the Institute.

The following papers were read:—

"On the demonstration of formulæ connected with interest and annuities." By Professor De Morgan.

"Observations upon the sickness and mortality experienced in Friendly Societies." By Mr. Tompkins.

Annual General Meeting.—Saturday, 1st July, 1854.

JOHN FINLAISON, Esq., President, in the Chair.

The circular convening the meeting was read.

The minutes of the last ordinary meeting were read and confirmed.

The following Report of the Council on the progress of the Institute during the past year was read, viz.:—

"REPORT OF THE COUNCIL.

"The events of the session just concluded are perhaps of less general interest than those which have occurred in preceding ones; it is, nevertheless, desirable to give a brief record of them.

"The financial account for the year ending 30th April last exhibits an income of £421. 1*s.*, and an outlay of £419. 3*s.* 7*d.* During the last five years the average of the former has been £417. 17*s.*, and of the latter £436. 17*s.* 5*d.* For the satisfaction of the members, the details of the account will be printed with the Report of the proceedings.

"In the course of the session 22 members have been elected; and 42 have ceased to be members, from deaths and other causes. The number now on the books is 245.

"The Institute will no doubt have witnessed with gratification the addition of M. Adolphe Quetelet's name to those on the honorary list, and also the election of M. Louis Pouget and Dr. Josiah Curtis as Corresponding Members—the former gentleman being editor of the French *Journal des Assurances.*

"The donations to the library continue to be on the usual liberal scale. The contribution by Professor De Morgan of the manuscripts of the late Francis Baily's works is one which calls for particular acknowledgment.

"The following papers have been read during the session, viz:—

1. 'On some points connected with the education of an actuary.' By H. W. Porter, Esq.
2. 'On the rates of mortality prevailing among the male and female lives assured in the Eagle Insurance Company during the 44 years ending 31st December, 1851.' By Charles Jellicoe, Esq.
3. 'On decimal numeration and decimal coinage.' By W. T. Thomson, Esq.
4. 'On a simple plan of classifying the policies of a Life Assurance Company, so as to possess at any time the means of forming a table of the mortality experienced in the Office.' By Samuel Brown, Esq.
5. 'On a method of approximating to the value of deferred and reversionary annuities, payable half yearly and quarterly.' By Holmes Ivory, Esq.
6. 'On the demonstration of formulæ connected with interest and annuities.' By Professor De Morgan.
7. 'Observations upon the sickness and mortality experienced in Friendly Societies.' By Henry Tompkins, Esq.

"All of these will be found in the *Journal* of the Institute, and notices of them also in the *Athenæum* and other periodicals. The paper on decimal coinage gave rise to an important discussion, which resulted in the presentation of a petition to Parliament supporting the plan recommended by the Select Committee of the House of Commons which sat last year on the subject.

"The intentions of the Government with reference to the legislation on Assurance Associations and Friendly Societies are still but imperfectly known. The Council, however, will not fail to watch the proceedings of the legislature on these points with all the care and scrutiny which their importance demands.

"The Council have for a long time been engaged in classifying the data obtained from the Companies contributing their 'extra risk experience'; and Mr. Cheshire, the Assistant Secretary, has devoted much time and attention to the deduction of the preliminary results. It is, however, matter of regret to the Council that they cannot hold out the expectation of any very satisfactory termination to their labours, on account of the insufficiency of the numbers involved in the inquiry. Without blending risks of a wholly incongruous nature, it has been found impracticable to class them into less than six divisions. Of these the largest is that of 'North America'; and the members will at once understand the unsuccessful character of the investigation, when they are informed that in this class the number of deaths at all ages is only 147. Some useful deductions may nevertheless be made, which the Council trust will appear very shortly in the *Journal* of the Institute; meanwhile no efforts will be spared to procure such additional data as will render what is already obtained available for useful purposes.

"At the suggestion of the Scottish members of the Institute, the Council have resolved to recommend an alteration in the mode of voting, the exact nature of which has been detailed in the circular calling the meeting.

"The Council will detain the members no further than to assure them of their anxious desire to promote the successful progress of the Institute, and to maintain its permanent welfare."

Resolved unanimously—

"That the Report be adopted and entered on the minutes.

"That it also be printed and circulated among the members."

An abstract of the receipts and payments for the financial year ended the 30th April last was read.

Resolved unanimously—

"That the financial statement be adopted and entered on the minutes"; also,

"That it be printed with the Report of the Council.

"That the following words be added to Clause 13 of the Constitution and Laws:—'*Any contributing member residing more than 20 miles from London, shall have the right to vote by proxy on elections, or any other question before the Institute of which notice shall have been given by the Council, such proxy to be held by a member qualified to vote, and no member to hold more than five proxies. The proxies to be in a form to be determined by the Council; and to be addressed to the Honorary Secretaries, and delivered at the rooms of the Institute three clear days before the day of meeting.*'"

The election of President, Council, and Officers for the year ensuing was proceeded with.

Mr. Porter and Mr. Norton were appointed scrutineers.

Resolved unanimously—

"That the thanks of the meeting be given to the President, Council, and Officers, for their services during the past year.

"That the thanks of the meeting be given to the Auditors."

The result of the ballot was then declared, by which it appeared that the List of President, Council, and Officers elected was as follows:—

President.—John Finlaison, Esq. (late Government Actuary.)

Vice Presidents.

Edwin James Farren, Esq.	Charles Jellicoe, Esq.
Holmes Ivory, Esq.	Robert Tucker, Esq.

Treasurer.—John Laurence, Esq.

Council.

*James Borthwick, Esq.	Holmes Ivory, Esq.
Samuel Brown, Esq.	Charles Jellicoe, Esq.
Charles James Bunyon, Esq., M.A.	John Laurence, Esq.
*Robert Christie, Esq.	William Lewis, Esq.
*Henry David Dickie, Esq.	Donald Lindsay, Esq.
Percy Matthew Dove, Esq.	*Jeremiah Lodge, Esq., B.A.
Edwin James Farren, Esq.	George Henry Pinckard, Esq.
John Finlaison, Esq.	John Reddish, Esq.
*Gilbert Laurie Finlay, Esq.	*W. Thos. Thomson, Esq., F.R.S.E.
Frederick Hendriks, Esq.	Robert Tucker, Esq.
William Barwick Hodge, Esq.	John Hill Williams, Esq.

Honorary Secretaries.—Samuel Brown, Esq.; John Hill Williams, Esq.

Resolved unanimously—

"That the thanks of the meeting be given to the scrutineers.

"That Mr. Curtis, Mr. Day, and Mr. Scoones, be elected Auditors for the year ensuing."

A vote of thanks was then passed to the President for his conduct in the chair, and the meeting separated.

Those marked thus * are new Members of the Council.

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On some Questions of Combination. *By* PROFESSOR DE MORGAN.

QUESTIONS of combination are so connected with an actuary's business, that no apology is necessary for offering solutions to the notice of the readers of this *Journal.*

When two systems, A and B, of combinations, each having its own individual cases, are so related that each case of A admits of being converted into a case of B, and into one only, by a rule which will not convert any other case of A into that same case of B, it may be said that the system A is *convertibly* within B. It is obvious that if A be convertibly within B, and B convertibly within A, the numbers of cases in the systems A and B are equal.

Let any system of combinations in each case of which m things are selected out of n be denoted by (m, n), which is rather a grammatical than an algebraical abbreviation. The n things combined may be the n numbers $1, 2, 3, \ldots n$. When (m, n) consists of simple combinations without repetition, let their number be denoted by m_n, which therefore represents $n(n-1) \ldots (n-m+1)$ divided by $1 \cdot 2 \cdot 3 \ldots m$. The obvious theorem $m_n = (n-m)_n$ is often used. In writing cases of combination, independent of permutation, a prescribed order is desirable, which may be the numerical, the alphabetical, or any other.

Any combination of m numbers has $m-1$ intervals, and each interval marks either a *break* or a *sequence*. Thus in 4 7 8 9 15 16 we see a break of two, two sequences, a break of five, and a sequence.

Any arrangement or disposition may be converted into any other by a succession of interchanges of contiguous components. Thus ABCD can be converted into CDBA by successively becoming BACD, BCAD, BCDA, CBDA, CDBA; each change being only interchange of neighbouring letters.

Required (m, n), so that in each combination there shall be a break of p or more in every interval. Take $(5, 25)$ and $p=3$ to reason on. Add $p(m-1)$, or 12, to n, or 25, and let A be the system $1, 2, 3 \ldots 25$, and let B be the system $1, 2, 3 \ldots 37$. Then $(5, 25)$ simply, and $(5, 37)$ with breaks of p or more in all the intervals, are each convertibly within the other. Thus—

Case of A	. .	2	3	10	12	13	4	5	6	11	12	21	22	23	24	25
Add	. . . .	0	3	6	9	12	0	3	6	9	12	0	3	6	9	12
Case of B	. .	2	6	16	21	25	4	8	12	20	24	21	25	29	33	37
Case of B	. .	1	8	16	25	33	7	11	15	19	23	18	23	27	31	37
Subtract	. .	0	3	6	9	12	0	3	6	9	12	0	3	6	9	12
Case of A	. .	1	5	10	16	21	7	8	9	10	11	18	20	21	22	25

By turning the A case *abcde* into $a(b+3)(c+6)(d+9)(e+12)$, we get a B case, obtainable by this rule in no other way; and, *vice versâ*, by turning the B case *abcde* into $a(b-3)(c-6)(d-9)(e-12)$. Hence, number in (m, n) with breaks of p or more $= m_{n-(m-1)p}$.

The number in (m, n) without any sequences $(p=1)$ is m_{n-m+1}. Thus $(3, 6)$, without sequences, gives 3_4 or 4 cases: they are, 1 3 5, 1 3 6, 1 4 6, 2 4 6.

The system (m, n) with breaks in *specified* intervals, is of the same number of cases, whatever the specified intervals may be. Take eight with breaks in the third, fifth, and sixth intervals, of which a case is 1 2 3 | 7 8 | 10 | 12 13. There is but one way of removing a break into a contiguous interval. Thus the first break can only be thrown forward by conversion of the case into 1 2 3 4 | 8 | 10 | 12 13, and backward only by conversion into 1 2 | 6 7 8 | 10 | 12 13. Hence two systems (m, n) with k breaks in intervals which have only one difference, and that difference affecting two contiguous intervals, are of the same number of cases. Hence, as before shown, the same is true if the k breaks be in any two sets of assigned intervals.

Required the number in (m, n) with k breaks of p or more in certain assigned intervals, and no other breaks. Let the first k intervals be taken. Then we have $k+1$ numbers without a sequence, followed by $m-k-1$ numbers with nothing but se-

quences, the $(k+1)$th and $(k+2)$th being also in sequence. The $(k+1)$th number cannot be higher than $n-(m-k-1)$; and, the $(k+1)$th being known, all the rest are determined. Hence we have merely to determine $(k+1,\ n-(m-k-1))$ with nothing but breaks of p or more. This, as shown, is $(k+1)_{n-(m-k-1)-kp}$, or $(k+1)_{n-m-(p-1)k+1}$. Thus (5, 15), with the first three intervals breaks of three or more ($m=5$, $n=15$, $k=3$, $p=3$), gives 4_5 or 5 cases. These cases are 1 5 9 13 14, 1 5 9 14 15, 1 5 10 14 15, 1 6 10 14 15, 2 6 10 14 15.

Required the number in (m, n) with k breaks of p or more in *any* intervals, and no other breaks. The number of ways of selecting k out of the intervals is k_{m-1}, the number belonging to the question for each such selection is as above; whence the answer is $k_{m-1} \times (k+1)_{n-m-(p-1)k+1}$.

If we make no limitation as to the character of the breaks, then $p=1$, and the number in (m, n) with breaks in k assigned intervals is $(k+1)_{n-m+1}$; while the number of cases with k breaks in any intervals is $k_{m-1} \times (k+1)_{n-m+1}$.

This last case may be simply verified. Since the cases for all values of k give the whole number of combinations, we have $m_n = 0_{n-1} \cdot 1_{n-m+1} + 1_{n-1} \cdot 2_{n-m+1} + 2_{n-1} \cdot 3_{n-m+1} + \ldots$ The second side is the coefficient of x^{-1} in the product of $0_{m-1} + 1_{m-1} \cdot x + \ldots$ and $0_{n-m+1} + 1_{n-m+1} \cdot x^{-1} + \ldots$ These factors are $(1+x)^{m-1}$ and $(1+x^{-1})^{n-m+1}$, and their product is $(1+x)^n : x^{n-m+1}$. The coefficient of x^{-1} in this is that of x^{n-m} in $(1+x)^n$; which is $(n-m)_n$, or m_n, the first side.

I shall now consider the solution of another class of problems, not, as will appear, essentially different from the first class. The system (m, n), when any attainable amount of repetition is allowed, has a number of cases which is obtained by altering the negative signs of m_n into positive ones. Thus $(4, n)$, allowing repetition, contains $n(n+1)(n+2)(n+3) : 1 \cdot 2 \cdot 3 \cdot 4$ cases. That is, (m, n) with repetition has m_{m+n-1} cases. The following is, I think, as easy a *direct* proof as this problem can admit of:—

Let the case be (12, 20) with repetition, of which I am to show that the system is convertibly within (12, 31) without repetition, and the converse.

Let system A be formed out of 20 letters, a, b, c, &c., with parcels of each for repetition. Let B be formed out of one of each of the 20 letters a, b, c, &c., and the numbers 1, 2, 3 ... 11. Each case in either system contains 12: each A case, all letters; each B case, letters, or letters and numbers.

First, A is convertibly within B. Take any case of A: if without repetition, it is itself a case of B; if with repetition, let it be *bbddddfgggll*. Over all but the first letter write 1, 2, 3 . . . 11. This gives

1 2 3 4 5 6 7 8 9 10 11
b b d d d d f g g g l l

Now discard every repetition, and put the number written over the discarded letter in its place. This gives *b* 1 *d* 3 4 5 *f g* 8 9 *l* 11, a case of B, the only one thus producible from the case of A, and not producible from any other.

Secondly, B is convertibly within A. Take any case of B: if without numbers, it is a case of A; if with numbers, let it be *a b m n p* 2 5 6 7 9 10 11. Begin writing 1, 2, 3, &c., over the letters; but as soon as we thus come to the first number in the case, write it *under* the letter, and continue writing under that same letter as long as the numbers of our case are in sequence. Write the first number we come to which is not in our case *over* the letter in use, and go on writing over the letters until we come to the next number which is in the case; and so on.

1	3	4	8	
a	*b*	*m*	*n*	*p*
	2		5	9
			6	10
			7	11

Now discard the numbers written over the letters, and for each number under any letter substitute that letter. This gives *a b b m n n n n p p p p*, a case of A, the only one producible from the given case of B, and not producible by this rule from any other. Hence each system is convertibly within the other, and the numbers of cases in the two are equal.

The two rules are real inverses of each other. Thus *abbmnnnnpppp* reverts, by the first rule, into *a b* 2 *m n* 5 6 7 *p* 9 10 11, from which it was obtained by the second rule. Nor can the numbers in the second rule ever *outrun* the letters. With *abmnp*, the latest possible number is 5: had the numbers been 5 6 . . . 11, all would have come under *p*; and any alteration of numbers throws us backward, none forward.

In the system with repetition, *m* may exceed *n*: thus (12, 3) with repetition is (12, 14) without repetition, or 12_{14}.

Every problem involving *partition with arrangement* is also a problem of *distribution of undistinguishables*, and a problem of *combination of distinguishables*. For instance, the common

permutation problem of the arrangements of the six numbers, a, b, c, d, e, f, may be represented in either of the following ways:—

Partition, &c. In how many ways can $a+b+\ldots+f$ be divided into the six parts, $a, b, \ldots f$?

Distribution, &c. In how many ways can $a+b+\ldots+f$ undistinguishable counters be distributed among six boxes, so that one box shall receive a, another b, &c.?

Combination, &c. Let every box be marked with a letter, P, Q, &c., and in every case of the last let each counter take the letter of the box it is in. Either of the preceding problems then is the following:—How many combinations exist of the letters P, Q, R, S, T, U, the condition being that some one of these letters shall occur a times, some other b times, &c.?

The answer in each of these three cases is the product $1 . 2 . 3 . 4 . 5 . 6$.

Now let it be asked, In how many arrangements the number m may be divided into n numbers? Write down m units, with $m-1$ intervals. To divide this into n lots in every possible way, we must insert in every possible way $n-1$ marks of partition, not more than one in any interval. The number of ways of doing this is $(n-1)_{m-1}$, which is the answer required. It also solves the following questions—In how many ways can m undistinguishable counters be placed in n boxes, one or more in each box? How many combinations are there of m letters out of n, allowing repetition, and no letter being entirely absent?

Since in the preceding there must be one unit in every place, deduct n units, place them, and distribute the rest: the answer then to the question, in how many ways $m-n$ can be arranged into n numbers *or zeros*, is $(n-1)_{m-1}$. Hence m can be divided into n numbers or zeros in $(n-1)_{m+n-1}$ or m_{m+n-1} ways. And this is also the number of ways in which m undistinguishable counters can be placed in n boxes, so that any boxes may be left empty: and the number of ways in which m combinations can be taken out of n letters, with any repetition; as before shown. This is the most simple proof of the formula for combinations with repetition.

In (m, n) with repetition, supposing $m \gtrless$ or $= pn$, in how many combinations does each of the n letters occur at least p times? This obviously asks the number, still with repetition, in $(m-pn, n)$; and the answer is $(m-pn)_{m-pn+n-1}$.

Again; in how many combinations of m letters do precisely l letters occur, each letter occurring at least p times? Choose certain l letters: the number of ways with these letters is

$(m-pl)_{m-pl+l-1}$. But the choice of l letters can be made in l_n ways: hence $l_n \times (m-pl)_{m-pl+l-1}$ is the number required.

This is the problem of breaks in another form. To make this clearer, let us take a more extensive B case to be converted into one of A. Let the system be (50, 80) with repetition, and let the B case have 16 letters and 34 of the 49 numbers. Let it be $a, b, \ldots p, 1, 2, \ldots 8|12|16, 17, \ldots 22|27, 28|31, 32, \ldots 42|44|47, 48, 49$. We have then

9	10	11	13	14	15	23	24	25	26	29	30	43	45	46	
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*
1			12			16				27		31	44		47
2						17				28		32			48
⋮						⋮						⋮			49
8						22						42			

$$a^9\, b\, c\, d^2\, e\, f\, g^8\, h\, i\, j\, k^3\, l\, m^{18}\, n^2\, o\, p^4$$

using a^9 in abbreviation of nine *a*'s, &c. Every break in the numbers carries us to a new letter; and having *six* breaks in the B case chosen, we have *seven* letters which undergo repetition in the A case deduced. So many ways, then, as we can take 34 out of 49 numbers with six breaks, in so many ways can we take combinations of 41 out of 16 letters, each combination having seven letters, all of which are repeated two or more times. Generally, in so many ways as we can take m out of n numbers with k breaks, in so many ways can we take $m+k+1$ out of $n-m+1$, each combination having $k+1$ letters, and each letter repeated at least twice. This latter, by the last problem, can be done in $(k+1)_{n-m+1} \times (m+k+1-2k-2)_{m+k+1-2k-2+k+1-1}$ ways, or $(k+1)_{n-m+1} \times (m-k-1)_{m-1}$, or $k_{m-1} \times (k+1)_{n-m+1}$ ways. And this is the result already obtained in the former problem.

This connection between problems involving combination with breaks and problems involving combination with repetitions might be carried much further, but what is here given will be sufficient to indicate the method.

There is no distinction more marked than that which exists between problems of partition in which different orders count as different ways, and problems in which all the arrangements of one partition are supposed undistinguishable. We may call these last *ordinate* partitions, choosing the order of magnitude as that in which to write down the parts. Thus the ordinate partitions of 5 into three numbers are only 3 1 1, 2 2 1; while among the inordinate partitions are also found 1, 3, 1, 1, 2, 2, &c.

I treated the question of ordinate partition of simple numbers

in the *Cambridge Math. Jo.*, vol. iv., p. 87, carrying the results up to the ordinate partition of the number x into *four*. Sir John Herschel subsequently turned his attention to the same subject, in a paper in the *Phil. Trans.* for 1850. This paper contains a much fuller discussion of the question (in the proportion of 24 pages quarto to three pages octavo), and a complete exhibition of the form of the final result, and the general formula for partition into five. To give the actuary some idea of his good fortune in having only to deal with questions connected with *inordinate* partition, I will copy my own result for ordinate quadripartition. Let β and γ stand for $\frac{1}{2}(-1+\sqrt{-3})$ and $\frac{1}{2}(-1-\sqrt{-3})$: the number of ordinate partitions of x into four is the 864th part of

$$6x^3+18x^2-27x-39+(27x+27)(-1)^x+32(\beta^{x-1}+\gamma^{x-1}-\beta^x-\gamma^x)$$
$$+54\left\{(\sqrt{-1})^x+(-\sqrt{-1})^x\right\}$$

No pure algebraical expression of a more simple character can be substituted for the above, but an arithmetical description of an easier working character can be given. The last of the following results is altered from Sir John Herschel's paper. The number of ordinate partitions of x is as follows:—

Two parts. The highest integer which does not exceed $\frac{1}{2}x$.

Three parts. The integer *nearest* (above or below) to $\frac{x^2}{12}$.

Four parts. The integer nearest to $\frac{x^3+3x^2}{144}$ or $\frac{x^3+3x^2-9x}{144}$, according as x is even or odd.

Five parts. The integer nearest to $\frac{x^4+10x^3+10x^2-120x}{2880}$ or $\frac{x^4+10x^3+10x^2-30x}{2880}$, according as x is even or odd.

This last is also the solution of the following problem:— How many combinations are there of x letters out of 5; each letter occurring once at least, and no one occurring oftener than the letter preceding?

On the Relation which should obtain between the Amount assured upon Lives and the Sum reserved at the Expiration of Given Terms to meet it. By CHAS. JELLICOE, *one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute, 27th November, 1854, and ordered by the Council to be printed.]

THE method of valuation once so universal—viz., that by means of tables of annuities involving the rates of premium charged—is now, I believe, admitted on all hands to be erroneous, and is, so far as I can learn, pretty generally abandoned. I have on more than one occasion endeavoured to demonstrate the fallaciousness of that method, and it is therefore needless again to draw attention to the peculiar consequences resulting from it; more especially as the great majority of the Companies now estimate their liabilities, as most actuaries agree that they should do, with the aid of tables based upon rates of interest and mortality approximating as closely as possible to those which observation and experience have shown to be the actually prevailing ones.

There is still, nevertheless, an important difference in the way in which this more accurate method is carried out. It is obvious that the rates of premium charged have no necessary connection with it, and that they may be altogether neglected; and in that case there can be no anticipation of the marginal addition made to the true premium for such contingencies as have to be provided for over and above the sum assured: *provided always, that the same elements are used in the valuation as those from which the mathematical or true premium was originally deduced.* For it is evident, that if in a valuation it be found necessary to depart from these elements and to adopt a lower rate of interest or higher rate of mortality, then the addition in question may be anticipated to any extent;* and if the fact of such departure be withheld, the Society so acting may take credit for the entire exclusion from its calculations of the marginal addition to its premiums, when in fact it has absorbed very nearly the whole of it. This may be done, of course, unwittingly as well as wilfully; and it is partly on account of the facility which is thus given to the production of delusive statements, and partly on account of the impropriety (as it appears to me) of

* For let the premium charged, as originally constructed, consist of $p' + \phi$, then it is evident that the adoption of either or both the alternatives mentioned will have the effect of increasing p', say, to $p' + \pi$; and ϕ is consequently diminished to $\phi - \pi$, where π may be any quantity greater or less than ϕ.

omitting all mention of so important a portion of the estimate as these additions make, that I have always advocated the valuation of them, and the introduction of their valde into the account as one of its principal items. On the other hand, I am aware that some inconvenience attaches to the introduction of this value, especially in the early stages of an Assurance Company, from the circumstance of the surplus being thus rendered in appearance very large, and presenting, as is thought, a temptation to all concerned to encroach upon it. But I cannot look upon this as of any importance at the present day, when the nature of life assurance business is so much better understood than formerly; and I must still maintain, that although on this score some trifling drawback may attend the method of valuation in question, it is nevertheless by far the best that can be adopted, simply because it is the most comprehensive and at the same time the most explicit, and because it more than any other precludes all possibility of subterfuge and evasion. The objection, however, above alluded to, is so strenuously insisted upon and is so frequently reiterated, that it becomes desirable to make some effort to remove it, and to see whether the nature of the thing cannot be placed in so simple a point of view as to enable the most casual observer to form a tolerably accurate judgment himself respecting it, and to arrive at nearly correct conclusions, let the state of affairs in any given instance be presented to him in what manner it may.

It is with this object that I now propose to show the relation which should obtain between the amount assured and the sum reserved to meet it; believing that, so far as it can be depended upon, it will serve to indicate the true state of affairs as simply and directly as it is possible under all the circumstances for the thing to be done.

In every life assurance contract the person assured undertakes to pay in effect two premiums—the one to provide for the sum assured merely, and the other to create a fund for expenses and extra contingencies. The security in either case is precisely the same, and we are therefore entirely justified in applying the same principles of valuation to each. It will be convenient to keep the two distinct; and for precision's sake I will denote the value of the true premium by the expression $p'_x(1+A'_x)$, and that of the extra premium (as we may for our present purpose call it) by $\phi_x(1+A'_x)$; and I will proceed to show what portion of these must be reserved at the end of one, three, five, and seven years, in respect of assurances effected simultaneously and upon the ordinary terms,

at ages 30, 35, 40, 45, and 50, assuming that the premiums are just due at the time of each valuation, that there are no new entrants, and that each assurance continues in force throughout the term. These conditions, it is true, do not precisely obtain in practice; but I think it will be conceded in the sequel that the inferences based upon them are nevertheless fairly deduced. Taking then the "experience" for the rate of mortality, and four per cent. for the rate of interest, the following table will exhibit the portion of the extra premium to be reserved in each case, and its mean value per cent. of the sum assured at the expiration of the terms above specified.

Age when Assurance effected.	Values of $\phi_{x+n}(1+A'_{x+n})$.			
	At end of 1 Year.	At end of 3 Years.	At end of 5 Years.	At end of 7 Years.
30	15·118	14·986	14·860	14·764
35	14·832	14·725	14·514	14·276
40	14·400	14·214	13·983	13·718
45	13·861	13·546	13·322	12·995
50	13·180	12·799	12·506	12·178
	71·391	70·270	69·185	67·931
Mean value per cent. of Sum assured ..	14·278	14·054	13·837	13·586

It will be seen by this table that the value of this portion of the reserve is never less than 12 per cent. of the sum assured, and that the least mean value is upwards of 13½ per cent. on that item. As years pass by, this mean value will no doubt diminish as respects any one class of entrants; but since in practice fresh ones are continually being added, whilst many of those first entering disappear, so that in some of the oldest Societies the average duration of the assurances has not hitherto exceeded eight or nine years, we may fairly conclude that the rate of reserve on this score, as above shown, must decline very slowly, and may often be nearly stationary for many years in succession.

What has been said applies with equal force, but inversely, to the other part of the liability, viz., that denoted by the expression $(p'_{x+n}-p'_x)(1+A'_{x+n})$, or the portion of the true premium to be set aside on account of the sum assured. Here, as will be seen by the following table, the values increase from year to year, and, as regards any one set of assurances, with great rapidity; but, for the reasons already adduced, it is probable that the mean

rate of 11·2 per cent. therein shown to prevail does not augment materially till an Office has been many years in operation. Much will of course depend on the ages at which the assurances are effected; but it will, I think, be admitted, that both this and the preceding table represent with sufficient accuracy the true state of the case in that respect.

Age when Assurance effected.	Values of $(p'_{x+n}-p'_x)(1+A'_{x+n})$.			
	At end of 1 Year.	At end of 3 Years.	At end of 5 Years.	At end of 7 Years.
30	·931	2·891	4·971	7·181
35	1·152	3·537	6·130	8·837
40	1·443	4·457	7·668	11·026
45	1·809	5·504	9·348	13·292
50	2·161	6·572	11·070	15·665
	7·496	22·961	39·187	56·051
Mean value per cent. of Sum assured ..	1·499	4·592	7·837	11·210

From what has preceded, it appears that whilst one portion of the liability of an Assurance Company is augmenting, the other has a tendency to decrease; and that, although the augmenting rate is the more rapid of the two, the consequent increase in the liability is retarded by the slow rate at which the average duration of the assurances progresses: so that there is thus reason to conclude that in many Assurance Societies the liability per cent. becomes, after a few years, nearly stationary. The following table, exhibiting the sum of the mean values given in those preceding it, will serve to illustrate this, on the supposition that the average duration of the assurances is just seven years, and that such duration has become stationary.

At end of	Values of $(p'_{x+n}+\phi_{x+n}-p'_x)(1+A'_{x+n})$.		
	Reserve to meet sum assured.	Reserve to meet extra contingencies.	Total.
1 Year	1·499	14·278	15·777
3 Years	4·592	14·054	18·646
5 „	7·837	13·837	21·674
7 „	11·210	13·586	24·796

By this statement it will be seen that the reserve to be made immediately on the establishment of a Company, when the value

of the premium charged is taken credit for, is very considerable, little less than 16 per cent. of the sum assured; and that this proportion increases slowly with the average duration of the policies, attaining nearly 25 per cent., or about one fourth of the total sum assured, when these last shall have become, one with another, about seven years old. The longest term hitherto reached in many of the oldest Societies is, I believe, only eight years and a half; so that the last-mentioned rate is probably not far, under ordinary circumstances, from a maximum one. It is possible, however, that assurances on young lives are more frequently dropped than those effected at more advanced ages; and if so, some allowance will have to be made on that score.

In these calculations, as I have said, the "experience" mortality has been assumed, and four per cent. as the rate of interest. If other elements be taken, the results are not so widely different as might be imagined. Thus, at three per cent. and with the "Carlisle" mortality, the table would present the following rates:—*

At end of	Values of $(p'_{x+n} + \phi_{x+n} - p'_x)(1 + \Delta'_{x+n})$		
	Reserve to meet sum assured.	Reserve to meet extra contingencies.	Total.
1 Year.....	1·677	13·002	14·679
3 Years	5·115	12·858	17·973
5 „	8·601	12·869	21·470
7 „	12·078	13·071	25·149

A little consideration makes the seeming discrepancies in these tables sufficiently intelligible. It will be seen that the higher the rate of interest realized, the less the rate of premium required for the sum assured; and the more this last rate is reduced, the more remains for extra contingencies. The total liability in either case is often nearly the same in amount, but it is of a different quality.

What has now been said will suffice, I think, to show that there is no foundation for the somewhat prevalent notion that the proportionate liabilities of an Assurance Company, as exhibited under this aspect, are trifling at its commencement and increase only materially after the lapse of many years: I think it will be seen that such is by no means the case. The absolute liability, of course, augments rapidly as the transactions entered into are

* These are the mean values in respect of assurances effected at each quinquennial interval from 25 to 55 inclusive.

multiplied; but as regards each one hundred pounds assured, it plain that where credit is taken for the value of the full premium, at least fifteen pounds should be reserved at the very commencement, and that this proportion must gradually increase till it becomes nearly double—when, if we may judge by our past experience, there is reason to think that a pause takes place, and that the proportion in question approximates to its maximum value. This, however, I need scarcely repeat, must depend, in any given instance, entirely on the facts of the case.

Of course what is here said applies exclusively to that system of valuation which regards the total liability, and which takes into account the value of the extra as well as that of the true premium. Where the former is disregarded, and the latter alone included in the estimate, the reserve should evidently be in correspondence with the per centages exhibited in the first of the three columns of the table at page 103, or of that at page 104. It is in this shape that the results of a valuation by the Northampton Table and other similarly constructed ones have been usually made to appear; and it may be remarked, that the results so obtained are true only when the rate of mortality and rate of interest actually prevailing correspond with those of the adopted table—in which case it is clear that the whole of the premium charged is absorbed, and that there is no provision for extra contingencies at all.

From these considerations, then, it appears that the reserve made by Assurance Societies may be and is exhibited in several ways; and since it is of some importance to make the due distinction between them, I will briefly point out some of them, and, for the sake of greater perspicuity, give them a general expression. We may thus represent the first by

Dr.	Cr.
$p_{x+n}(1+A_{x+n})$	$p_x(1+A_{x+n})$,

where p_x represents the premium charged, and A_x the annuity corresponding with it, and where the results would consequently be those obtained by the use of tables formed as the Northampton and some few others are. For the second mode we may take the formula

Dr.	Cr.
$p'_{x+n}(1+A'_{x+n})$	$p'_x(1+A'_{x+n})$

in which the true premium alone is involved, and the additions altogether disregarded. Here the rates of reserve would correspond with those shown in the first column of per centages, at pages 103 and 104. For the third may be selected the expression

DR.	CR.
$(p' \pm \pi)_{x+n}(1+A'_{x+n})$	$(p' \pm \pi)_x(1+A')_{x+n}$

which will serve to indicate the results when the premium originally adopted as the true one has been departed from, and the value of the additions enhanced or encroached upon, whether disregarded or otherwise, accordingly as the sign + or − is really applicable. A fourth method may be exhibited by the formula

DR.	CR.
$(p' + \phi)_{x+n}(1+A'_{x+n})$	$p'_x(1+A'_{x+n})$

The rate of reserve will here be similar to that in the last column of the tables at pages 103 and 104, and will comprise the value of the future additions as well as that of the liability under the sums assured. Lastly, a mode adopted by some few Societies whose manner of distributing the surplus is peculiar to them may be denoted by the expressions

DR.	CR.
$p'_{x+n}(1+A'_{x+n})$	$(p' + \phi)_x(1+A'_{x+n})$

where, as will be observed, the true value of the whole premium charged is set against the true value of the sums assured, and where, in consequence, the amount reserved appears to be out of all proportion to what it should be. It is proper however to mention, that in the Societies adopting this arrangement the value of the $\phi_x(1+A'_{x+n})$ is, in fact, reserved, and forms the fund out of which the reductions in the future premiums are provided for: that is to say, the additions originally made to the true premium being found to be worth so much, and that sum being forthcoming over and above what is required for other purposes, it would be argued that such additions might be dispensed with, and the true premium alone accepted on account of the future payments to be made.

If a Society could be conducted without expenses of any kind, it would be reasonable to expect *à priori* that the true premiums alone would suffice; or, in other words, that the reductions allowed would be exactly equal to the additions originally made. As however expenses must be incurred, and, under ordinary circumstances, to a considerable amount, such a result is hardly to be looked for —unless, indeed, the funds of the Society are augmented from other sources than the usual ones, or unless the premium assumed in the outset to be the true one be ascertained by subsequent investigations to be more than sufficient for the purpose, and a reduced rate be substituted in after valuations.

On the Construction of Survivorship Assurance Tables.

By PETER GRAY, ESQ., F.R.A.S.

[Read before the Institute, 26th April, 1852, and ordered by the Council to be printed.]

IN a Postscript to the introductory matter of the *Assurance and Annuity Tables*, compiled and published by Messrs. Smith, Orchard, and myself in April, 1851 (and which Postscript was written by me), it is stated that I had "devised a new table, which, while answering all, and more than all, the purposes of Tables V. and VI. [which show the single and annual premiums for a survivorship assurance of £1 for every combination of two ages], by means of a *single* value for each pair of ages, and thus occupying little more space than one of them, would also be very much less laborious in its construction than the former of those tables. The new tables would, in fact, consist of series of columns supplementary to Mr. Jones's *Commutation Tables for Two Lives*, and possessing all the properties, *mutatis mutandis*, with respect to the formation of temporary and deferred, as well as present, whole life benefits and payments, that give to tables of the form of those mentioned their great value." The form of the table to which reference is here made suggested itself to me while occupied in superintending the printing of the work which contained the foregoing announcement. Having, however, had just then, as I thought, quite enough to do with survivorship assurances for a time, I contented myself with making the announcement cited, and communicating my ideas in a general way to my coadjutors, Messrs. Smith and Orchard.

My attention has recently been recalled to the subject—first, by a suggestion in the January Number of the *Assurance Magazine* that I ought to make my method public, and, secondly, by the reading of a paper by Mr. David Chisholm, at the meeting of the Institute of Actuaries on the 23rd of February last, in which a method of constructing the same tables that I had in view is very fully described and exemplified. Mr. Chisholm's paper appeared to me to be an exceedingly able one; and being occupied with a subject to which I had devoted great attention, I listened to the reading of it with much interest. In particular, I was gratified by the recognition, on the part of a gentleman of so much experience, of the value of, and the desirableness of possessing, tables of the kind I was the first to suggest.

Mr. Chisholm's method of construction differs from mine. First, his method is essentially logarithmic, while mine admits of being applied both in numbers and in logarithms; and of the two modes I think the former is the preferable. Again, Mr. C.'s method is not *continuous*—that is, each tabular value is formed independently of other tabular values; and as a consequence, to ensure accuracy, either the whole work must be subjected to a rigid scrutiny, or the entire computation must be performed in duplicate. My method, on the other hand, in both its applications, *is* continuous. Each succeeding value is deduced, by the simplest conceivable operation, from a preceding value, so that verification at a few points suffices to verify the whole. It seems to me, therefore, that it may not be unacceptable if I proceed now to explain and exemplify the method I had in view. Those who may be disposed to undertake the improving exercise of forming the tables in question, will at all events have thus a choice of methods presented for their adoption.

It is known that

$$\tfrac{1}{2}(p_{x\,.\,n-1}-p_{x\,.\,n})(p_{y\,.\,n-1}+p_{y\,.\,n})v^n,$$

which may be also written thus,

$$\frac{\tfrac{1}{2}(l_{x+n-1}-l_{x+n})(l_{y+n-1}+l_{y+n})v^n}{l_{x\,.\,y}},$$

denotes the present value of £1 to be received at the end of n years, provided (x) shall have died in the nth year, and (y) have been alive at the instant of (x)'s death. Now to adapt this expression to Mr. Jones's tables, we must introduce in both numerator and denominator such a power of v as will make the latter equal to $D_{x.y}$. This will be either v^x or v^y, according as x or y is the greater. We shall thus have for the value of the contingency under consideration in the two cases—

$$\underset{(x\,7\,y)}{\frac{\tfrac{1}{2}(l_{x+n-1}-l_{x+n})(l_{y+n-1}+l_{y+n})v^{x+n}}{D_{x.y}}};\quad \underset{(x\,\angle\,y)}{\frac{\tfrac{1}{2}(l_{x+n-1}-l_{x+n})(l_{y+n-1}+l_{y+n})v^{y+n}}{D_{x.y}}}.$$

The denominators of these expressions, as they now stand, being already formed (in Jones's tables), it is obvious that we have now to occupy ourselves with the formation of the numerators only, for the requisite number of cases of variation of x, y, and n.

Looking, then, at the numerators, involving three variables, x, y, and n, it might at first sight seem to be requisite to form

them for every value of each of these quantities. But, in the first place, since of the two quantities x and y, neither takes any values different from those taken by the other, and since in the first expression each value of x is combined only with those values of y that do not exceed it, and in the second each value of y is combined only with those values of x that do not exceed it, it follows that the number of formations is thus at once reduced one half. And, in the second place, since, *owing to the peculiar structure of the Commutation Tables*, in both expressions the same effect is produced by any variation given to n as by an *equal* variation given to both x and y, it further follows, that it will suffice to compute the two expressions for all the values of x and y, and for a single value of n.

It is most convenient to make $n=1$, in which case the numerators of the two expressions (which will then denote the value of the contingencies in respect of the first year) will respectively become

$$\underset{(x \gtrdot y)}{\tfrac{1}{2}(l_x - l_{x+1})(l_y + l_{y+1})v^{x+1}}, \text{ and } \underset{(x \lessdot y)}{\tfrac{1}{2}(l_x - l_{x+1})(l_y + l_{y+1})v^{y+1}}.$$

And, in accordance with what has just been said, the values of these expressions that will have to be computed, are (supposing the Carlisle Table to be used), of the first, those in which x receives every value from 104 to 0, and y every value from x to 0; and, of the second, those in which y receives every value from 104 to 0, and x every value from y to 0. I proceed now to inquire how these values may be most advantageously formed.

The expressions deduced above will be simplified, if for $\frac{1}{2}(l_x - l_{x+1})$ and $l_y + l_{y+1}$, we write d_x and s_y respectively. It might seem preferable to combine the constant factor $\frac{1}{2}$ with the quantity $l_y + l_{y+1}$, inasmuch as d_x and s_y would then denote, the former the number dying between the ages x and $x+1$, and the latter the number living at age $y+\frac{1}{2}$. But the other arrangement will be found to be the more advantageous, since, the values of $l_y + l_{y+1}$ being generally very much greater than those of $l_x - l_{x+1}$, we are less inconvenienced, in the case of this latter function, by the additional figure which arises from the division of an odd number by 2. The expressions with which we have to deal will therefore now be denoted as follows:—

$$\underset{(x \gtrdot y)}{d_x s_y v^{x+1}}, \text{ and } \underset{(x \lessdot y)}{d_x s_y v^{y+1}}.$$

The following table, formed from the Carlisle Table of Mortality by a process too obvious to need explanation, contains all the terms of the series s_y and d_x, with their differences, for which we shall have occasion in the subsequent examples.

x & y	s_y	Δs_y	d_x	Δd_x
104	1	3	·5	·5
103	4	4	1·0	·0
102	8	4	1·0	·0
101	12	4	1·0	·0
100	16	4	1·0	·0
99	20	5	1·0	·5
98	25	7	1·5	·5
97	32	9	2·0	·5
96	41	12	2·5	1·0
95	53	17	3·5	1·5
94	70	24	5·0	2·0
93	94	35	7·0	3·5
92	129	51	10·5	4·5
91	180	67	15·0	3·5
90	247	76	18·5	1·0
89	323	90	19·5	6·0
88	413	115	25·5	6·5
87	528	135	32·0	3·5
86	663	149	35·5	3·5
85	812	162	39·0	3·0
84	974	178	42·0	5·0
83	1152	196	47·0	4·0
82	1348		51·0	

At this point the two methods which I propose to develope part company, and must henceforth be treated separately. I therefore commence with—

I.—*The Construction in Numbers.*

The process is to be continuous. To see, therefore, how succeeding values can be most readily deduced from each other, taking first the case $x \nearrow y$, write down in order a few of the leading terms of the series belonging to adjoining differences of age, thus:—

$x-y=0$	$x-y=1$	$x-y=2$	$x-y=3$
$d_{104}s_{104}v^{105}$	$d_{104}s_{103}v^{105}$	$d_{104}s_{102}v^{105}$	$d_{104}s_{101}v^{105}$
$d_{103}s_{103}v^{104}$	$d_{103}s_{102}v^{104}$	$d_{103}s_{101}v^{104}$	$d_{103}s_{100}v^{104}$
$d_{102}s_{102}v^{103}$	$d_{102}s_{101}v^{103}$	$d_{102}s_{100}v^{103}$	$d_{102}s_{99}\,v^{103}$

and so on. Now what we have to ascertain is, how we can most easily pass from any one term to an adjoining term. Examine, then, the succession of values in the several *columns*. We see that in passing from term to term all the three factors vary; the

addends consequently would obviously be characterized by a degree of complexity that would quite unfit them for our purpose. But examine the succession in the several *rows*. Here we see that only one factor varies in passing from term to term; and that the terms consequently are nothing else than a series of multiples by this varying factor of the constant product of the two remaining factors. Farther, the differences of these terms will also be a series of multiples of the same constant product; so that, if this last series be formed, we shall be able, by continuous addition of its terms to the initial value, namely, that corresponding to $x-y=0$, to form, with great facility, the terms belonging to each row.

To make this more plain, denote for a moment $d_{104}v^{105}$, $d_{103}v^{104}$, $d_{102}v^{103}$, &c., by P, Q, R, &c., respectively; then will the above-written terms take the following form:—

$x-y=0$	$x-y=1$	$x-y=2$	$x-y=3$
Ps_{104}	Ps_{103}	Ps_{102}	Ps_{101}
Qs_{103}	Qs_{102}	Qs_{101}	Qs_{100}
Rs_{102}	Rs_{101}	Rs_{100}	Rs_{99}

and so on. And their differences, taken from the several rows, will be

$P(s_{103}-s_{104})$,	$P(s_{102}-s_{103})$,	$P(s_{101}-s_{102})$,
$Q(s_{102}-s_{103})$,	$Q(s_{101}-s_{102})$,	$Q(s_{100}-s_{101})$,
$R(s_{101}-s_{102})$,	$R(s_{100}-s_{101})$,	$R(s_{99}-s_{100})$;

or,

$P\Delta s_{104}$,	$P\Delta s_{103}$,	$P\Delta s_{102}$,
$Q\Delta s_{103}$,	$Q\Delta s_{102}$,	$Q\Delta s_{101}$,
$R\Delta s_{102}$,	$R\Delta s_{101}$,	$R\Delta s_{100}$.

It thus appears that the values occupying the first row in the foregoing scheme will be formed by the continuous addition to the initial term, Ps_{104}, of the products of the quantity P (or $d_{104}v^{105}$), by the successive terms of the series Δs_x, which is one of those of which a portion is tabulated on page 110. In like manner, the values occupying the second row will be formed by the continuous addition to *its* initial term, Qs_{103}, of the products of the quantity Q (or $d_{103}v^{104}$), by the successive terms of the same series Δs_x; and so also for the third and following rows.

A like property holds with respect to the function belonging to the case $x \angle y$, as may be very briefly shown.

$x \nearrow y$

$x-y$	s_y	$x=104$	$x=103$	$x=102$	$x=101$	$x=100$
0	1	·0224420	·1849220	·3809394	·5885513	·8082771
	3	673260	·1849220	·1904697	·1961838	·2020693
1	4	·0897680	·3698440	·5714091	·7847351	1·0103464
	4	897680	·1849220	·1904697	·1961838	·2525866
2	8	·1795360	·5547660	·7618788	·9809189	1·2629330
	4	897680	·1849220	·1904697	·2452297	·3536212
3	12	·2693040	·7396880	·9523485	1·2261486	1·6165542
	4	897680	·1849220	·2380871	·3433216	·4546559
4	16	·3590720	·9246100	1·1904356	1·5694702	2·0712101
	4	897680	·2311525	·3333219	·4414135	·6062078
5	20	·4488400	1·1557625	1·5237575	2·0108837	2·6774179
	5	·1122100	·3236135	·4285568	·5885513	·8587944
6	25	·5610500	1·4793760	1·9523143	2·5994350	3·5362123
	7	·1570939	·4160745	·5714090	·8337810	1·2124157
7	32	·7181439	1·8954505	2·5237233	3·4332160	4·7486280
	9	·2019779	·5547061	·8094961	1·1771026	1·7681062
8	41	·9201218	2·4502166	3·3332194	4·6103186	6·5167342
	12	·2693039	·7859186	1·1428181	1·7166080	2·5763833
9	53	1·1894257	3·2361352	4·4760375	6·3269266	9·0931175
	17	·3815139	1·1095321	1·6666097	2·5013430	3·3846604
10	70	1·5709396	4·3456673	6·1426472	8·8282696	12·4777779
	24	·5386078	1·6180677	2·4284884	3·2860781	3·8393163
11	94	2·1095474	5·9637350	8·5711356	12 1143477	16 3170942
	35	·7854697	2·3577557	3·1903671	3·7274916	4·5465588
12	129	2·8950171	8·3214907	11·7615027	15 8418393	20·8636530
	51	1·1445416	3·0974438	3 6189239	4·4141348	5·8094918
13	180	4·0395587	11·4189345	15·3804266	20·2559741	26·6731448

$x \angle y$

$y-x$	d_x	$y=104$	$y=103$	$y=102$	$y=101$	$y=100$
0	·5	·0224420	·1849220	·3809394	·5885513	·8082771
	·5	224420	0	0	0	0
1	1·0	0448840	·1849220	·3809394	·5885513	·8082771
	·0	0	0	0	0	·4041386
2	1·0	·0448840	·1849220	·3809394	·5885513	1·2124157
	·0	0	0	0	·2942757	·4041386
3	1·0	·0448840	·1849220	·3809394	·8828270	1·6165543
	·0	0	0	·1904697	·2942757	·4041386
4	1·0	·0448840	·1849220	·5714091	1·1771027	2·0206929
	·0	0	924610	·1904697	·2942757	·8082771
5	1·0	·0448840	·2773830	·7618788	1·4713784	2·8289700
	·5	224420	924610	·1904697	·5885513	1·2124157
6	1·5	·0673260	·3698440	·9523485	2·0599297	4·0413857
	·5	224420	924610	·3809394	·8828270	1·6165543
7	2·0	·0897680	·4623050	1 3332879	2·9427567	5·6579400
	·5	224420	·1849220	·5714090	1·1771026	2·8289699
8	2·5	·1122100	·6472270	1·9046969	4·1198593	8·4869099
	1·0	448840	·2773830	·7618787	2·0599296	3·6372471
9	3·5	·1570940	·9246100	2·6665756	6·1797889	12·1241570
	1·5	673260	·3698440	1·3332877	2·6484809	2·8289699
10	5·0	·2244200	1·2944540	3·9998633	8·8282698	14·9531269
	2·0	897680	·6472271	1·7142271	2·0599296	·8082771
11	7·0	·3141880	1·9416811	5·7140904	10·8881994	15·7614040
	3·5	·1570940	·8321491	1·3332877	·5885513	4·8496627
12	10·5	·4712820	2·7738302	7·0473781	11·4767507	20·6110667
	4·5	·2019779	·6472271	·3809394	3·5313078	5·2538013
13	15·0	·6732599	3·4210573	7·4283175	15·0080585	25·8648680

$x \nearrow y$

$x-y$	s_y	$x=99$	$x=98$	$x=97$	$x=96$	$x=95$
0	20	1·0406568	2·0097684	3·5329050	5·8279131	10·8635143
	5	·2601642	·5627352	·9936295	1·7057307	3·4845235
1	25	1·3008210	2·5725036	4·5265345	7·5336438	14·3480378
	7	·3642299	·7235166	1·3248394	2·4164518	4·9193273
2	32	1·6650509	3·2960202	5·8513739	9·9500956	19·2673651
	9	·4682956	·9646889	1·8768558	3·4114613	7·1740189
3	41	2·1333465	4·2607091	7·7282297	13·3615569	26·4413840
	12	·6243941	1·3666425	2·6496787	4·9750478	10·4535704
4	53	2·7577406	5·6273516	10·3779084	18·3366047	36·8949544
	17	·8845583	1·9293777	3·8641148	7·2493553	13·7331219
5	70	3·6422989	7·5567293	14·2420232	25·5859600	50·6280763
	24	1·2487882	2·8136758	5·6305673	9·5236629	15·5778696
6	94	4·8910871	10·3704051	19·8725905	35·1096229	66·2059459
	35	1·8211494	4·0999276	7·3970198	10·8029609	18·4474772
7	129	6·7122365	14·4703327	27·2696103	45·9125838	84·6534231
	51	2·6536748	5·3861794	8·3906493	12·7929800	23·5717764
8	180	9·3659113	19·8565121	35·6602596	58·7055638	108·2251995
	67	3·4862003	6 1096961	9·9362952	16·3465856	27·6712158
9	247	12·8521116	25·9682082	45·5965548	75·0521494	135·8964153
	76	3·9544958	7·2351664	12·6963772	19·1894700	30·5408234
10	323	16·8066074	33·2013746	58·2929320	94·2416194	166·4372387
	90	4·6829556	9·2449348	14·9044428	21·1794892	33·2054589
11	413	21·4895630	42·4463094	73·1973748	115·4211086	199·6426976
	115	5·9837766	10·8527496	16·4500887	23·0273640	36·4850104
12	528	27·4733396	53·2990590	89·6474635	138·4484726	236·1277080
	135	7·0244334	11·9782199	17·8853313	25·3016716	40·1745059
13	663	34·4977730	65·2772789	107·5327948	163·7501442	276·3022139

$x \angle y$

$y-x$	d_x	$y=99$	$y=98$	$y=97$	$y=96$	$y=95$
0	1·0	1·0406568	2·0097684	3·5329050	5·8279131	10·8635143
	·5	·5203284	·6699228	·8832262	2·3311652	4·6557919
1	1·5	1·5609852	2 6796912	4·4161312	8·1590783	15·5193062
	·5	·5203284	·6699228	1·7664525	3·4967479	6·2077225
2	2·0	2·0813136	3·3496140	6·1825837	11·6558262	21·7270287
	·5	·5203284	1·3398456	2·6496787	4·6623305	10 8635144
3	2·5	2·6016420	4 6894596	8·8322624	16·3181567	32 5905431
	1·0	1·0406568	2·0097684	3 5329050	8 1590784	13·9673756
4	3·5	3·6422988	6·6992280	12·3651674	24·4772351	46·5579187
	1·5	1·5609852	2·6796913	6·1825837	10·4902436	10·8635144
5	5·0	5·2032840	9·3789193	18·5477511	34·9674787	57·4214331
	2·0	2·0813136	4·6894597	7·9490361	8·1590784	3·1038612
6	7·0	7·2845976	14·0683790	26·4967872	43·1265571	60·5252943
	3·5	3·6422988	6·0293053	6·1825837	2·3311652	18·6231675
7	10·5	10 9268964	20·0976843	32·6793709	45·4577223	79·1484618
	4·5	4·6829556	4·6894597	1·7664525	13·9869915	20·1750981
8	15·0	15·6098520	24·7871440	34·4458234	59·4447138	99·3235599
	3·5	3·6422988	1·3398456	10·5987148	15·1525741	10·8635144
9	18·5	19·2521508	26 1269896	45·0445382	74·5972879	110·1870743
	1·0	1·0406568	8·0390738	11·4819411	8·1590784	10·8635144
10	19·5	20·2928076	34·1660634	56·5264793	82·7563663	121·0505887
	6·0	6·2439408	8·7089966	6·1825837	8·1590784	9·3115837
11	25·5	26·5367484	42·8750600	62·7090630	90 9154447	130·3621724
	6·5	6 7642692	4·6894597	6·1825837	6·9934957	15·5193062
12	32·0	33·3010176	47·5645197	68·8916467	97·9089404	145·8814786
	3·5	3·6422988	4·6894597	5·2993574	11·6558263	12·4154450
13	35·5	36·9433164	52·2539794	74·1910041	109·5647667	158·2969236

$y-x=0$	$y-x=1$	$y-x=2$	$y-x=3$
$d_{104}s_{104}v^{105}$,	$d_{103}s_{104}v^{105}$,	$d_{102}s_{104}v^{105}$,	$d_{101}s_{104}v^{105}$,
$d_{103}s_{103}v^{104}$,	$d_{102}s_{103}v^{104}$,	$d_{101}s_{103}v^{104}$,	$d_{100}s_{103}v^{104}$,
$d_{102}s_{102}v^{103}$,	$d_{101}s_{102}v^{103}$,	$d_{100}s_{102}v^{103}$,	$d_{99}s_{102}v^{103}$,

and so on. Calling for a moment $s_{104}v^{105}$, $s_{103}v^{104}$, $s_{102}v^{103}$, &c., P′, Q′, R′, &c. respectively, the foregoing terms may be written thus:—

$P'd_{104}$,	$P'd_{103}$,	$P'd_{102}$,	$P'd_{101}$,
$Q'd_{103}$,	$Q'd_{102}$,	$Q'd_{101}$,	$Q'd_{100}$,
$R'd_{102}$,	$R'd_{101}$,	$R'd_{100}$,	$R'd_{99}$;

and their differences, taken from the rows, will be

$P'\Delta d_{104}$,	$P'\Delta d_{103}$,	$P'\Delta d_{102}$,
$Q\Delta d_{103}$,	$Q'\Delta d_{102}$,	$Q'\Delta d_{101}$,
$R'\Delta d_{102}$,	$R'\Delta d_{101}$,	$R'\Delta d_{100}$.

The operation here, then, it thus appears, will be altogether analogous to that in the former case. The terms in the several rows, being multiples of the quantities P′, Q′, R′, &c. respectively, will be formed by the continuous addition to an initial term of series of multiples of the same quantities, by the successive values of Δd_n, which also is a tabulated series.

The preceding example (pages 6 and 7) shows the practical application of the foregoing precepts, the rate of interest being 3 per cent.

The example consists of two portions, corresponding to the relative magnitudes of x and y, x being greater than y in the former, and less than y in the latter. In the first portion, x is constant in each column, and decreases by a unit in passing from column to column. On the other hand, y is equal to x in the first row, and decreases by a unit in descending from each row to the next. As a consequence, the relation between x and y in each row is that indicated by the values in the column headed $x-y$. Thus, in the first row, proceeding from left to right, we have the values of the function corresponding to $x=104$, $y=104$; $x=103$, $y=103$, and so on; in the second row we have the values corresponding to $x=104$, $y=103$; $x=103$, $y=102$; and so of the succeeding rows. The column headed s_n, *which is supposed to be written on a separate piece of paper or card*, and consequently to be moveable, contains the successive values of s_n, taken from the table on page 110, with their differences, *i. e.*, the values of Δs_n on alter-

nate lines. In the present position of this slip the values upon it have, opposite to each, in the column headed 104, the multiple by that value of the quantity we have called P, namely, $d_{104}v^{105}$, it being understood that the initial value and the multiples corresponding to the values of Δs_y only have been independently formed, continuous addition of these giving the multiples corresponding to the values of s_y, the accuracy of which there is consequently the power of determining at any point. If the slip be now conceived to be moved up one step, and so applied to the column headed 103, the same thing will hold. Each value on the slip will have opposite to it in this column the multiple of the quantity called Q, namely, $d_{103}v^{104}$, corresponding to that value; of which multiples, as before, the initial value and those corresponding to the successive values of Δs_y only have been independently formed, the others being derived from these by continuous addition. It will thus be understood, that in passing from each column to the next the slip has to be moved up one step, and in the position thus attained it will indicate the multiples to be inserted in the adjoining column. A second representation of it is introduced, in connection with the column headed 99, to show the position it will have assumed when the first five columns have been passed over.

The foregoing description applies, with but little change, to the remaining portion of the example. In regard to variation in passing from column to column and from row to row, y here takes the place of x in the former portion; and generally it will be found that in corresponding values the values of x and y are merely interchanged. The moveable slip here contains the successive values of d_x and their differences, and the multiples in the several columns are those of the quantities P′, Q′, R′, &c. Here, too, the slip is shown in two different positions.

It seems hardly necessary to remark, that each portion of the computation will consist of 105 columns and 105 rows, and that each column, in passing from left to right, and each row in passing from upper to lower, will contain one value fewer than the preceding; so that both the last column and the last row will contain each but a single value.

It remains to describe the method of forming the addends, which, as has been shown, consist of a series of multiples of a quantity, which is the same for each column, but different for different columns, the multipliers being in the one case $(x \nearrow y)$,

the terms of the series Δs_y, and in the other ($x \angle y$), the terms of the series Δd_x.

Refer to the series Δs_y. We notice, first, that none of the values (with the exception of the last two) consist of more than three figures. The required multiples then can be easily formed by the aid of a small table of the multiples corresponding to the first nine natural numbers. Secondly, a good many of the values occur twice, and some of them oftener, so that the number of multiples to be formed will always be less than the number of values in the column to be constructed. And, thirdly, if the values be arranged once for all, *in the order of their magnitude*, with space between them for their differences, on a separate slip, similar to that already described, it is evident that then the multiples wanted may be very easily formed by the continuous addition of still smaller multiples.

Similar remarks apply to the other series, Δd_x, corresponding to the terms of which, also, multiples have to be formed. The formation in this case, however, is much easier. Only seven terms of the series consist of more than two figures; as many as twenty of them are zero; most of the others are frequently repeated, and of nearly the whole of these the constant difference is ·5. There are, in fact, in no case more than twenty-four distinct multiples to be formed. It is to be noted, moreover, in reference to this series, that a good many of its terms are negative. The multiples corresponding to these terms must consequently be *subtracted* instead of *added;* and this change of affection should be indicated in setting them down by prefixing the negative sign, or by writing them in red ink.

To return now to the example. The values formed, it will be borne in mind, have reference each to a single year only, and are consequently not those that are requisite to be tabulated. The values for tabulation are those having reference to the *whole after lifetime at each age.* These values obviously consist of the successive sums formed by continuous addition of the values formed as above, in which x and y have a common difference. The formation of a few of these final values, for the first six differences of x and y, is shown on the opposite page.

This construction needs little explanation. The addends in the several columns will be found in the successive rows in the previous formation. The terms in the present formation, in which the values of x and y are interchanged, will be found occupying corresponding positions in the two portions; and addition of any two of these

$x \gtrless y$

x	$x-y=0$	$x-y=1$	$x-y=2$	$x-y=3$	$x-y=4$	$x-y=5$
104	·0224420 ·1849220	·0897680 ·3698440	·1795360 ·5547660	·2693040 ·7396880	·3590720 ·9246100	·4488400 1·1557625
103	·2073640 ·3809394	·4596120 ·5714091	·7343020 ·7618788	1·0089920 ·9523485	1·2836820 1·1904356	1·6046025 1·5237575
102	·5883034 ·5885513	1·0310211 ·7847351	1·4961808 ·9809189	1·9613405 1·2261486	2·4741176 1·5694702	3·1283600 2·0108837
101	1·1768547 ·8082771	1·8157562 1·0103464	2·4770997 1·2629330	3·1874891 1·6165542	4·0435878 2·0712101	5·1392437 2·6774179
100	1·9851318 1·0406568	2·8261026 1·3008210	3·7400327 1·6650509	4·8040433 2·1333465	6·1147979 2·7577406	7·8166616 3·6422989
99	3·0257886 2·0097684	4·1269236 2·5725036	5·4051836 3·2960202	6·9373898 4·2607091	8·8725385 5·6273516	11·4589605 7·5567293
98	5·0355570 3·5329050	6·6994272 4·5265345	8·7011038 5·8513739	11·1980989 7·7282297	14·4998901 10·3779084	19·0156898 14·2420232
97	8·5684620 5·8279131	11·2259617 7·5336438	14·5524777 9·9500956	18·9263286 13·3615569	24·8777985 18·3366047	33·2577130 25·5859600
96	14·3963751 10·8635143	18·7596055 14·3480378	24·5025733 19·2673651	32·2878855 26·4413840	43·2144032 36·8949544	58·8436730 50·6280763
95	25·2598894	33·1076433	43·7699384	58·7292695	80·1093576	109·4717493

corresponding terms will give the M corresponding to the values of x and y that belonging to those terms. Thus—

$$\begin{array}{r} 4{\cdot}0435878 \\ 1{\cdot}9783178 \\ \hline \end{array}$$

$$M_{101\,.\,97} = 6{\cdot}0219056$$

To show the final arrangement of the terms now formed, I give the concluding portion of the table corresponding to difference of ages five years, as it will appear when completed. The columns D and N are borrowed from Jones.

Ages.		D	N	E	F	M
90	95	256·9646	312·6071	109·471749	130·903360	240·375110
91	96	141·4307	171·17636	58·843673	73·481927	132·325600
92	97	76·75788	94·41848	33·257713	38·514449	71·772162
93	98	41·73244	52·68604	19·015690	19·966698	38·982387
94	99	23·58128	29·10476	11·458961	10·587778	22·046739
95	100	14·048867	15·05589	7·816662	5·384494	13·201156
96	101	8·133289	6·922603	5·139244	2·555524	7·694768
97	102	4·414135	2·508468	3·128360	1·084146	4·212506
98	103	1·999932	·508536	1·604603	·322267	1·926870
99	104	·508536	·000000	·448840	·044884	·493724

The new columns here are E and F; M, which contains the sums of the corresponding terms in E and F, being also now added for the first time to a joint life table. And it will by this time, I suppose, be sufficiently understood that the *fundamental* properties of these columns are that, taking any pair of the tabulated ages, say 95.100,

$\frac{E_{95\,.\,100}}{D_{95\,.\,100}}$ = the present value of an assurance of £1 on (100) against (95);

$\frac{F_{95\,.\,100}}{D_{95\,.\,100}}$ = " (95) " (100);

$\frac{M_{95\,.\,100}^{*}}{D_{95\,.\,100}}$ = { " " on the joint duration of (95) and (100);

and the other properties, requisite for the formation of temporary, deferred, and intercepted assurances, necessarily follow from the possession of these. By the continuous summation of the terms in these columns, also, other columns may be formed, *ad infinitum*, which shall possess like properties in regard to assurances increasing from year to year.

* If $N_{94\,.\,99}$ take the place of $D_{95\,.\,100}$ in these expressions, they will denote the annual premiums for the same benefits, payable during the joint duration of (95.100).

If in the foregoing construction v be made equal to unity (which leads to $r=0$, *i. e.*, to the hypothesis of money bearing no interest), the construction will be much facilitated. The results of columns E and F will then indicate probabilities of survivorship, while column M will become identical with column D, since the number of failures *after* a specified age is necessarily equal to the number alive *at* the same age.

II.—*Construction in Logarithms.*

Recurring to the symbolical expressions on page 110 for the leading terms of several adjoining columns belonging to the case $x \nearrow y$, namely,

$x-y=0$	$x-y=1$	$x-y=2$	$x-y=3$
$d_{104}s_{104}v^{105}$,	$d_{104}s_{103}v^{105}$,	$d_{104}s_{102}v^{105}$,	$d_{104}s_{101}v^{105}$,
$d_{103}s_{103}v^{104}$,	$d_{103}s_{102}v^{104}$,	$d_{103}s_{101}v^{104}$,	$d_{103}s_{100}v^{104}$,
$d_{102}s_{102}v^{103}$,	$d_{102}s_{101}v^{103}$,	$d_{102}s_{100}v^{103}$,	$d_{102}s_{99}\,v^{103}$,

we have to examine now, not the differences, but the ratios of those terms. Take a column—the first. Here we have

$$\frac{d_{103}s_{103}v^{104}}{d_{104}s_{104}v^{105}}=\frac{d_{103}}{d_{104}}\cdot\frac{s_{103}}{s_{104}}(1+r);\quad \frac{d_{102}s_{102}v^{103}}{d_{103}s_{103}v^{104}}=\frac{d_{102}}{d_{103}}\cdot\frac{s_{102}}{s_{103}}(1+r);$$

and we should find similar expressions for the ratios in the other columns. It will thus appear that the addends by the use of which, setting out with the logarithms of the initial terms, we should form continuously the logarithms of the successive terms in each column, are

$(x-y=0)$, $\Delta\log.d_{104}+\Delta\log.s_{104}+\log.(1+r)$, $\Delta\log.d_{103}+\Delta\log.s_{103}+\log.(1+r)$, &c.
$(x-y=1)$, $\Delta\log.d_{104}+\Delta\log.s_{103}+\log.(1+r)$, $\Delta\log.d_{103}+\Delta\log.s_{102}+\log.(1+r)$, &c.
$(x-y=2)$, $\Delta\log.d_{104}+\Delta\log.s_{102}+\log.(1+r)$, $\Delta\log.d_{103}+\Delta\log.s_{101}+\log.(1+r)$, &c.

and so on. There is nothing formidable here. Additions of three lines will suffice, since the constant, log. $(1+r)$, can be combined once for all with the terms of one or other of the series $\Delta\log.d_x$ or $\Delta\log.s_y$. But I use this method only for verification. There is an easier method for the principal formation.

Examine the ratios in the several rows. From the first row we have—

$$\frac{d_{104}s_{103}v^{105}}{d_{104}s_{104}v^{105}}=\frac{s_{103}}{s_{104}};\quad \frac{d_{104}s_{102}v^{105}}{d_{104}s_{103}v^{105}}=\frac{s_{102}}{s_{103}};\quad \frac{d_{104}s_{101}v^{105}}{d_{104}s_{102}v^{105}}=\frac{s_{101}}{s_{102}},\ \&c.;$$

from the second—

$$\frac{d_{103}s_{102}v^{104}}{d_{103}s_{103}v^{104}}=\frac{s_{102}}{s_{103}};\quad \frac{d_{103}s_{101}v^{104}}{d_{103}s_{102}v^{104}}=\frac{s_{101}}{s_{102}};\quad \frac{d_{103}s_{100}v^{104}}{d_{103}s_{101}v^{104}}=\frac{s_{100}}{s_{101}},\ \&c.;$$

and so on. Hence, taking the logarithms of these ratios, the addends for the continuous formation of the logarithms of the values occupying the several rows will be the terms of the series $\Delta \log. s_y$.

A like procedure to that employed above will show, that in the case $x \angle y$ the addends for the formation in columns are series of terms of the same form as those obtained for the case $x \nearrow y$, the only difference being that the terms of the series $\Delta \log. d_x$ and $\Delta \log. s_y$ are interchanged; and that for the formation in rows the addends are the terms of the series $\Delta \log. d_x$.

The following table contains the portions of the several auxiliary series here requisite, which come into operation in the examples to be presently given.

x & y	Log. s_y	Δ log. s_y	∇ log. s_y	Log. d_x	Δ log. d_x	∇ log. d_x
104	0·0000000	6020600	6148972	$\bar{1}$·6989700	3010300	3138672
103	·6020600	3010300	3138672	0·0000000	0000000	0128372
102	·9030900	1760912	1889285	·0000000	0000000	0128373
101	1·0791812	1249388	1377760	·0000000	0000000	0128372
100	·2041200	0969100	1097472	·0000000	0000000	0128372
99	·3010300	0969100	1097472	·0000000	1760913	1889285
98	·3979400	1072100	1200473	·1760913	1249387	1377760
97	·5051500	1076339	1204711	·3010300	0969100	1097472
96	·6127839	1114920	1243292	·3979400	1461280	1589652
95	·7242759	1208221	1336593	·5440680	1549020	1677392
94	·8450980	1280299	1408672	·6989700	1461280	1589653
93	·9731279	1374618	1502990	·8450980	1760913	1889285
92	2·1105897	1446828	1575200	1·0211893	1549020	1677392
91	·2552725	1374245	1502617	·1760913	0910804	1039176
90	·3926970	1165055	1293428	·2671717	0228629	0357002
89	·5092025	1067476	1195848	·2900346	1165056	1293428
88	·6159501	1066838	1195210	·4065402	0986098	1114470
87	·7226339	0988796	1117168	·5051500	0450784	0579156
86	·8215135	0880425	1008798	·5502284	0408362	0536735
85	·9095560	0790030	0918402	·5910646	0321847	0450219
84	·9885590			·6232493		

The formation of this table is sufficiently obvious,* with the exception of the columns headed $\nabla \log. s_y$ and $\nabla \log. d_x$. The values in these columns are the corresponding values in the immediately preceding columns respectively, increased by log. $(1+r)$ = ·0128372. The succeeding figures of this constant logarithm being 247, or say 25, the requisite correction is made by

* $\Delta \log. s_y$ and $\Delta \log. d_x$ must not be confounded with log. Δs_y and log. Δd_x. The *difference of the logarithm* of a number is by no means the same thing as the *logarithm of the difference* of that number.

increasing the third and every fourth following term of the series in which this logarithm enters by a unit in the last place. But for the necessity for this correction, it would suffice to combine the constant logarithm with the terms of only one of the series $\Delta \log. s_y$, $\Delta \log. d_x$. The corrections would in such case fall in the proper places in the values formed for verification, only when the differences of age are of the form $4n$.

The initial and verification values are formed as follows :—

Initial and Verification Values.

$x \mathbin{7} y$				$x \angle y$			
$x-y=0$		$x-y=13$		$y-x=0$		$y-x=13$	
Log. v^{105}	6520914	Log. v^{105}	6520914	Log. v^{105}	6520914	Log. v^{105}	6520914
„ d_{104}	6989700	„ d_{104}	6989700	„ s_{104}	0000000	„ s_{104}	0000000
„ s_{104}	0000000	„ s_{91}	2552725	„ d_{104}	6989700	„ d_{91}	1760913
104.104	3510614	104.91	6063339	104.104	3510614	91.104	8281827
∇ log. d_{104}	3138672	∇ log. d_{104}	3138672	∇ log. s_{104}	6148972	∇ log. s_{104}	6148972
Δ „ s_{104}	6020600	Δ „ s_{91}	1374245	Δ „ d_{104}	3010300	Δ „ d_{91}	0910804
103.103	2669886	103.90	0576256	103.103	2669886	90.103	5341603
∇ log. d_{103}	0128372	∇ log. d_{103}	0128372	∇ log. s_{103}	3138672	∇ log. s_{103}	3138672
Δ „ s_{103}	3010300	Δ „ s_{90}	1165055	Δ „ d_{103}	0000000	Δ „ d_{90}	0228629
102.102	5808558	102.89	1869683	102.102	5808558	89.102	8708904
∇ log. d_{102}	0128373		0128373		1889285		1889285
Δ „ s_{102}	1760912		1067476		0000000		1165056
101.101	7697843	101.88	3065532	101.101	7697843	88.101	1763245
∇ log. d_{101}	0128372		0128372		1377760		1377760
Δ „ s_{101}	1249388		1066838		0000000		0986098
100.100	9075603	100.87	4260742	100.100	9075603	87.100	4127103
∇ log. d_{100}	0128372		0128372		1097472		1097472
Δ „ s_{100}	0969100		0988796		0000000		0450784
99.99	0173075	99.86	5377910	99.99	0173075	86.99	5675359

Verification of the above Values.

Log. v^{100}	7162775	Log. v^{100}	7162775	Log. v^{100}	7162775	Log. v^{100}	7162775
„ d_{99}	0000000	„ d_{99}	0000000	„ s_{99}	3010300	„ s_{99}	3010300
„ s_{99}	3010300	„ s_{86}	8215185	„ d_{99}	0000000	„ d_{86}	5502284
	0173075		5377910		0173075		5675359

The initial values are, of course, the same in both compartments. I have, nevertheless, formed them separately for the sake of symmetry, and to show more distinctly how the addend series vary in passing from column to column.

I now introduce an example of the actual formation :—

$x \nearrow y$

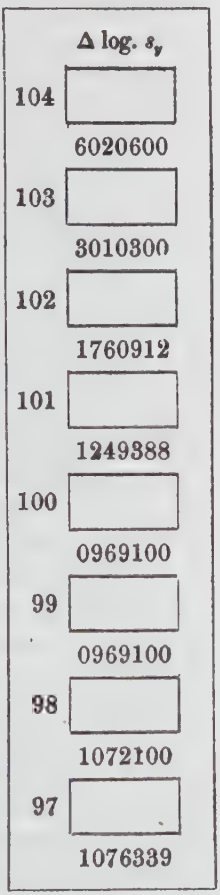
$x-y$	$x=104$	$x=103$	$x=102$	$x=101$	$x=100$	$x=99$
0	3510614 6020600	2669886 3010300	5808558 1760912	7697843 1249388	9075603 0969100	0173075 0969100
1	9531214 3010300	5680186 1760912	7569470 1249388	8947231 0969100	0044703 0969100	1142175 1072100
2	2541514 1760912	7441098 1249388	8818858 0969100	9916331 0969100	1013803 1072100	2214275 1076339
3	4302426 1249388	8690486 0969100	9787958 0969100	0885431 1072100	2085903 1076339	3290614 1114920
4	5551814 0969100	9659586 0969100	0757058 1072100	1957531 1076339	3162242 1114920	4405534 1208221
5	6520914 0969100	0628686 1072100	1829158 1076339	3033870 1114920	4277162 1208221	5613755 1280299
6	7490014 1072100	1700786 1076339	2905497 1114920	4148790 1208221	5485383 1280299	6894054 1374618
7	8562114 1076339	2777125 1114920	4020417 1208221	5357011 1280299	6765682 1374618	8268672 1446828
8	9638453 1114920	3892045 1208221	5228638 1280299	6637310 1374618	8140300 1446828	9715500 1374245
9	0753373 1208221	5100266 1280299	6508937 1374618	8011928 1446828	9587128 1374245	1089745 1165055
10	1961594 1280299	6380565 1374618	7883555 1446828	9458756 1374245	0961373 1165055	2254800 1067476
11	3241893 1374618	7755183 1446828	9330383 1374245	0833001 1165055	2126428 1067476	3322276 1066838
12	4616511 1446828.	9202011 1374245	0704628 1165055	1998056 1067476	3193904 1066838	4389114 0988796
13	6063339	0576256	1869683	3065532	4260742	5377910

$x \swarrow y$

$y-x$	$y=104$	$y=103$	$y=102$	$y=101$	$y=100$	$y=99$
0	3510614 3010300	2669886 0	5808558 0	7697843 0	9075603 0	0173075 1760913
1	6520914 0	2669886 0	5808558 0	7697843 0	9075603 1760913	1938988 1249387
2	6520914 0	2669886 0	5808558 0	7697843 1760913	0836516 1249387	3183375 0969100
3	6520914 0	2669886 0	5808558 1760913	9458756 1249387	2085903 0969100	4152475 1461280
4	6520914 0	2669886 1760913	7569471 1249387	0708143 0969100	3055003 1461280	5613755 1549020
5	6520914 1760913	4430799 1249387	8818858 0969100	1677243 1461280	4516283 1549020	7162775 1461280
6	8281827 1249387	5680186 0969100	9787958 1461280	3138523 1549020	6065303 1461280	8624055 1760913
7	9531214 0969100	6649286 1461280	1249238 1549020	4687543 1461280	7526583 1760913	0384968 1549020
8	0500314 1461280	8110566 1549020	2798258 1461280	6148823 1760913	9287496 1549020	1933988 0910804
9	1961594 1549020	9659586 1461280	4259538 1760913	7909736 1549020	0836516 0910804	2844792 0228629
10	3510614 1461280	1120866 1760913	6020451 1549020	9458756 0910804	1747320 0228629	3073421 1165056
11	4971894 1760913	2881779 1549020	7569471 0910804	0369560 0228629	1975949 1165056	4238477 0986098
12	6732807 1549020	4430799 0910804	8480275 0228629	0598189 1165056	3141005 0986098	5224575 0450784
13	8281827	5341603	8708904	1763245	4127103	5675359

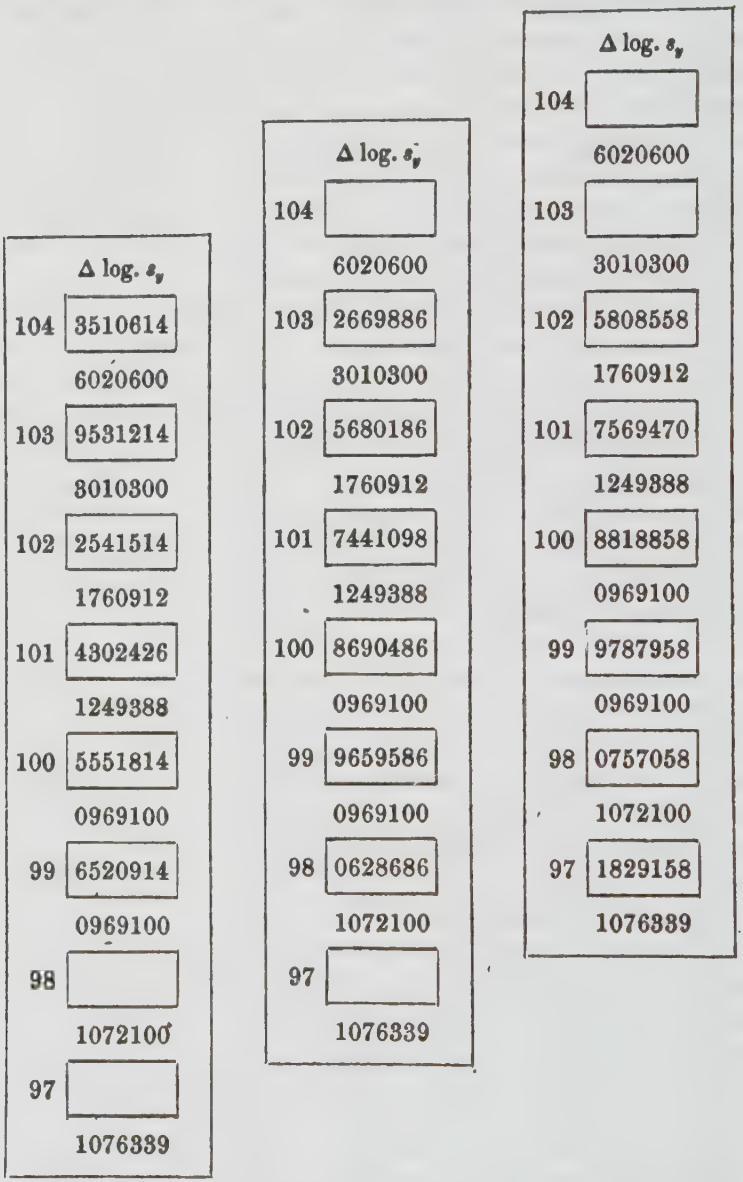
After what has been said, this example seems to require but little explanation. The arrangement of the values is precisely the same as in the preceding formation in numbers. In the first compartment x is constant in each column, and in the second y is constant, while in both the differences of age are constant in the several rows. The values here formed are consequently the logarithms of the values occupying corresponding positions in the preceding formation. In practice it will be proper to leave the alternate columns unoccupied, to receive the natural numbers, which it is unnecessary here to take out, since, as just mentioned, we already have them in the first formation.

Refer to either portion of the present example—say the first. The addends in the several columns are the successive terms of the same series (Δ log. s_y), commencing in each column with the term immediately following that with which a commencement was made in the preceding column. Each term consequently may be traced, from its occurrence in the first column, in a diagonally ascending line, till it appears for the last time in the first row. It is comparatively easy then, when the addends have been properly inserted in the first column, to insert them throughout, since a single reference for each value will suffice for every occurrence of that value on the same opening of the paper made use of. But the labour of writing the addends may be spared by writing them once for all upon one or more perforated cards of the form here shown.



The manner of using this card will be obvious from comparison of it with the first portion of the example just given. For the first column it will be placed with the first opening over the initial value in that column. Continuous addition then will, it is apparent, produce precisely the same values as already appear in the column in question. To form the second column, the card will be placed so that the initial value in that column shall appear through the second opening, opposite 103, and continuous addition will, as before, produce the required values. To make this more clear,

the card is here shown in three different positions, adapted to the formation of the leading terms of the first three columns.



In a previous part of this paper I intimated my preference for the first of the two methods of construction that have now been described. The reason of this preference may be now briefly stated. It is simply this—that when the method in question is

employed we have the benefit of a continuous process, and consequently the power of thorough verification, up to a later stage than when the logarithmic method is made use of. By the first method we form continuously the *actual values* of the functions, of which, by the other method, we form only *the logarithms;* and as the operation of taking out the numbers corresponding to these logarithms is one to which an *infallible* check cannot be applied, it seems to follow that we have the greatest security against error by employing the method of construction in numbers. It is true that this method is attended with *a little* more labour than the other; but, were the additional labour very much more than it is, I should consider it amply compensated by the advantage in regard to security against error which, as we have just seen, this method possesses.

Another distinction between the two methods deserves to be mentioned. When the logarithmic method is employed we can obtain no more than seven figures in the several results. Seven figures may or may not be sufficient. On this point different computers will entertain different views. Those who desire a greater number, however, or to ensure greater accuracy with the same number, can have their desire gratified to the fullest extent by employing the method of construction in numbers. The number of figures that may be obtained true by this method is, for all practicable purposes, unlimited.

And now, before I close, I would direct attention to a mechanical aid, the use of which imparts an almost magical facility to the most complex and extensive computations, and in the absence of which even the best devised methods of applying formulæ at all complex, or requiring the employment of many figures, become exceedingly perplexing and irksome. I refer to the employment of paper, ruled in squares, each square being adapted to receive a single figure. By this means not only does each figure necessarily fall in its proper place, and so facilitate subsequent proceedings, but values that occur *periodically*, and have to be *worked up to*, as it were, in the course of continuous or other operations, can be inserted in their places in the outset, so as to avoid all necessity for interruption in the work. But one must try this paper to be able fully to appreciate its great advantages, and the facilities it affords. I believe I am warranted in saying, that by the employment of it in such operations as have been described in the present paper, the labour that would otherwise be necessary will be found to be reduced one half, while accuracy will be most materially

promoted. The paper may be obtained by order of any stationer. That which I use (foolscap) has about 70 horizontal lines and 46 vertical lines on a page.

I am quite aware that this paper is known to and used by a good many actuaries, as well as in astronomical computations; but I have recently had occasion to become acquainted with so many instances in which it is not known, that I believe I shall be doing good service in thus prominently directing attention to its advantages.

I have now finished. I am quite prepared to learn that the practised computer, whatever opinion he may entertain of the methods I have endeavoured to develope, will think that I have been unnecessarily minute in my details as to their application. I admit it. *For him* I have gone into unnecessary detail. But I have had chiefly in view the younger members of the Institute, whom I would fain stimulate to enter upon the exercise of the construction of tables, and I have been anxious to remove everything that might prove an obstruction in their way. If two, three, or four of our younger members would unite for the construction of a complete set of tables of the kind now described, the labour would be comparatively light; and, besides the benefit they would thus confer on the Profession generally, I feel very sure that, at the conclusion of their task, they would be ready to acknowledge that the exercise had been the most beneficial to themselves of any they ever entered upon. They would find they had acquired such an intimate acquaintance with the structure and properties of the tables, that they could apply them to practical purposes with a facility and confidence which without this preparation long experience only could have imparted. I should be glad to afford, to any of our friends disposed to act on this recommendation, all the assistance in my power in the way of information or suggestion.

In conclusion, I think it necessary to mention that the methods developed in the foregoing paper are identical in principle with methods I have elsewhere described for like purposes.

On the Laws of Mortality and Sickness of the Labouring Classes of England. *By* THOMAS ROWE EDMONDS, *Esq., Actuary to the Legal and General Life Assurance Society.*

THE mortality, according to age, of the general population of England in all localities, was first satisfactorily determined for the period of seven years, ending with the year 1844. The observations and results were published by the Registrar-General in the year 1849; and in the following year I was allowed the opportunity of exhibiting in *The Lancet* (vol. i. 1850, pp. 297–330) the principal results in new combinations, with my conclusions therefrom. The knowledge thus obtained of the mortality of the general population (with distinction of town from country districts) is valuable for testing the correctness of independent observations made on the mortality of classes of the labouring population, such as are contained in "Friendly Societies," whose mortality and sickness it is now proposed to discuss.

Within the last few years, a very great addition has been made to the previously existing small stock of observations on the mortality and sickness experienced by members of Friendly Societies. In *The Lancet* of April, 1839 (vol. i., p. 185), was published an observation made by me on the members of a metropolitan Benefit Society, in which observation was comprehended the deaths and sickness, distributed according to age over 30,000 years of life. At that time there existed only two similar observations—one of these was contained in the report of the Highland Society, and embraced 85,000 years of life; the other was made by Mr. Ansell, and embraced 25,000 years of life. Since the time mentioned, we have had large additions made to this department of knowledge, from three sources, to the extent of two and a half millions of years of life observed. The first of these additions is due to Mr. Neison, and embraces more than one million years of life, observed during the five years ending with 1840. The next addition proceeded from a very large Friendly Society, called the "Manchester Unity of Odd Fellows," and embraced about 600,000 years of life, observed during the three years ending with 1848—this observation having been made by the corresponding secretary of that Society, Mr. Ratcliffe. The third and last addition to this department of knowledge has been made by authority of the British Government, the results having been published within the last half year. Mr. Finlaison, jun., has been employed to make this observation, which extends over the five years ending with 1850, and

embraces about 800,000 years of life. This last observation, like that of Mr. Neison, is founded on official returns, which all registered Friendly Societies are required by Act of Parliament to make once every five years.

The three observations on Friendly Societies, made respectively by Mr. Neison, Mr. Ratcliffe, and Mr. Finlaison, jun., yield each its specific law of mortality and sickness. The three laws of mortality obtained are in accordance with the general law, which in all other observations regulates, in adult life, the increase of mortality according to age. The general law is this—that the rate of mortality increases, from puberty to the age of about 53 years, at the rate of 3 per cent. (·02991) for every advance of one year in age; and after that age, increases at the rate of 8 per cent. (·07969) for each year of age until the end of life. There exist innumerable specific laws of mortality, appropriate to particular masses of population; but all these specific laws differ from each other only in two characteristic points—one characteristic being the particular age at which the period of "florescence" terminates and the period of "senescence" begins, and the other being the absolute mortality at any specified age.

Previously to the publication of the three observations now under consideration, there existed grounds for believing that, in adult life, the increase according to age in the amount or duration of sickness suffered in each year of life was regulated by the same two numbers which regulate the increase of mortality in the periods of "florescence" and "senescence" respectively. In the year 1832, I published sickness tables, founded upon such unity of law. The present three observations confirm the view then acted upon, and may be said to establish the fact of identity between the constants of mortality and the constants of sickness. There is, however, a remarkable discrepancy between the three observations as to the particular age at which the period of florescence is divided from the period of senescence. In the three observations on mortality, and the three observations on sickness, the limiting age is either at 53 years or at 46 years. According to Mr. Finlaison's observation, the limit dividing the two periods is at the age of 53 years, both for mortality and sickness; according to Mr. Ratcliffe's observation, the line of separation is at the age of 46 years, both for mortality and sickness; but according to Mr. Neison's observation, the limits of division for mortality and sickness are different, that for mortality being at 53 years, and that for sickness being at 46 years of age. This difference of seven

years in the position of the limit dividing the period of florescence from that of senescence, occasions a difference of 40 per cent. in the relative mortality or relative sickness at ages above 53 years compared with ages below 46 years. For example: assuming (as is not far from the truth) the mortality and sickness stated by all three observers to be equal at ages below 46 years, it will ensue, from the difference in the position of the limits just mentioned, that at any specified age above 53 years the mortality observed by Mr. Ratcliffe is 40 per cent. greater than the mortality observed by Mr. Neison and Mr. Finlaison, and that the sickness observed by Mr. Ratcliffe is equal to that observed by Mr. Neison, but 40 per cent. greater than the sickness observed by Mr. Finlaison.

The discrepancies just mentioned are of such magnitude as to detract considerably from the value of the results of all three observations at any age exceeding 53 years. In order to extract a useful result from the apparently contradictory observations, it is necessary to devise some principle of reconciliation which will explain the inconsistencies. Such a principle is found in the assumption, that one particular class of facts has been treated in three different ways by the three observers, whether designedly or not. The class referred to is that of members retired on the superannuation or sick pension list. It appears highly probable that Mr. Ratcliffe, in his observation, has treated all on this list as sick, and added their deaths to the general amount of deaths. On the other hand, it appears highly probable that Mr. Finlaison has treated nearly the whole of this class as pensioners in good health, and removed them from observation, whether as regards sickness or mortality.* To reconcile the third observation with the two just mentioned, it is necessary to assume that Mr. Neison has designedly reckoned all superannuation-time as sick-time; and that he has unavoidably omitted the deaths of the same class of members, because they were omitted in the returns from Friendly Societies submitted to him.

It may be useful here to remark, that Friendly Societies in general make these returns very reluctantly, and are careless as to their correctness or completeness. A very large proportion of the total returns from these Societies has been rejected by Mr. Neison and Mr. Finlaison as unavailable, through obvious defects arising from negligence, wilful or otherwise. As regards the adopted residue of the returns containing no obvious defects, the results which they indicate cannot be entitled to much confidence, unless

* The interpretation here suggested of "sickness" has been confirmed by Mr. Finlaison, in the second part of his official report, since published.

they are consistent with one another, and corroborated by other observations founded upon facts better attested. It is fortunate for the public that there exists such an extensive and well authenticated observation as that of Mr. Ratcliffe, with which can be compared the observations of Mr. Neison and Mr. Finlaison. From his position in connection with the "Manchester Unity," we have, in Mr. Ratcliffe, responsibility for facts as well as conclusions; whilst Mr. Neison and Mr. Finlaison are responsible for arithmetical conclusions only, founded upon alleged facts, whose correctness they had no power of testing.

There is another way of treating the discrepancies mentioned, which is, to assume that the statements of all three observers are of equal weight, and that the true statement is the mean of all three statements. This plan of proceeding is found highly successful, as will be seen on inspection of Table III. (p. 145). It will there be perceived that the results of the combined observations can be closely represented by assuming the age of 48 years to be the point of separation of the period of "florescence" from that of "senescence," for mortality as well as for sickness. Out of twenty-four points of comparison of sickness and mortality, at decennial intervals of age, there will be seen twenty-three points of agreement; whilst the only point of divergence (which is, in "city" mortality, between the ages of 55 and 65 years) is a specialty hereafter explained, in which all three observations agree in departing from the theoretical numbers.

In the "Manchester Unity," at ages above 53 years, the mortality (shown in Table I., p. 143) is 15 per cent. greater than the mortality of the total male population of England. This higher relative mortality is such as might have been anticipated from this Society, containing less than the average proportion of members resident in rural districts, and more than the average proportion of members resident in town and city districts. According, however, to the observations of Mr. Neison and Mr. Finlaison, at ages above 53 years the mortality in Friendly Societies (containing the due proportion of rural, town, and city members) is 20 per cent. less than the mortality of the total male population of England. This is an unexpected result, which is open to doubt, more especially from the fact that there is a discrepancy in the statements of sickness at the same ages respectively made by Mr. Neison and Mr. Finlaison, which discrepancy, when converted into its proportional amount of deaths, would be just sufficient to make up the suspected deficiency of 20 per cent. The sickness suffered at ages exceeding 53 years is 40 per cent. greater, according to Mr. Neison, than it was according to

Mr Finlaison. The alleged mortality being equal in the two cases, it would follow that the sickness of a population can be greatly increased without the deaths being simultaneously increased—a conclusion opposed to all experience. It is nearly certain that the extra 40 per cent. of sickness observed by Mr. Neison represents the life-time of all superannuated members, which life-time is altogether excluded from his observation by Mr. Finlaison. If one half of all superannuation-time be true sickness, Mr. Neison's observation, as compared with Mr. Finlaison's, will contain an extra 20 per cent. of true sickness, without any extra per centage of deaths. If, however, an addition to the deaths (for omissions in the returns) be made, proportionate to the extra per centage of the true sickness observed by Mr. Neison, the resulting mortalities of all three observations will be in harmony with each other, and conformable to expectation.

It is also to be remarked, that if the mortality of members of Friendly Societies is influenced by "selection," as is probably the fact, there will be additional reason for doubting the truth of the statement in question; for the statement of Mr. Neison and Mr. Finlaison is, that in Friendly Societies the mortality in the period of florescence, as well as in the period of senescence, is 20 per cent. less than that of the total male population of England of the same age. The former part of this statement is probably true; but the latter part is inconsistent with the former part, and the known ordinary effects of selection of good lives from the general mass. In Friendly Societies the great majority of members are admitted below the age of 35 years; so that the effect of selection, being limited to about 15 years from admission, does not extend to the period of age exceeding 53 years. Assuming the reduction of 20 per cent. in the mortality during the period of florescence to be caused by "selection," there ought to be no difference, in the period of senescence, between the mortality of members of Friendly Societies and the mortality of the classes from which the members have been selected. It may be useful here to remark, that a reduction through selection of 20 per cent. in the mortality for the period of florescence is equivalent in appearance to a retrogression of four years in the limit at which the period of senescence commences. If, as is the fact, this limit in the total male population of England is at the age of 53 years, the corresponding position of this limit in Friendly Societies affected by selection in the degree mentioned would be at the age of 49 years.

In the three several observations the members are distributed into three classes, according to their residence in "rural," "town,"

or "city" districts; and the rates of mortality and sickness at different ages are separately stated for each class. The general result of all the observations is, that the mortality of the town is 25 per cent. greater than the mortality of the rural districts; and the mortality of the city 25 per cent. greater than that of the town districts. In Mr. Ratcliffe's observation, the mortality of the aggregate population is greater that the mortality of his town population; but in Mr. Neison's and Mr. Finlaison's observations, the mortality of the aggregate is considerably less than that of their town population. These variations are accidental, and arise from the absence of any settled principle regulating the distribution of the members into the three classes. Mr. Ratcliffe has distributed his numbers equally amongst the three classes of rural, town, and city residents, and has formed his rural class from residents of townships or parishes whose total population is less than 5,000 of all ages and both sexes. Mr. Neison (who has been followed by Mr. Finlaison) has distributed his numbers so that the rural, town, and city residents are in the proportions (nearly) of 4, 2, and 1, respectively; and Mr. Finlaison (with Mr. Neison probably) has formed his rural class from residents in townships or parishes whose total population is less than 3,000. Instead of these classifications, without any apparent principle, it appears to me that the preferable course would have been to comprehend in the "town" class about one half of the total numbers, and so to have adjusted the limits of magnitude of the places designated as "towns" as to render the mortality of the class resident in towns coincident in amount with the mortality of the aggregate population observed. Having thus fixed the intermediate "town class," the two extremes of rural and city classes would be simultaneously fixed. I acted upon the opinion now expressed, in forming a system of theoretical life tables, which I published twenty-two years ago. I divided the population into three classes, and gave to the three degrees of mortality the designations "village," "mean," and "city."* The names of the two extreme classes were indicative of extreme rarity or extreme density of population; the intermediate term was used to indicate either a population of mean density, or the aggregate population independent of density.

Both Mr. Neison and Mr. Finlaison have, in their observations, made an important subdivision of the residents of rural districts into two classes, whose mortalities differ from one another in as great a degree as the mortality of town districts differs from that of rural districts in general. One of these two subdivisions is a class consisting chiefly of agricultural labourers; the other sub-

division is a class consisting of the residue of all residents in rural districts. According to both observations, the mortality of agricultural labourers is less than the mortality of the residue of the rural class in the proportion of 4 to 5, which is the relation of rural to town and of town to city mortality. Attempts have been made to subdivide the "town" and the "city" members of Friendly Societies into classes suffering similarly well marked different degrees of mortality; but, apparently, hitherto without success. This failure of the investigation so far as regards town and city districts, affords however no ground for doubting, what is evidently the fact, that the range of mortality between classes occupying the extremes of good health and bad health is much greater in city than it is in rural districts.

The mortality experienced by members of Friendly Societies resident in "city" districts, according to the observations of Messrs. Neison, Finlaison, and Ratcliffe, approaches nearly to the mortality of the male population of large towns of England of the healthiest class, such as York, Sunderland, Plymouth, Brighton, &c. (*See* Table II., p. 144; and the *Lancet*, vol. i. 1850, p. 380.) The result has been obtained by combining together the members of Friendly Societies resident in large towns of the first, second, and third degrees of health. Liverpool and Manchester represent large towns or cities of the third or lowest degree of health in England—the mortality of these large towns being 50 per cent. greater at most ages than that of the healthiest class of large towns just mentioned; and 100 per cent. greater than the mortality of the four healthiest Registrar's divisions, which comprehend one third part of the total population of England. It may also be seen, on comparing together the first and last columns of Table II., that the least mortality observed between the ages of 35 and 65 years is one third part only of the greatest mortality observed in England at the same interval of age—the least mortality being that of agricultural labourers, observed by Mr. Neison. The true range of mortality is, however, considerably greater than the observed range of 1 to 3; for it is nearly certain that, in all towns of great magnitude, there exist two large classes, occupying the extremes of good and bad health, one of which suffers a mortality one third less, and the other a mortality one third greater, than the average mortality of the whole population of the town; whence we are justified in concluding that, in the least healthy class of large towns of England, there exists a considerable portion of the population suffering a mortality four times as great as that which has been observed to exist in a class formed by selection from agricultural labourers.

All three observers, in their statements of "city" mortality at ages above 55 years, agree in presenting a similar anomaly. According to all three statements, city mortality is considerably greater than town mortality at all ages under 55 years, whilst at ages above 55 years the relation is reversed, though in a less degree. On inspection, however, of the three last columns of Table II., exhibiting the rates of mortality according to age of three classes of the largest towns of England, arranged according to their degrees of healthiness, there will be found no such anomalous appearance. It will there be seen, that at ages above 55 years the unhealthiness of either class of towns is as strongly indicated by increased mortality, as it is at ages below 55 years. There is thus no countenance afforded to the paradox presented by Messrs. Neison, Finlaison, and Ratcliffe—that as the unhealthiness of large towns increased, the mortality of the population at ages above 55 years proportionally diminished.

The anomalous appearance in question is probably attributable to the mixing together in one observation of classes resident in cities, whose mortalities are widely different. All three observers have formed their "city" class out of residents in large towns whose mortalities, for their total populations, differed from one another as much as 50 per cent., as already stated. In the case of such combination of two classes differing greatly in mortality, the result exhibited at the early ages will be the mean of the rates of mortality for the healthy and the unhealthy parts of the population. At the higher ages, however, the rate of mortality belonging to the healthier portion of the population will alone appear, by reason of the previous extinction by death of the portion subject to the higher of the two rates of mortality. The error arising from defective classification is much greater in city than it is in town or rural populations; but it can never be altogether avoided. In deducing the mortality of the total population of England, a defect of the nature alluded to must exist through the difficulty of forming classes of people whose mortalities are nearly on an equality. The mortality of a combination of correctly constituted classes will never increase according to age so rapidly as the mortality of the classes observed separately would increase. In the total male population of England, the line which apparently divides the period of florescence from that of senescence is at the age of 53 years. It is not improbable that the true line of division for the separate classes is at 51 years of age, the difference of two years being the effect of the combination of various classes whose amalgamation is necessarily imperfect.

It has already been stated that sickness, in duration for each

year of life, increases with the age, according to the same law which regulates the increasing proportion of deaths to the number living at successive ages. From the unity of law between sickness and death in relation to life, it necessarily follows that the sickness endured at any age bears a constant proportion to the deaths at any specified age. This proportion is commonly found to be, that of two years of sickness to one death. Occasionally this proportion is as low as one year and three quarters, and as high as two years and a half, of sickness to each death. In addition to the law determining the duration of sickness at each age, there is another law, which connects together the number of attacks or cases of sickness with the number of deaths, and consequently with the number of living, and with the duration of sickness at each year of age. The law which regulates the attacks of sickness was first communicated to the public in *The Lancet* of the year 1836 (vol. i., p. 855), in an article wherein I showed, from recent observations, that the fatality of cases or attacks of sickness increased with the age according to the law which regulates the increase of mortality and the increase of duration of sickness for each year of life. I then stated it to be an inevitable consequence of the identity of the numbers regulating the proportion of attacks of sickness to deaths, of duration of sickness to life, and of deaths to life at each age, that the number of annual attacks of sickness for a given number living was the same at all ages, and that the duration of each attack of sickness increased with the age in the same degree as the mortality increased.

The most interesting and remarkable of the above laws of sickness is that which makes the proportion of annual attacks of sickness to the number living constant at all ages. Notwithstanding its extreme simplicity, the existence of such a law does not appear to have been suspected previous to its announcement; nor did there exist, at that time, any facts which could be appealed to as supplying direct evidence of its existence. The public were subsequently indebted to Dr. Farr for adducing facts directly proving the constancy of the proportion of the numbers attacked by sickness to the number living at various ages of adult life. In the returns from Friendly Societies published by Mr. Neison and Mr. Finlaison, a great multitude of facts have been collected in further corroboration of the proportion of attacks of sickness being constant, and independent of age—at least in the period of florescence. According to Mr. Neison, the constant proportion of living persons attacked by sickness, in any year between 20 and 50 years of age, is 22 per cent. According to Mr. Finlaison, the constant proportion is 24

per cent. in each year. In both observations there is a depression of about 2 per cent. in the attacks near the age of 33 years, which is the age at which the greatest number of admissions of new members occurs; and which consequently is the age at which "selection" might be expected to have the greatest disturbing effect, whether on sickness or on mortality.

In the period of senescence, or at ages exceeding 50 years, the facts bearing on the laws of sickness (whether of attacks or of duration) are everywhere imperfectly recorded, on account of the difficulty of distinguishing cases of sickness from cases of debility from old age. Judging from all observations hitherto made, there appears to be no ground for doubting that the law of attacks, like the law of duration, of sickness, is continuous throughout the two periods of "florescence" and "senescence," or from the earliest to the most advanced age of adult life. The returns of Mr. Neison are not available for the purpose of determining whether the proportion of attacks of sickness is constant at advanced ages, as well as at ages under 50 years, because no attempt has been made to distinguish therein sickness from old age; but the returns of Mr. Finlaison, which professedly represent pure sickness, free from cases of debility from old age, are sufficient for the present purpose. There is, however, a preliminary correction to be made in the numbers which Mr. Finlaison takes to represent attacks of sickness, which correction is of small amount at ages below 50 years, but is of great magnitude at very advanced ages. Mr. Finlaison assumes the number of persons who have been on the sick list in the course of the year to represent the number of new attacks of sickness in the year. This assumption is obviously incorrect, for in such list is included the cases of sickness remaining at the end of the previous year, and transferred to the new year. The number of such transferred old cases of sickness is represented by the proportion of living constantly sick at the particular year of age, which is the correction to be applied in diminution of Mr. Finlaison's alleged numbers of attacks. For example: according to Mr. Finlaison's tables, and taking decennial intervals of age, the proportions constantly sick at the ages 50 and 80 years, respectively, are $3\frac{1}{4}$ and 26 per cent. Also, according to the same tables, the annual proportion of persons on the sick list is $26\frac{1}{4}$ per cent. at the age of 50 years, and 56 per cent. at the age of 80 years. If we subtract from the latter numbers the corrections previously given, we obtain the numbers 23 and 30 per cent. as the corrected per centage of new annual attacks at the respective ages of 50 years and 80 years. These two numbers approach sufficiently near to each other to

warrant the presumption that a defect in the observation is the cause of the failure to obtain an exact coincidence. The apparent excess of attacks at the age of 80 years is in all probability owing to the complication of debility from old age with cases of sickness. Seven out of thirty cases of sickness would not have had the visible effect of producing incapacity for labour, if the man of 80 had not been subject to a debility from which the man aged 50 was comparatively free.

On inspection of Table III., it will be seen that at the mean age of 40 years the rate of sickness obtained by combining all three observations is 1·17 weeks to one year (or 52 weeks) of life, which is the same thing as 2·24 years of sickness to 100 years of life, or 2·24 per cent. of the living constantly sick. From the same table it will be seen that the mean number of deaths at the same age, according to the same combined observations, was 1·02 in one year out of every 100 people living. There was consequently 2·24 years of sickness to 1·02 deaths, or 2·20 years of sickness to one death. This relation of sickness to death remains constant when other ages are compared in the assumed observation representing the mean of all three observations.

The relation of sickness to death, which is constant for different ages in the same observation, is subject to variation in different observations. For example: in rural districts, the proportion of sickness to one death, at every age, is found to be 2·52 years, when the average of all three observers is taken; whilst in town and city districts, the averages similarly obtained are 2·29 and 1·92 years of sickness to one death. This result, from combining the three observations on each of the three districts, is not, however, in accord with the separate observations, for Mr. Finlaison states the disproportion between rural and city sickness to one death to be much greater than the disproportion stated by the other two observers.

The present three observations, when closely investigated, do not however either of them make good the proposition, that the quantity of true sickness to one death is less in cities than it is in rural districts. Mr. Finlaison, in showing the effect of "heavy" labour in increasing the apparent sickness, has supplied a cause sufficient to account for the apparent excess of rural sickness over town and city sickness. The apparent sickness, according to Mr. Finlaison, depends in a great measure on the occupation or labour of members being "heavy" or "light." He has shown, that at any given age, whether in rural or city districts, whether the mortality is high or low, the apparent sickness is 20 per cent. greater

for heavy than it is for light labour. In the two cases, the real amount of sickness is probably the same; but the incapacity for labour, or apparent sickness, is 20 per cent. greater in the case of heavy labour. This excess probably arises from minor degrees of sickness, which are sufficient to incapacitate for heavy, but not for light, labour. The proportion of "heavy" labour is much greater in rural than in city districts. We have thus a reason for expecting more apparent sickness in the former than in the latter districts. The cause now mentioned, when taken in combination with the opposite methods of treating superannuation, is also sufficient to account for the disproportion of rural to city sickness being greater according to Mr. Finlaison than it is according to the other two observers; for a considerable proportion (one fourth part) of the sickness exhibited in the tables of Mr. Neison and Mr. Ratcliffe consists of the life-time of superannuated members, who do not engage in labour, whether heavy or light. On the other hand, Mr. Finlaison, in his tables, takes no account of superannuation, except to exclude it from any connection with sickness. It thus happens that all Mr. Finlaison's recorded sickness is affected by the distinction of heavy and light labour, whilst three fourths only of the sickness recorded by Mr. Neison and Mr. Ratcliffe is so affected.

The laws of mortality, of sickness, and of health, may be said to be the three chief regulators of human life. It has been already shown, that there exists such a parallelism between the two laws of mortality and sickness (in duration), that when the mortality at any age is known, the sickness at the same age is also known, by means of the existing constant relation of sickness to death at all ages (about two years of sickness to one death). It remains to be seen that the degrees of health at different ages also bear a simple and constant relation to the mortality. It is first to be noticed, that health-time, being that portion of a man's life-time which is not sick-time, will of course decrease with the age as sick-time increases; so that, at very advanced ages, sick-time and health-time will form nearly equal portions of a man's life-time. At the early ages, however, of adult life, the proportions of health-time to life-time decrease so slowly, that, for the purpose of estimating the amount of health enjoyed at any age between 20 and 50 years, the element of time may with propriety be neglected, and the degree or intensity of health be alone regarded. On reflection it will be easily perceived that the degree of health possessed at any age is identical or synonymous with the vital force possessed at that age. The mortality is the inverse measure of the vital force, and is consequently the inverse measure also of the intensity of health.

For the purpose of illustrating the combined effects of the laws of mortality, sickness, and health, let any two ages in the period of "florescence," differing from each other by the interval of 24 years, itself. For example: if the mortality, sickness, or health, be numerically expressed for the age 20 years, we have only to multiply be taken, this being the interval in which the mortality doubles such mortality or such sickness by 2, in order to obtain the numbers expressing the mortality and sickness respectively as existing at the age 44 years. Similarly, the intensity of health at the age 44 years is obtained by dividing by 2 the number expressing intensity of health enjoyed at the age of 20 years. If, instead of comparing people of different ages in the same class of population, we compare people of the same age in different classes of population, whereof the mortalities differ in the proportion of 1 to 2, we obtain the same results as those just stated; if the mortality is increased 100 per cent., the sickness will be simultaneously increased 100 per cent., and the intensity of health reduced 50 per cent. Sickness and death are unquestionable evils, which increase as the force of life or health decreases; additions to intensity of life or health are unquestionable benefits, because they have the effect of diminishing unquestionable evils. Independently of these unquestionably beneficial effects, there is ground for assuming that life or health is intrinsically a benefit: whence it would follow, that additions to intensity of life, or robustness of health, must also be benefits. If the intensity of life or health be doubled, there is nothing to oppose the conclusion that the intrinsic benefit, pleasure, or happiness, is also doubled.

All three observers have devoted much of their attention to investigations of the mortality of members of Friendly Societies, classed according to occupation, with a view to discover the effect of any particular branch of labour or occupation in prolonging or shortening the duration of life. So far as regards the great majority of branches of labour examined, the comparative results elicited by the three observers are unsatisfactory, since they fail to exhibit any principle of classification applicable to trades in general. In one important respect, however, the three observers are nearly agreed, viz., as to the two classes which occupy the extremes of the scale of vitality. The highest rank is occupied by agricultural labourers; the lowest rank is occupied by those in the occupation of clerks. The highest degree of vitality belongs to the class whose corporeal powers are most exercised; the lowest degree of vitality belongs to the class whose corporeal powers are least exercised. According to the observations of both Mr. Neison and Mr. Rat-

cliffe, the vitality of clerks is lower than that of painters, potters, and miners; that is to say, an occupation involving no corporeal labour is apparently more detrimental to life than occupations involving corporeal labour in a poisoned atmosphere.

According to the observation of Mr. Finlaison, "heavy" labour in rural districts is attended by increased vitality, whilst in town and city districts heavy labour has no similar advantage over light labour. This opposition of results is open to question on the ground of imperfect classification of the trades taken as the constituents of heavy labour and light labour respectively. A large proportion of the trades carried on in towns and cities has been classed by Mr. Finlaison as heavy labour, although confined to the hands and arms, and involving little exercise of the chief or lower limbs of the body. Such labour cannot, with reason, be considered as the counterpart of the unmistakably "heavy" labour of the agriculturist. It is highly probable that, if the distinction between heavy labour and light labour were as well marked in towns and cities as it is in rural districts, a similar advantage would be shown to exist in all three cases in favour of the vitality of the classes whose labour is "heavy."

The amount of bodily labour undergone, or muscular force exerted, by any particular class of labourers, appears to afford a correct index of the vitality of that class. This principle, which has been shown to be applicable to the opposite extremes (agriculturists and clerks) in the scale of vitality, would probably be found applicable to the intermediate classes, if they could be arranged in well marked grades of labour undergone. This principle, when applied to the aggregate of trades collected in rural, town, and city districts respectively, will serve to explain why the vitality of rural is greater than that of town populations, and that of town greater than the vitality of city populations; for it cannot be doubted that the average amount of labour undergone by each man is greater in rural than in town, and in town greater than in city, districts. The disparity in the vitalities of the rural, town, and city populations would be greater than that observed if the native residents alone of rural, town, and city districts had been compared; for there is a great interchange of population by migration between rural and town districts, and between rural and city districts, whilst at the same time the balance of gain by migration is always against rural districts, and in favour of town and city districts. This transfusion of the rural element unduly elevates the vitality belonging to the native populations of towns and cities, at the expense of the vitality belonging to natives of rural districts. If such recruiting

from rural districts were stopped, the population of a large city, in however healthy a locality, would, in no long period of time, become unable to maintain its members by propagation.*

When classes of people engaged in different occupations are compared, there appears to be no reason for doubting that the vitality of each class is proportional to the amount of labour performed by that class. It is only when single individuals of the same or different classes are compared that the principle is found to be inapplicable. The vitality of an average agricultural labourer is greatly superior to that of an average clerk; but there are many clerks who possess a greater degree of vitality than that of an average agricultural labourer. An agricultural labourer commonly exercises his power of labour to its full extent; whilst many a clerk, possessing a great power of labour, does not exercise that power. In the former case, labour undergone and capacity for labour are almost exact measures of one another; but in the latter case, the labour of an individual is no index of his capacity for labour. Amongst individuals engaged in the same occupation, it may be said, with great appearance of truth, that the vital force of each is proportional to his capacity for labour, Having arrived at this principle, governing the vitality of the members of any one trade, there appears to be no obstacle to the extension of the principle to individuals of different trades, when compared together, and to say generally, that the vitality of any individual is proportional to his capacity for labour. In the majority of occupations (excluding that of agricultural labourers), the capacity for labour is generally in excess of the labour actually performed; but the proportional excess is probably nearly the same in most trades, whilst the excess itself has a continual tendency to diminution. This tendency is a consequence of the general law, that capacity for labour increases with the full exercise of that capacity, but diminishes when that capacity is insufficiently exercised.

The vital force of any human being is closely and almost inseparably connected with the capacity for muscular exertion. The greater the muscular force, the greater the vital force—a conclusion which might be arrived at without the aid of statistical observations of the number living and dying. Nearly all the known elements of animal vitality are included or involved in the capacity for hard labour.

* Since the above remarks were written, the Commissioners of the Census of 1851 have published their Report, exhibiting for the first time the birthplaces of the English population. It is there shown, that out of every 100 adult persons resident in London and 61 other large towns of England, only 40 were resident in the towns wherein they were born; whilst of the remaining 60 immigrants, 20 were born in other parts of the counties in which they reside, 30 were born in other counties of England, and 10 were born in Ireland and Scotland.

The animal organization is tasked to its full extent by hard labour; if any of the principal organs be unsound, hard labour will prove the defect. The capacity for long continued hard labour cannot exist without powerful digestive and respiratory organs, a great strength of muscular fibre, and nearly all other known elements of strong vitality.

The vital force of mankind (and probably of animals in general) is subject to a composite law, consisting of three parts, one of which is immutable, and the other two variable, and capable of being influenced by human agency. The constant annual rate of increase of the vital force throughout the period of "childhood," the constant annual rate of decay of the vital force during the period of "florescence," or reproduction, and the greater constant annual rate of decay of the vital force during the period of "senescence," are in all probability immutable quantities which, like the law of gravitation, form part of the foundations of the universe. The second part of the composite law consists in the extent of the two periods of childhood and florescence respectively. In different masses of population, and in the same mass at different times, the extent of the period of childhood has been known to vary so much as three years, and the extent of the period of florescence so much as ten years. The cause of these variations is yet unknown. If the cause, when discovered, should prove to be under the control of human agency, the extent of these periods may be permanently prolonged, of which the effect would be more than a proportional prolongation of human life. The third and last part of the composite law of vital force consists in the absolute amount of that force possessed at birth, or at any specified age, by the average individual of a class. This amount, in England, has been shown to vary so greatly, that there exist classes of population whose mortalities at the same age differ from each other in the high proportion of one to three. These great variations have been shown to have a close connection with degrees of bodily labour, and are apparently, to a great extent, under the control of human agency. Without any interference of man, the laws of nature operate to promote the increase of the classes possessing the higher degree of vitality, and to retard the increase of the classes possessing the lower degrees of vitality. It is, however, within the power of man greatly to aid the operations of nature, by offering additional rewards to the higher degrees of labour, and affording additional facilities for the increase of that part of the population which is now ascertained to possess the highest degree of vitality.

TABLE I.

Showing, for Quinquennial Intervals of Age, the Rates of Mortality and Sickness experienced by Members of Friendly Societies, according to three several Observations; with which are compared the Results from Five Theoretical Tables, one of which represents the Annual Mortality of the Total Male Population of England, according to the latest Observations.

Age.	MORTALITY (per Cent. per Annum).						SICKNESS (in Weeks to One Year of Life).				
	Finlaison.	Neison.	Ratcliffe.	Theoretical. Limit at 53 Years of Age.	Theoretical. Limit at 46 Years of Age.	Theoretical. Limit, 53. Total Male Population of England.	Finlaison.	Neison.	Ratcliffe.	Theoretical. Limit, 53.	Theoretical. Limit, 45.
							Weeks.	Weeks.	Weeks.	Weeks.	Weeks.
20 to 25	·73	·68	·76	·61	·65	·77	·98	·86	·70	·72	·72
25—30	·76	·73	·81	·71	·75	·89	·99	·90	·77	·84	·84
30—35	·80	·80	·88	·82	·87	1·03	·98	·92	·86	·97	·97
35—40	·95	·88	·93	·95	1·01	1·19	1·12	1·07	·98	1·12	1·12
40—45	1·11	1·03	1·23	1·10	1·17	1·38	1·25	1·34	1·23	1·30	1·30
45—50	1·36	1·26	1·48	1·28	1·47	1·60	1·50	1·72	1·62	1·51	1·70
50—55	1·73	1·68	2·18	1·52	2·14	1·90	1·86	2·32	2·20	1·79	2·49
55—60	2·52	2·22	2·83	2·13	3·14	2·66	2·32	3·28	3·48	2·51	3·66
60—65	2·98	2·93	4·58	3·12	4·61	3·90	3·35	5·50	5·77	3·68	5·37
65—70	4·81	4·49	6·86	4·58	6·76	5·72	5·06	11·13	6·93	5·40	7·88
70—75	6·66	7·09		6·72	9·92	8·40	8·47	18·40	13·55	7·92	11·55
25 to 35	·78	·77	·84	·76	·81	·96	·98	·91	·82	·90	·90
35—45	1·03	·95	1·08	1·03	1·09	1·28	1·18	1·21	1·10	1·21	1·21
45—55	1·54	1·47	1·83	1·40	1·81	1·75	1·68	2·02	1·91	1·65	2·09
55—65	2·75	2·57	3·70	2·62	3·87	3·28	2·84	4·39	4·62	3·09	4·51
65—75	5·74	5·79	..	5·65	8·34	7·06	6·76	14·77	10·24	6·66	9·71

TABLE II.

Showing, for Decennial Intervals of Age, the Rates of Mortality observed amongst the Members of Friendly Societies resident in Rural, Town, and City Districts; with which are compared the Mortality of the whole and of parts of the Male Population of England.

Age.	Labourers in Rural Districts.	Out-door Heavy Labour in Rural Districts.	Theoretical. Limit at 51 Years of Age.	Rural Districts.			All in Rural Districts, except Labourers.	In-door Light and Heavy Labour in Rural Districts.	Male Population of England.		Theoretical. Limit at 53 Years of Age.
	Neison.	Finlaison.		Neison.	Finlaison.	Ratcliffe.	Neison.	Finlaison.	Four Healthiest Registrar's Divisions. Nos. 2, 4, 5, & 11.	Total Population. Eleven Registrar's Divisions.	
25 to 35	·57	·64	·64	·71	·72	·76	·78	·85	·95	·97	·96
35—45	·70	·78	·86	·80	·86	·91	·86	·98	1·07	1·25	1·28
45—55	1·03	1·24	1·20	1·24	1·32	1·50	1·41	1·61	1·48	1·78	1·75
55—65	1·75	2·34	2·40	2·22	2·51	3·44	2·60	2·75	2·65	3·14	3·22
65—75	4·58	4·84	5·17	5·27	5·34	..	5·55	6·08	5·83	6·61	6·78

TABLE II.—*Continued.*

Age.	Theoretical. Limit at 48 Years of Age.	Town Districts.			City Districts.			Theoretical. Limit at 53 Years of Age.	Population of Cities of England.		
		Neison.	Finlaison.	Ratcliffe.	Neison.	Finlaison.	Ratcliffe.		Twelve of Healthiest Class.	Eight of less Healthy Class.	Liverpool and Manchester.
25 to 35	·77	·75	·80	·76	·98	1·02	·98	1·06	1·08	1·12	1·28
35—45	1·03	·97	1·10	1·04	1·43	1·54	1·38	1·41	1·43	1·62	2·07
45—55	1·57	1·67	1·71	1·66	1·99	2·04	2·51	1·92	2·06	2·42	3·20
55—65	3·32	3·36	2·94	4·06	3·20	3·22	2·94	3·54	3·55	4·26	5·27
65—75	7·15	7·69	6·48	..	6·61	5·62	..	7·46	7·06	8·47	10·39

TABLE III.

Showing, for Decennial Intervals of Age, the Rates of Sickness in Rural, Town, and City Districts, according to each of three Observers; also showing the Mean Rates of Sickness and Mortality in each of the same three Districts, for comparison with the Results of Theoretical Tables, in all of which the period of "Senescence" commences at the Age of 48 Years.

Age.	Rural Districts.			Town Districts.			City Districts.			Mean of Three Observations.		
	Finlaison.	Neison.	Ratcliffe.	Finlaison.	Neison.	Ratcliffe.	Finlaison.	Neison.	Ratcliffe.	Rural.	Town.	City.
	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.	Weeks.
25 to 35	1·02	·87	·78	·96	·89	·83	·90	1·11	·84	·89	·89	·95
35—45	1·19	1·08	·99	1·18	1·30	1·08	1·18	1·50	1·28	1·09	1·19	1·32
45—55	1·61	1·65	1·76	1·79	2·62	1·80	1·68	2·46	2·26	1·67	2·07	2·13
55—65	2·78	4·08	4·54	3·00	5·17	4·10	2·63	4·63	5·58	3·80	4·09	4·28

TABLE III.—*Continued.*

Age.	Sickness.—Observed. Mean of Three Observations of all Three Districts.	Sickness.—Theoretical. Corresponding to Mean of Three Observations.			Mortality.—Observed. Being Mean of the Three Observations.				Mortality.—Theoretical. Corresponding to Mean of Three Observations.			
		Rural.	Town.	City.	Rural.	Town.	City.	Three Districts.	Rural.	Town.	City.	Three Districts.
	Weeks.	Weeks.	Weeks.	Weeks.								
25 to 35	·90	·84	·92	1·02	·73	·77	·99	·80	·64	·77	1·02	·77
35—45	1·17	1·13	1·24	1·37	·86	1·04	1·45	1·02	·86	1·03	1·37	1·03
45—55	1·87	1·73	1·88	2·09	1·35	1·68	2·18	1·61	1·31	1·57	2·09	1·57
55—65	3·95	3·65	3·98	4·43	2·72	3·45	3·12	3·01	2·77	3·32	4·43	3·32

October, 1854.

On a Method of substituting Francs and Centimes for the present English Metallic Currency. By JAMES YATES, ESQ., M.A., F.R.S.

[Read before the Statistical Section of the British Association for the Advancement of Science, at Liverpool, on the 25th September, 1854.]

HAVING for some years been an advocate for the abolition of the measures, weights, and coins, now used in Great Britain, and the adoption in their stead of the *système métrique*, employed throughout a great part of the Continent, I have had occasion to consider the various difficulties which present themselves in the prosecution of this object.

One of the first of these difficulties in the order of time, and one of the first also in the magnitude of the obstacles to be removed, is the presumed necessity of calling in the existing English coins, and substituting new ones in their place. This is not a difficulty attendant upon the adoption of francs and centimes, or of any particular kind of money rather than another. In the evidence given before the late Decimal Coinage Committee, and more especially in the evidence of Sir John Herschel, the Master of the Mint, it is represented as a most formidable obstacle to the introduction of the plan, which is recommended by that Committee. Sir John Herschel, after explaining his ideas upon the best method of effecting the change, says—"I should feel disposed to assign somewhere about twenty years from its commencement as a probable term for the completion of the process, and the introduction of a totally new coinage."* The inference in the minds of many of those who have paid attention to the subject has been, that it is impossible to adopt in this country a decimal coinage upon any plan whatsoever.

It will be my object to offer some suggestions respecting the best method of obviating this difficulty.

The principal agency which I should employ, and which I believe has never yet been proposed, would be the issue of a paper currency for this particular purpose. I wish therefore to observe, as a preliminary remark, that I am not more friendly than others to the use of paper money, but have always been of opinion, both that all credit should be based upon a solid foundation of real property, and that promissory notes, bills of exchange, and all other written representations of value, should only be resorted to in cases of great and manifest advantage to all the parties concerned. But in this case, I both propose to make the notes payable after a very short period of time, and to issue them in order to accomplish an

* Report from the Select Committee on Decimal Coinage, 1853, p. 50.

immense national benefit, which will probably never be attained in any other way.

The lowest of these notes would be of the value of a franc; the higher would represent the values of all the intended coins above the value of a franc.* They would therefore bear some resemblance to the paper money now current throughout all Germany, which the Germans very commonly take in preference to metallic currency. There is some reason to apprehend that this new currency might become rather too acceptable in our own country; and the only well grounded objection to my plan which I can anticipate is, that if this currency were once introduced, it might be so popular, that it would remain longer than its presence would be necessary.

The value of francs and centimes having been accurately ascertained in relation to pounds, shillings, pence, and farthings, every note should express the amount paid for it in the present currency, the amount to be received for it in francs and centimes, and the period within which it must be presented for payment. In adjusting the rate of exchange, the advantage of minute fractions should be given to the holder of the note; so that, if the exchange cannot be made with perfect exactness, the holder of the note may receive, in addition to what is due to him, such a fraction of a centime as may be necessary to make even money. For example: having paid $9\frac{1}{2}d.$, he would receive a franc, although a franc might be worth half a centime or a quarter of a centime more than $9\frac{1}{2}d.$ Thus every holder of English coin would receive his due, or more than his due, and the Government would be repaid for these small outgoings by the interest of the money represented by the paper, or by the profits accruing from a well regulated system of coinage.

In order to prevent forgery, the pattern of the notes, and the mode of manufacturing them, ought to be kept secret until they are issued; and if it were required to present them for payment within a sufficiently short time, there would be no danger of fraudulent imitation. They might, however, be reissued with proper marks and securities.

When we consider the present issue of stamps by the Government, at the rate of several millions per day, or when we reflect upon the rapidity with which the London newspapers are printed and circulated, we can imagine no impediment on account of the number of notes which might be required. Let us suppose ten millions to be issued in a day. Reckoning those reissued as if they were fresh issues, this would allow for 260 millions in a month.

* *See* scheme of coinage, p. 151.

On endeavouring to form a judgment from the quantity and value of the coins now in circulation, I am inclined to think that this amount of notes would be sufficient to call in the whole. If so, the issue of the notes necessary for this object might be accomplished in a month. To cash these notes would require a considerably longer time. The operation might, perhaps, in order to avoid all danger of confusion or unnecessary expense, be extended over a year; but I cannot imagine any reason why it should occupy a longer period.

In regard to the substitution of the new coinage, so far as it would be required, in place of the old, the following suggestions have occurred to me—which I beg to offer by beginning with the copper coinage, proceeding to the silver, and ending with the gold.

It would be necessary to have in readiness, before the first day fixed for issuing the notes, an abundant supply of coins of the two lowest denominations, viz., two-centime pieces, and centimes. I am not aware that the ordinary form of the two-centime piece, as we now find it current throughout the west of Europe, can be materially improved; but I think it would be a great advantage to issue centimes in a form resembling that coin which has long been more abundant, and has been found perhaps in actual practice more useful, than any other—viz., the tchien, or cash, of the Chinese. Being made of brass or a similar mixed metal, cheaper than copper, and yet durable, and agreeable to the eye, it would be larger than the copper centime. It would also be distinguishable, even in the dark, by the square hole in the centre, which is used in China for the purpose of stringing these coins together. If every shopkeeper among us had upon his counter a column of such centimes transfixed by an upright wire, they would be extremely convenient for change, superseding the use of those very objectionable methods of balancing accounts, by giving a dram of spirits, a pinch of snuff, or some other useless or pernicious commodity, which are now extensively adopted, because our present coinage does not descend low enough to afford the means of exact payment. It appears probable that all danger of forgery would be prevented by the cheapness of this coin, which, being made for the Government by contract, and in immense quantities, would be manufactured at far less cost by such contractors than would be possible in the case of any private person who might attempt to issue it on his own account. It appears desirable that in issuing these coins, and the pieces of twice their value, there should be no delay. Let them be provided in exchange for the coin now current as soon

as the adoption of the entire system is enacted by law. A better method of teaching the use of a decimal coinage cannot be imagined. Every child, and every rustic more obtuse than a child, might be most readily initiated into the use of decimal accounts by becoming familiar with these coins, and would be prepared in a few weeks or months to use all the coins of higher denominations. They would, moreover, afford great facilities in the adoption of the paper currency, already described, as soon as it might be issued.

If we were to adopt francs and centimes in virtue of treaties with other countries, the coins of which I am speaking would be required for exportation as well as home consumption. If so, 100 tons of them would not be too much for the first issue; it would be absorbed immediately. The preparation of these coins would require a degree of activity from which alarmists might shrink, but in which the Birmingham manufacturers would rejoice; nor would the merchants of Liverpool object to forward them in chests and hogsheads to all parts of the world.

I would suggest, that our pence, halfpence, and farthings, should continue in circulation until they were worn out, although nothing need prevent the issue of the corresponding pieces of the value of ten, five, or two centimes, as speedily as may be found convenient. But it must be remembered that these English coins are only tokens, and that their intrinsic value is far below that for which they pass as money. All that appears necessary in a legal and constitutional point of view is, that the Government fulfil the virtual engagement into which it has entered by issuing them at a certain assigned value. By exchanging them at that exact value through the medium of the paper currency, already described, the Government discharges its obligation. But the Government is not bound to allow these tokens to remain in circulation for an indefinite time, and, whenever the holder pleases, to exchange them at their original price. It is enough to have given the holder a reasonable time for the exchange; and if he neglects the opportunity, the Government may put upon them such a new value as it may deem expedient. The copper coins which have been called in may be issued with a stamp to indicate this fact; and those which have not been called in may be declared to have the new values either by Act of Parliament or by proclamation. Thus every penny, without changing its name, may pass for ten centimes; every halfpenny for five, and every farthing for two, centimes.

Thus far the process would be extremely easy, as there could be no difficulty whatever in teaching the masses of the people, in

every place, that ten of the new centimes were equal to a penny, five to a halfpenny, and so forth.

The introduction of the system would be attended with more complexity in regard to silver. It might be found desirable, perhaps it would be indispensable, to adopt silver as the standard of value. If so, the weight of all the silver coins must be attended to with much greater precision, since they would cease to be tokens; they would possess the intrinsic value belonging to their position in the scale. Hence it would become necessary that the English franc should contain the same quantity of silver as the continental franc—viz., five grammes of standard silver, of which a tenth part is copper. This would not, however, cause any insuperable difficulty, although it might be necessary, or at least expedient, to withdraw some of our present silver from circulation. The most abundant by far, and the most important, of our silver coins, is the shilling, which is a token representing the twentieth part of a pound. Payment at this rate must therefore be offered to those who hold shillings:—Having been recalled, they may either be issued at their real value, or recoined. In this manner fifty millions of francs would be supplied at once; for if we weigh our shillings with francs, we find that, even making allowance for less alloy,* a large proportion of them, perhaps one sixth, are of no greater intrinsic value than francs. Many more will soon be reduced to the same weight by wearing; and others are so little heavier than francs that they would not bear the expense of recoining. Hence, having been recalled, and stamped as francs, they might be reissued.

In like manner, the sixpences would pass as half-francs, and the threepenny pieces as quarterfrancs; and in the case of these smaller silver coins, as well as of the copper coins, their new value would be declared by law or by royal proclamation; and probably the shilling would retain its old name, together with the penny and the halfpenny. Even the five-shilling pieces and the halfcrowns, although their greater size has contributed to their preservation, are so frequently on a par with an equal reckoning of francs in regard to the quantity of silver contained in them, that they might very well be treated in the same manner; the five-shilling piece, if not fit to be recoined on account of its state of preservation, being uttered as five francs, and the halfcrown as $2\frac{1}{2}$ francs.

It only remains that we consider the case of the gold coinage, which ought to be called in through the medium of the same

* The shilling contains one sixtieth less alloy than the franc.

paper currency, and might then be reissued at its real value. This should be ascertained by weighing gold with gold. The value of a sovereign, and consequently of a halfsovereign, should be declared by law, or by proclamation. It might be stamped upon every coin; or, if this were considered objectionable as a matter of taste, the public announcement of it would probably be sufficient. Thus all our gold coinage might be continued until the pieces were so much worn as to require renewal; and as they would probably, under the altered circumstances, be tokens, in this respect, taking the place of silver, they might be allowed to remain in circulation much longer than at present.

If such a plan as that which I have now endeavoured to delineate shall be adopted, nine tenths of the quantity of pieces, and a far larger proportion of the amount of our present coinage in both assigned and intrinsic value, will remain in circulation, and will be known to a great extent by the same denominations. Hence, it appears probable that the prices both of labour and of commodities will undergo far less derangement than might have been expected; for although the same denominations will be applied to coins of a little lower value, yet, the depreciation occurring (at least in the most common cases) in equal proportion in regard to all objects and all transactions, it appears not unlikely that the disturbance attendant on the change may be almost imperceptible.

SCHEME OF COINAGE.

Denominations.	Values in Francs.
GOLD.	
Royal	100·
Half Royal, or Double Sovereign	50·
Sovereign	25·
Louis, or Napoleon	20·
Ten-franc Piece	10·
SILVER.	
Five-franc Piece, Crown, or Dollar (*This may be gold.*)	5·
Two-franc Piece, Florin, or Guilder	2·
Franc	1·
Half Franc	·50
Quarter Franc	·25
PAPER, BRASS, COPPER.	
Blue Postage Stamp, Twopenny Piece (*This piece may be silver.*)	·20
Red ditto, and Receipt Stamp, Penny	·10
Halfpenny	·05
Two-centime Piece	·02
Centime	·01

NOTES AND QUERIES.

MR. CHARLES GABRIEL SHAW, an Associate of the Institute, sends the following ingenious method of obtaining formulæ for the present value and for the amount of an annuity for n years, commencing at a and increasing by c yearly, analogous to those given by Mr. E. H. Galsworthy in the last Number of the *Magazine*.

Let r equal the yearly interest of £1, and let $v=\frac{1}{1+r}$, and $A_n=\frac{1-v^n}{r}$.

The present value of the annuity will be, $av+(a+c)v^2+(a+2c)v^3+$, &c. $+(a+\overline{n-1}c)v^n$.

When $n=\infty$, or is infinite, and the annuity becomes a perpetuity, this will be $\frac{a}{r}+c\{v^2+2v^3+3v^4+$, &c., *ad infinitum*$\}$.

Calling the series within the brackets S, then $(1+r)S=\{v+2v^2+3v^3+4v^4+$, &c.$\}$, and $rS=\{v+v^2+v^3+v^4+$, &c.$\}=\frac{1}{r}$, and $S=\frac{1}{r^2}$; then the perpetuity equals $\frac{a}{r}+\frac{c}{r^2}$.

When n is finite, the value of all the terms of the perpetuity after the nth must be subtracted from the above. At the beginning of the $\overline{n+1}$st year, the next payment due being $(a+nc)$, and the future increase still c per annum, the value of the perpetuity will be, by the above formula, $\frac{a+nc}{r}+\frac{c}{r^2}$, which must be multiplied by v^n to bring it to the *present* value of all that part of the perpetuity which is payable after the expiry of n years. Effecting this subtraction, we find the value as required of the first n payments,

$$\frac{a}{r}+\frac{c}{r^2}-\left(\frac{a+nc}{r}+\frac{c}{r^2}\right)v^n=\left(\frac{1-v^n}{r}\right)a+\left(\frac{1-v^n}{r^2}\right)c-\frac{nv^nc}{r}$$

$$=A_na+\left(\frac{A_n-nv^n}{r}\right)c.$$

As to the amount of the annuity, it must of course be the same as that of the present value accumulated for n years; and if we multiply that value by $(1+r)^n$, and make $M_n=\frac{(1+r)^n-1}{r}$, we have for this amount $M_na+\left(\frac{M_n-n}{r}\right)c$.

THE following is the problem referred to at page 134, Vol. IV., as sent by Mr. JAMES MEIKLE, of the Scottish Provident Institution:—

To determine the rate of interest in a life annuity, the table of mortality and age being given.

$A={}_1av^1+{}_2av^2+{}_3av^3+\ldots.+{}_nav^n$.

It is evident that any series which represents the value of v will converge very slowly; but by making $v=1-d$, we get

$$\begin{aligned}A.&={}_1a(1-d)^1+{}_2a(1-d)^2+{}_3a(1-d)^3+\ldots.+{}_na(1-d)^n\\&={}_1a+{}_2a+{}_3a+\ldots.+{}_na\\&\quad -d\{{}_1a+2{}_2a+3{}_3a+\ldots.+n{}_na\}\\&\quad +d^2\left\{{}_2a+3{}_3a+6{}_4a+\ldots.+\frac{n.n-1}{1.2}{}_na\right\}\\&\quad -d^3\left\{{}_3a+4{}_4a+10{}_5a+\ldots.\frac{n.n-1.n-2}{1.2.3}{}_na\right\}\\&\quad +,\ \text{\&c.}\end{aligned}$$

But

$$\left.\begin{aligned}{}_1a+{}_2a+{}_3a+\ldots.+{}_4a &= \frac{N_x}{D_x}\\ {}_1a+2{}_2a+3{}_3a+\ldots.+n{}_na &= \frac{S_x}{D_x}\\ {}_2a+3{}_3a+6{}_4a+\ldots.\frac{n.n-1.}{1.2}{}_na &= \frac{\Sigma S_{x+1}}{D_x}\\ \text{\&c.}\qquad\qquad \text{\&c.}&\end{aligned}\right\}\quad\begin{aligned}&\text{where } D_x=\text{No. living at age } x\\ &N_x=\text{Sum of living at } x \text{ and upwards}\\ &S_x=\text{Sum of } A_x\\ &\Sigma S_{x+1}=\text{Sum of } S_{x+1},\ \text{\&c.}\end{aligned}$$

$$\therefore\ A=\frac{A_x}{D_x}-\frac{dS_x}{D_x}+\frac{d^2\Sigma S_{x+1}}{D_x}-\frac{d^3\Sigma^2 S_{x+2}}{D_x}+,\ \text{\&c.}$$

Now the coefficients of d are constant for any value of d, and independent of it.

Let $\frac{N_x}{D_x}-A=a$; $\frac{S_x}{D_x}=b$; $\frac{\Sigma S_{x+1}}{D_x}=c$; $\frac{\Sigma^2 S_{x+2}}{D_x}=d$, &c. Then the series may assume the form of $a-bd+cd^2-dd^3+ed^4-fd^5$, &c.$=o$; or, putting $d=y$, for the sake of distinction, $a-by+cy^2-dy^3+ey^4-fy^5+$, &c.$=o$; from which the value of y may be found by Lagrange's theorem.

$$\begin{aligned}&u=f(y)\\ &y=z+x\phi(y)\end{aligned}\left\{\therefore u=f(z)+\frac{x}{1.2}\phi(z)\frac{df(z)}{dz}+\frac{x^2}{1.2}\frac{d}{dz}\left\{\overline{\phi(z)}|^2\frac{df(z)}{dz}\right\}\right.$$
$$\left.+\frac{x^3}{1.2.3}\frac{d^2}{dz^2}\left\{\overline{\phi(z)}|^3\frac{dfz}{dz}\right\}+,\ \text{\&c.}\right.$$

$$f(z)=z=\frac{a}{b}$$

$$\phi(z)=\frac{cz^2-dz^3+ez^4-,\ \text{\&c.}}{b},\ \text{\&c.}$$

$$y=\frac{a}{b}+\frac{cz^2}{b}+\frac{2c^2-bd}{b^2}z^3+\frac{5c^3-5cdb+eb^2}{b^3}z^4+\frac{bceb^2-fb^3-21c^2db+14c^4+3d^2b^2}{b^4}z^5$$
$$+\frac{gb^4-7cfb^3-7deb^3+28c^2eb^2+28cd^2b^2-84c^3db+42c^5}{b^5}z^6+,\ \text{\&c.}$$

$$\therefore$$

$$d=\frac{a}{b}+\frac{ca^2}{b^3}+\frac{2c-bd}{b^5}a^3+\frac{5c^3-5cdb+eb^2}{b^7}a^4+\frac{6ceb^2-fb^3-21c^2db+14c^4+3d^2b^2}{b^9}a$$
$$+\frac{gb^4-7cfb^3-7deb^3+28c^2eb^2+28cd^2b^2-84c^3db+42c^5}{b^{11}}a^6+,\ \text{\&c.}$$

This series converges rapidly, as will be seen from the following example:—

Find the rate of interest in the sum of £4·18289, which is the value of a life annuity on age 80, by Carlisle Table of Mortality.

$$\begin{aligned}
\frac{a}{b} &= \cdot0340012\\
\frac{ca^2}{b^3} &= \cdot00388575\\
\&c. &= \cdot00049785\\
\&c. &= \cdot000065931\\
\&c. &= \cdot0000088845\\
\&c. &= \cdot0000012315\\
d &= \cdot0384608470\\
\text{nearly} &= \underline{0384615}
\end{aligned}$$

$$1-d = v = \cdot961548 = \frac{1}{1+r}$$

$$\therefore r = \frac{1-v}{v} = \frac{d}{v} = \frac{\cdot0384615}{\cdot9615385} = \cdot04$$

∴ Rate of interest=4 per cent.

MR. MEIKLE also furnishes the following:—

Assurances are effected either by premiums payable for the whole of life or for a term of years. A person opens a policy on the whole life scale, and, after a few years, proposes to change to the other scale. What sum ought he to pay, to compensate for the previous deficiencies? and of what is it composed?

Let the case be illustrated by an example:—Entrant of 30, paying 17·5541 (Carlisle 4 per cent. pure), at age 35 proposes to change to the 21 years' scale, the premium on which would have been 23·6069.

$\frac{17\cdot5541(1+a_{35})}{\overline{16}|a_{35}+1-{}_{16}av^{16}}$ = Future life premiums of 17·5541 converted into a term of 16 years . . £26·7535

But he only intends paying . . . . . 23·6069

and therefore must redeem this . . . . . 3·1466

which will be the sum required, $3\cdot1466(\overline{16}|a_{35}+1-{}_{16}av^{16})=35\cdot184$.

Or, analytically—Let [·]=life premium; [21]=21 years' premium. Then sum required=$S=[\cdot](1+a_{35})-[21](\overline{16}|a_{35}+1-{}_{16}a_{35}v^{16})$.

$$S\,{}_5a_{30}v^5=[\cdot](1+a_{35})\,{}_5a_{30}v^5-[21](\overline{16}|a_{35}+1-{}_{16}a_{35}v^{16})\,{}_5a_{30}v^5$$

$$=[\cdot](1+a_{30})-[\cdot](\overline{5}|a_{30}+1-{}_5a_{30}v^5)-[21]\,\overline{21}|a_{30}+1-{}_{21}a_{30}v^{21}+[21](\overline{5}|a_{30}+1-{}_5a_{30}v^5)$$

$$S=\frac{([21]-[\cdot])\,\overline{5}|a_{30}+1-{}_5a_{30}v^5}{{}_5a_{30}v^5}$$

$$=([21]-[\cdot])\left\{\frac{{}^0a_{30}v^0+{}_1a_{30}v^1+{}_2a_{30}v^2+{}_3a_{30}v^3+{}_4a_{30}v^4}{{}_5a_{30}v^5}\right\}$$

$$=([21]-[\cdot])\left\{\frac{1}{{}_5a_{30}v^5}+\frac{1}{{}_4a_{31}v^4}+\frac{1}{{}_3a_{32}v^3}+\frac{1}{{}_2a_{33}v^2}+\frac{1}{{}_1a_{34}v^1}\right\}$$

Proving that each yearly deficiency is increased by the chance of death during nonpayment, besides the ordinary interest.

Demonstration of the expressions $\frac{(1+i)^n-1}{i}$ *the Amount, and* $\frac{1-(1+i)^{-n}}{i}$ *the Present Value, of* £1 *per Annum for n Years.*—Since $(1+i)^n-1$ evidently represents the amount of an annuity of i pounds in n years, the first formula above given is clearly that for the *amount* of £1 per annum for the same term. For as $i : (1+i)^n-1 :: 1 : \frac{(1+i)^n-1}{i}$; in like manner, $\frac{1-(1+i)^{-n}}{i}$ is the *present value* of £1 per annum for n years. For as $(1+i)^n : 1 :: \frac{(1+i)^n-1}{i} : \frac{(1+i)^n-1}{i.(1+i)^n} = \frac{1-(1+i)^{-n}}{i}$. —Ed. *A. M.*

FOREIGN INTELLIGENCE.

Germany.—*New Insurance Companies in Germany.*—The General Railway Insurance Company in Berlin (*Allgemeine Eisenbahn Versicherungs Gesellschaft*), registered by Order of Council of His Majesty the King of Prussia, of the 26th September, 1853, with a subscribed capital of Prussian thalers 1,000,000 (£166,000 sterling), in 1,000 shares, of which 20 per cent. is paid up, insures, according to the deed of constitution—

Class 1. To railway managers, for any loss and damage on moveable and immovable property, or on articles and goods of all kinds to be forwarded by trains or to be kept at the stations.

Class 2. To passengers and railway officers, for any personal injury or loss of life, and for loss on luggage.

The prospectus issued treats only of this second class, and I extract the following particulars.

Insurance on persons is granted to railway passengers, either for single days, for one, three, and six months, for one or five years, or for the duration of life, along all the railways of Europe; to railway officers, only for the period of one year, at the premium of 1 per cent.

The maximum to be insured by a passenger is Pr. th. 7,000 (£1,150); by an officer, Pr. th. 1,000 (£160).

Claims are settled according to the following rates—

100 per cent.	in case the accident	causes	death immediately, or within 30 days.
Up to ⅓ „	do.	do.	incapacity to work for 8 days.
„ 6⅔ „	do.	do.	„ for a longer time.
„ 33⅓ „	do.	do.	loss of limb, or lasting serious damage to health.
„ 66⅔ „	do.	do.	total incapacity to work.

The premiums are—

Per Mille.	Sum Insured. Pr. Th. 3,000.	Sum Insured. Pr. Th. 7,000.
For one day	14/630	12/630
„ „ month	140/630	120/630
„ three months	315/630	225/630
„ six „	420/630	360/630
„ one year	630/630	570/630
„ five years	2,100/630	1,980/630

For the duration of life, according to the age—Age 20 years, 10 per mille; 45 years, 6⅔; 55 years, 5; 60 years, 4.

Insurance on luggage is only granted for the period of one year, and includes all luggage of the passengers insured, which is delivered to the administration of the railway, and for which a receipt is given. The sum assured and the claims are fixed according to the weight of the luggage, each pound either at three thalers or at five thalers, according to the will of the passenger.

The premiums are—

	At Three Thalers each Pound.		At Five Thalers each Pound.
100 lbs.	Th. 300	2⅔ per mille	Th. 500	2 per mille.
300 lbs.	„ 900	1 2/7 „	„ 1,500	1⅓ „
500 lbs.	„ 1,500	1⅓ „	„ 2,500	1⅓ „

The Thuringia Railway and General Reassurance Society, in Erfurt (*Eisenbahn und Allgemeine Ruckversicherungs Gesellschaft*), registered by Order of Council of His Majesty the King of Prussia, of 26th September, 1853, with a subscribed capital of Prussian thalers 2,000,000 (£332,000 sterling), in 1,000 shares, of which 20 per cent. are paid up, insures, according to the deed of constitution—

a. To railway managers, for any loss and damage on moveable and immovable property, or on articles and goods of all kinds to be forwarded by the train or to be kept at the stations.

b. To railway officers and workmen, their wives and children, and likewise to railway passengers, for any personal injury or for loss of life.

c. To Insurance Associations of all branches the Society will grant reassurance.

The prospectus treats only of class *b*, and resembles in some points the before-mentioned Company in Berlin. Passengers may insure for one or two days, for one, three, or twelve months; the maximum sum is 6,000 th. (£1,000). Officers are divided in four classes, and may insure only for one year.

	Maximum Sum.
Locomotive conductors, firemen, &c. Train conductors, &c.	Th. 1500 (£240).
Directors, engineers, railway post officers, &c. Station officers, booking officers, telegraphists, &c.	Th. 6,000 (£1,000).

The indemnifications given are specified as follows:—

In case of injury, the expenses of medical attendance and nursing, and 50 per cent. of the lost earnings, are given, up to 10 per cent. in case of exterior *or* interior injury; and up to 15 per cent. in cases of exterior *and* interior injury.

In case of mutilation—

75 per cent. for loss of both arms or hands, or both feet, or total loss of eyesight.
60 „ „ of the right arm or hand, or the right eye.
50 „ „ of the left arm or hand, or one foot.
30 „ „ of the left eye.
100 „ in case the accident causes death immediately, or within two months.

Even the particular indemnification for different combinations of the enumerated losses is fixed in the prospectus.

The premiums are, for passengers—

	Sum Insured, 1,000 Thalers.	Sum Insured, 3,000 Thalers.	Sum Insured, 6,000 Thalers.	
For one day	3/180	3/180	3/180	per mille.
„ two days	6/180	6/180	6/180	„
„ one month	..	50/180	45/180	„
„ three months	150/180	100/180	90/180	„
„ one year	240/180	200/180	180/180	„

The Concordia Life Assurance Society in Cöln, registered by Act of Council of His Majesty the King of Prussia, of 26th September, 1853, with a subscribed capital of Pr. th. 10,000,000 (£1,600,000), in 10,000 shares, of which 20 per cent. is paid up, will effect all kinds of life assurance—annuities, Tontine Associations, and assurances against loss of life and personal injury of travellers, caused by the accidents of travelling.

The prospectus is divided into four parts, and the travellers' insurance has a separate prospectus besides these four. The insurance is granted for all journeys of the insured during a certain period, within the bounds of Europe, at sea and on land, by steam navigation, by railway, or any other conveyance.

100 per cent. of the sum insured is paid in case the accident causes death or total incapacity to work; up to 50 per cent. in case the injured is obliged to choose another profession, or in case of lasting consequences; and up to 25 per cent. in cases of less importance.

The premiums are—For one month, $\frac{1}{4}$ per mille; three months, $\frac{1}{2}$ per mille; one year, 1 per mille.

The prospectus No. 1 treats of life assurance payable at death; in case of suicide, the value of the policy is paid to the survivor. The payment of premiums ceases at the age of 85 years; and policies in force longer than five years are bought by the Company at three quarters of their value.

No charges for stamp, postage, &c., except one thaler for each policy. The prospectus contains four tables—

1. Premiums for common life assurance for 100 thalers:—

	Single Premiums.	Annually.		Single Premiums.	Annually.
Age 10 years	26·62	1·27	Age 40 years	45·52	2·83
„ 20 „	31·14	1·40	„ 50 „	56·20	4·21
„ 30 „	37·30	2·04	„ 60 „	68·30	6·70

2. Premiums for life insurance if death happens within one, three, five, seven, and ten years.

3. Single and annual premiums to insure a capital of 100 thalers, payable at death, or if the person insured reaches the age of

Age of the Insured.	55 Years.	60 Years.	65 Years.	70 Years.
10	1·68	1·53	1·42	1·35
20	2·32	2·03	1·83	1·71
30	3·53	2·92	2·53	2·29
40	6·45	4·78	3·86	3·33
50	..	..	7·03	5·49
55	..	..	..	7·58

4. Premiums for survivorship assurance.

The prospectus No. 2 treats of annuities: no expenses are charged, only the stamps. It contains also four tables:—

1. Ordinary annuities. The price for an annuity of 10 thalers yearly is fixed—

Age 5 years	Th. 216·	Age 50 years	Th. 138·80
„ 10 „	„ 218·37	„ 60 „	„ 106·37
„ 20 „	„ 204·67	„ 70 „	„ 76·47
„ 30 „	„ 190·20	„ 75 „	„ 66·60
„ 40 „	„ 169·53		

2. Deferred annuities.

3. Annuities for two lives, payable till the death of the last.

4. Annuities for two lives, one half only payable after the death of the first.

The prospectus No. 3 treats of savings'-bank assurance, for deposits. With one year's notice, the Company grants 3½ per cent. compound interest; with six months' notice, 3 per cent.; with three months' notice, 2 per cent.

The four tables are—1. The amount of capital, with 3½ per cent. compound interest from 5 to 50 years, for single payment; 2. Ditto ditto, for an annual payment; 3. The premiums for assurance of 100 thalers, payable if the insured reaches a certain age; 4. Ditto ditto at a certain time at all events; the payment of premiums ceasing with the death of the assured.

The prospectus No. 4 treats of Tontine assurance. Children born in the same year are united in one class of Tontine, and the accumulated capital is divided when the children have passed the age of 21 years. The first division will take place in 1867, among persons born in 1845. The Company charges 5 per cent. for expenses of management, deducting it from the payments, and grants compound interest at 3½ per cent. The other arrangements are made according to the plan of the French Tontines, which is generally known.

I am an advocate of Tontine Institutions; but I cannot suppress the remark, that I think it very unfair to promote the business by saying in the prospectus, "One of the eldest Societies of this kind, which cannot be without experience, assures that a child entering the Tontine in the first year of life, may expect at the age of 21 years, for an annual contribution of 10 thalers, the amount of at least 1,250 thalers; but considerably more if the circumstances are favourable."

I know that this assertion is made in the prospectus of *La Caisse Paternelle* and other French Societies, but I do not consider this an excuse for the directors of the *Concordia*. I consider them to blame either for copying such assertions without a close examination, or for repeating it after knowing that they are untenable. A very simple calculation will show that an expectation of 600 instead of 1,250 thalers might be deemed rather high.

The tables annexed to this prospectus give the single or annual payments for children born from 1845 to 1854.

We have been favoured with the following summary by Herr Rath G. Hopf, the able Manager of the Gotha Life Assurance Bank, and Foreign Correspondent of the Institute of Actuaries.

New Business and Position of the Life Assurance Companies of Germany in the Year 1853.

Established.	Name of Company.	Year.	Assurances existing at the beginning of the Year.		New Assurances during the Year.		Assurances existing at the end of the Year.		Income (Premiums and Interest).	Claims paid.		Expenses of Management.			Assurance Fund.				Share Capital.	
															In General.		Reserve and advanced Premiums.	Clear Surplus.		
			Persons.	Sums.	Persons.	Sums.	Persons.	Sums.		Persons.	Sums.	Absolute.	Per Cent. of Income.	Per £1,000 of Assured Sums at the end of the Year.	Amount.	Per Cent. of Assured Sums at the end of the Year.			Nominal.	Paid up.
				£.		£.		£.	£.		£.	£.			£.		£.	£.	£.	£.
1829	Gotha	1853	17,715	4,004,057	1,324	307,700	18,427	4,159,314	187,911	390	90,514	7,397	3·94	1·78	1,043,778	25·09	846,708	175,895	Mutual	Mutual
1829	Lubeck	1853	2,170	498,977	713	168,632	2,714	609,094	27,909	63	12,874	3,646	13·06	5·98	70,498	11·58	..	..	72,857	7,286
1831	Leipsig	1853	4,552	747,457	256	41,200	4,645	761,443	34,332	97	16,200	1,968	5·73	2·58	194,424	25·53	167,613	22,011	Mutual	Mutual
1831	Hanover ..	1853–4	2,913 Policies	241,400	97 Pol.	8,543	2,920 Policies.	243,428	6,763	60	4,786	452	6·68	1·85	38,370	15·11	..	..	Mutual	Mutual
1834	Trieste Generali	1853	2,520	541,000	350	77,143	2,700	585,714	37,854	..	..	..	..	..	97,492	..	84,236	13,257	200,000	20,000
1836	Berlin	1853	7,225	1,198,600	600	91,443	7,470	1,244,014	61,573	186	29,178	3,670	5·96	2·95	251,092	20·18	199,035	52,056	142,857	28,571
1836	Munich	1853	1,644	172,824	283	29,224	1,806	189,609	7,143	24	3,200	..	..	..	31,086	16·39	..	..	Capital of the Loan Bank.	
1840	Vienna	1853	2,388	167,771	571	23,439	2,872	181,833	8,109	35	3,778	1,143	14·09	6·29	22,506	12·38	..	..	Mutual	Mutual
1842	Brunswick..	1853	715	54,743	74	5,528	766	58,314	1,914	11	1,200	..	..	..	6,395	10·97	..	..	Mutual	Mutual
1845	Frankfort ..	1853	803	154,873	183	38,794	908	176,063	7,887	10	2,774	792	10·00	4·49	36,016	..	..	None	244,898	24,490
1846	Hammonia At Hamburg	1853	1,230	84,724	293	43,314	1,268	108,900	3,778	42	3,643	1,333	35·28	12·24	None	None	None	None	3,605	3,605
1847	Janus At Hamburg	1853	2,945	357,704	661 Pol.	83,649	3,370 Policies.	411,267	15,011	39	5,380	2,375	16·77	6·12	29,469	7·17	..	..	71,429	7,143
1852	Teutonia At Leipsig	1853	..	..	153 Pol.	21,244	153 Policies.	21,244	..	..	..	..	..	..	..	..	..	..	85,714	..
	Total ..	..	46,820	8,224,130	5,558	939,853	50,019	8,750,237	400,184	957	173,527				1,821,126				821,360	91,095

A corresponding statement for the year 1852 will be found in the *Assurance Magazine*, Vol. IV., p. 136. The only addition to the previous list is the Teutonia, established at Leipzig in 1852. The number of policies or lives assured in 12 Companies existing at the end of the year 1852 is there stated as 46,980; in the present account, 46,820. The difference arises in the Lubeck Company, which in the former report was only given by conjecture. The new assurances in the year 1853, including the new Company, show an increase of 322 policies or lives, and £98,011 sums assured, over the new business of 1852; whilst the total assurances existing at the end of the year are in excess of those at the end of the year 1852 by 4,039 lives or policies and £526,108 sums assured, and amount to £8,750,237, assured on 50,019 lives or policies. The income from premiums and interest has increased £22,556 per annum in the course of the year, and the total assurance fund has increased from £1,680,314 to £1,821,126, notwithstanding that the claims paid in the latter year (£173,527) have exceeded those of the former by £11,554. To those who look upon Life Assurance Institutions as evidences of prosperity, as well as the growth of an unselfish and yet prudent spirit in the mass of the population, these results cannot but be highly satisfactory. We rejoice to perceive that a system which has in the course of a century been the means of such blessing to this kingdom, should begin to be appreciated as it deserves in other countries. (S. B.)

Frauds in Life Assurance.—Extract of a Letter from Herr Rath G. Hopf, 3rd October, 1854.—In respect to the frauds committed at Hildesheim, heavy punishments have been inflicted on the impostors; and two agents and a medical reporter, who deceived their Societies, have likewise been severely punished. May their example be of a deterring effect. The more the benefits of life assurance are acknowledged on one side, and the further they extend, the more impostors speculate to abuse them to the purpose of blameable speculations, and the Offices must employ every precaution to guard themselves against such abuse. At present a lawsuit is going on at Berlin on account of deceits practised upon several German and English Companies by Jews, who, though much advanced in age, were assured as much younger than they were by means of false certificates of birth. The registers of the births of the Jews were formerly kept, in most German countries, not by the officers of the State, but by the rabbins. This is still the case in some countries. In others, as in Prussia, the registers are kept by the magistrates; but neither by them nor by the rabbins with that accuracy and sureness which are shown relative to the Christian population, by the clergy. Thus it has been possible to obtain false certificates of birth by bribery. In some cases the magistrate himself was agent of the cheated Company. No doubt these frauds will be visited with heavy punishments, as well at Berlin as at Hildesheim. (S. B.)

Fire Insurance in Hamburg.—Nearly all German Fire Insurance Companies, and a great part of the English ones, have agencies in Hamburg, and the greatest part of Hamburg property is insured with them. All Hamburg buildings must be insured with a mutual Institution called the "General Feuer Casse." But Hamburg has also an Insurance Company exclusively for fire insurance, "The Feuer Assecuranz Compagnie von 1843," the balance-sheet of which for 1853 I annex; and three of the Marine Insurance Companies have also extended their operations to fire

insurance. These are—the Patriotische Feuer Assecuranz Compagnie; Neue Fünfte Assecuranz Compagnie; See und Feuer Assecuranz Compagnie. The latter discontinued the business of fire insurance since 1852, and now acts only for marine insurance, under the name of "Neptunus Assecuranz Compagnie."

A summary of the fire insurance business of these three Companies, extracted from the balance-sheets, will be, I hope, not without interest.

The sums insured diminish from year to year, and the average premium also becomes less. Both facts may be considered as a consequence of the increasing number of agencies of foreign Societies. The results of the operations of these Companies are very satisfactory, considering the small extent of their business.

*Summary of the business of Fire Assurance Companies in Hamburg, 1843 to 1852 inclusive.**

Name of Company.	Year.	Sum Insured.	Premium.	Claims Paid.	Average Premium per Cent.	Claims per Cent. on Sums Insured.
		£.	£.	£.		
Patriotische Assecuranz Compagnie in Hamburg (Patriotic Insurance Company in Hamburg).	1843	2,314,737	8,796	976	0·38	0·042
	1844	2,125,575	7,818	306	0·36	0·014
	1845	2,065,641	6,603	4,536	0·32	0·220
	1846	1,695,792	5,292	1,071	0·31	0·063
	1847	1,788,360	5,526	..	0·31	0·000
	1848	1,602,042	5,556	2,469	0·35	0·154
	1849	1,953,234	5,367	..	0·27	0·000
	1850	1,980,372	5,742	3,291	0·29	0·166
	1851	1,651,206	5,050	622	0·31	0·038
	1852	1,630,008	5,775	635	0·35	0·039
	1853	1,614,057	4,725	512	0·29	0·032
Neue Fünfte Assecuranz Compagnie in Hamburg (New Fifth Insurance Company in Hamburg).	1843	2,558,838	13,344	4,050	0·52	0·159
	1844	2,140,637	9,940	1,152	0·46	0·054
	1845	2,069,925	9,266	6,570	0·45	0·317
	1846	1,632,700	8,277	2,580	0·51	0·158
	1847	1,548,330	6,927	885	0·45	0·057
	1858	1,445,000	6,061	3,396	0·42	0·235
	1849	1,330,425	5,751	2,328	0·43	0·175
	1850	1,449,054	5,900	1,440	0·41	0·098
	1851	1,217,988	5,310	390	0·44	0·032
	1852	1,072,566	4,740	1,363	0·44	0·127
	1853	1,020,159	4,854	1,008	0·48	0·099
See und Feuer Assecuranz Compagnie in Hamburg (Marine and Fire Insurance Company in Hamburg).	1843	946,780	4,023	1,404	0·42	0·148
	1844	884,450	3,807	93	0·43	0·010
	1845	812,565	2,785	2,258	0·34	0·278
	1846	497,550	1,755	156	0·35	0·032
	1847	553,030	1,725	438	0·31	0·079
	1848	455,805	1,434	501	0·31	0·110
	1849	363,910	1,275	32	0·35	0·008
	1850	403,560	1,251	603	0·31	0·150
	1851	287,835	783	343	0·27	0·120
	1852	99,300	276	150	0·28	0·152

* Banco marca, 13⅓ = £1.

Hamburg, August, 1854. Wilhelm Lazarus.

CORRESPONDENCE.

ON THE VALUE OF CONTINGENT REVERSIONARY INTERESTS.

To the Editor of the Assurance Magazine.

SIR,—In one of the first papers read before the Institute of Actuaries, Mr Jellicoe pointed out "the method of procedure which should be observed in determining the value of life contingencies, where the risk was of an isolated character, as distinguished from cases in which the usual considerations of average presented themselves." The paper was not, I believe, printed for the use of the members, but a short account of it is given in the *Post Magazine* of the 10th March, 1849. This was followed by one from Mr. Hardy, "On the values of annuities which are to pay certain given rates of interest on the purchase money during the whole term of their continuance, and to replace their original values, on their expiration, at certain other given rates"—where for the first time also is given, in an intelligible form, an expression for the value of an annuity certain under such conditions; in accordance with this a set of tables was constructed, and appended to the paper read before the Institute 25th November, 1850.

In Vol. II. page 159 of the *Assurance Magazine* will be found a further communication from Mr. Jellicoe on contingent reversions, where, after alluding to the labours of Mr. Sang on the same subject, he speaks of marketable securities *as constituting investments to be made not subject to any contingency whatever, but as securing to the holder of them, in any case, a given rate of interest so long as he retains them, and reproducing the capital invested when such interest shall cease to be paid or to accrue.* In this point of view it is clear that the price a purchaser should pay for an annuity or reversion would be such as to enable him to get rid of the contingencies, if he wished to do so, without losing any of the usual advantages derivable from the purchase.

It may appear, and probably is, unnecessary to attempt to add anything to what has been so well done on this interesting subject; but, considering the importance of it to actuaries of the present day, owing to the frequency with which applications for loans on reversionary interests are now made to Insurance Offices, I have thought it may be useful once more to advert to it.

I propose to work out the two cases of contingent reversions given by Mr. Jellicoe, and to append a short table to each showing the marketable value of such reversions—that is, securing at the expiration of the contingency the capital advanced, and providing in the interval for the interest stipulated for.

The formula given by Mr. Griffith Davies for determining the value of an annuity on an isolated life is $\frac{1}{d+p}-1$. That given by Mr. Jellicoe as the present value of £1 annuity payable during the life of A, to commence at the death of B, is $\frac{1}{d+p}-(1+\mathrm{AB})$, the truth of which is sufficiently evident without demonstration.

The annexed table, constructed from this formula, is made up of Nos. 3 and 5 of Davies's *Tables of Life Contingencies*—No. 3 giving $\frac{1}{d+p}-1$

at 5 and 6 per cent. with the Northampton premium, as charged by the Equitable; No. 5, the Equitable experience at $3\frac{1}{2}$ per cent. The former may be adapted to any other rates of premium, by selecting the nearest age in the Northampton Table corresponding with the premium proposed to be charged; the latter is taken to represent the fair value of an annuity on the joint existence of two lives, and which is intended to form part of the sum advanced.

If $s=\frac{1}{d+p}-1-\mathrm{AB}$, then $s+\mathrm{AB}=\frac{1}{d+p}-1$, and $s+\mathrm{AB}+a=$ the sum to be assured; $p(s+\mathrm{AB}+a)=$the annual premium for ditto; $s+\mathrm{AB}+p(s+\mathrm{AB}+a)=$the purchase-money, the price of an annuity during the joint lives, and the first year's premium, making together the sum advanced; $i\{s+\mathrm{AB}+p(s+\mathrm{AB}+a)\}=$annual interest; and consequently $p(s+\mathrm{AB}+a)+i\{s+\mathrm{AB}+p(s+\mathrm{AB}+a)\}=a$.

Suppose A, aged 25 years *next* birthday, to be entitled to an estate, producing £1,000 per annum, on the death of B, aged 65 years *last* birthday, and to require an immediate advance of £1,000, by way of deferred annuity. To determine the amount of such annuity, so as to allow the lender 5 per cent. interest besides the Northampton premium,

$\frac{1}{d+p}-1=12{\cdot}958$

AB = 8·331 (Equitable $3\frac{1}{2}$ per cent.)

4·627 = Value of A's interest per £.

·216123 = Reciprocal of ditto, or ratio of annuity

1000

216·123 = Deferred annuity to be granted.

216·123 × 8·331 =	1800·521	= Price of annuity during joint lives
133·8	1000·	= Sum paid to borrower
	216·123	= One year's annuity
1728984		
64837	3016·644	= Sum to be insured.
6484		
216		
1800·521		

3016·644 × ·02404 =	72·520	= Annual premium
40420·	1000·	
	1800·521	
6033288		
1206658	2873·041	= Sum advanced
12066	05	
72·52010	143·65205	= Interest on ditto
	72·520	= Annual premium
	216·172	= Annuity as above (very nearly).

TABLE I.—*Showing the value of an Annuity of £1, on A, after the death of B; allowing the purchaser a given rate of interest on the sum advanced, besides the premium necessary to secure his capital by a life assurance according to the Northampton 3 per Cent. Table.*

Age of A.	Age of B.	Interest, 5 per cent.		Interest, 6 per cent.	
		Value of £1 Annuity on A after B.	Annuity which £1 will purchase.	Value of £1 Annuity on A after B.	Annuity which £1 will purchase.
20	50	1·351	·7402		
	55	2·469	·4050	·817	1·224
	60	3·655	·2736	2·003	·4993
	65	4·798	·2009	3·326	·3007
	70	6·373	·1569	4·721	·2118
	75	7·816	·1279	6·164	·1622
	80	9·327	·1072	7·675	·1303
25	55	2·188	·4570	·632	1·5823
	60	3·336	·2998	1·780	·5618
	65	4·627	·2161	3·071	·3256
	70	5·993	·1669	4·437	·2254
	75	7·411	·1349	5·855	·1708
	80	8·903	·1123	7·347	·1361
30	60	2·980	·3356	1·528	·6545
	65	4·233	·2362	2·781	·3596
	70	5·570	·1795	4·119	·2428
	75	6·964	·1436	5·512	·1814
	80	8·436	·1185	6·984	·1432
35	65	3·771	·2652	2·431	·4114
	70	5·074	·1971	3·734	·2678
	75	6·443	·1552	5·103	·1960
	80	7·897	·1266	6·557	·1525
40	70	4·528	·2208	3·312	·3019
	75	5·846	·1711	4·630	·2160
	80	7·278	·1374	6·062	·1650
45	75	5·226	·1914	4·139	·2416
	80	6·619	·1511	5·533	·1807
50	80	5·909	·1692	4·960	·2016

It will be seen from the foregoing table that a difference of not less than thirty years is taken between the age of A, "the tenant in reversion," and that of B, "the tenant in possession." Where the difference is less than thirty years, the annuity would either have a negative value, or be so small as almost to preclude the possibility of a loan being carried through upon the conditions here stated.

The other formula given by Mr. Jellicoe in the paper alluded to is, $1-(p+d)(1+AB)$, as the present value of £1 to be paid on the decease of B if A survive; p being here the premium of insurance on A against B.

If we have the value of £1 payable upon A dying before B, and £1 upon B dying before A, the two are together equal to the value of £1 payable on the failure of their joint lives. The expression for this, in Mr.

Milne's notation, is, $\mathfrak{A}\mathfrak{B}=\underset{1}{\mathfrak{A}\mathfrak{B}}+\underset{1}{\mathfrak{B}\mathfrak{A}}$ ∴ $\underset{1}{\mathfrak{B}\mathfrak{A}}=\mathfrak{A}\mathfrak{B}-\underset{1}{\mathfrak{A}\mathfrak{B}}$; and since $\mathfrak{A}\mathfrak{B}=1-(1-v)(1+\mathrm{AB})=1-d(1+\mathrm{AB})$, and $\underset{1}{\mathfrak{A}\mathfrak{B}}$, the single premium for insuring £1 upon A dying before B, $=p(1+\mathrm{AB})$ ∴ $\underset{1}{\mathfrak{B}\mathfrak{A}}=1-(p+d)(1+\mathrm{AB})$.

The formula $\mathfrak{A}\mathfrak{B}-\underset{1}{\mathfrak{A}\mathfrak{B}}$ is better adapted for calculation than $1-(p+d)(1+\mathrm{AB})$, because the value of $\mathfrak{A}\mathfrak{B}$ may be found for any given rate of interest from Orchard's *Assurance Premiums*, and $\underset{1}{\mathfrak{A}\mathfrak{B}}$ (where the Northampton, or Carlisle 3 per Cent. premiums are used) can be obtained at once from Davies's *Tables*, and from those by Messrs. Gray, Smith, and Orchard. But then, by using the formula $\mathfrak{A}\mathfrak{B}-\underset{1}{\mathfrak{A}\mathfrak{B}}$, the condition cannot be fulfilled which should be stipulated for in transactions of this kind—that an annuity equal in amount to the reversionary annuity should be purchased during the joint lives of A and B, which can only be done by estimating the value of such annuity at the same rate of interest as is required for the loan.

We have seen that in the example given of the value of a reversionary annuity two rates are introduced, 5 per cent. being required for the advance of money, and $3\frac{1}{2}$ per cent. only allowed for the joint life annuity. But by using the expression $1-(p+d)(1+\mathrm{AB})$, any two rates may be introduced—AB may be taken, as above, at $3\frac{1}{2}$ per cent., and d at 5 per cent.

Taking the same ages as before, what sum should A pay at the death of B for an immediate advance of £1,000, allowing the lender 5 per cent. interest, besides the Northampton premium for insuring the life of A against that of B?

The value will be $\dfrac{1000}{1-(p+d)(1+\mathrm{AB})}$

$p=\cdot01637$ $\quad 1+\mathrm{AB}=9\cdot331$ (Equitable $3\frac{1}{2}$ per cent.)
$d=\cdot04762$

$\cdot06399$
$133\cdot9$

$\cdot57591$
1920
192
6

$\cdot59709$

$\cdot40291=1-(p+d)(1+\mathrm{AB})$

$\dfrac{1000}{\cdot4029}=2482$, the sum required.

$\dfrac{2482-1000}{9\cdot331}=158\cdot825$, the amount of annuity to be purchased during the joint lives.

Then proceeding as in the former example—

158·825 × 8·331 = 1323·171 = Price of joint life annuity
133·8 1000·000 = Sum paid to borrower
158·825 = One year's annuity
1270600 2481·996 = Sum to be insured.
47647
4765
159
1323·171

2482 × ·01637 = 40·62 = Annual premium
73610· 1000·
1323·171
2482 2363·791 = Sum advanced
1489 ·05
74
17 118·18955 = Interest on ditto
40·62 40·62 = Annual premium
158·809 = Annuity as above.

Taking AB at 5 per cent. and using the formula $\mathfrak{A}\mathfrak{B} - \underset{1}{\mathfrak{A}\mathfrak{B}}$, we find, from Orchard's *Tables* (AB being = 7·528), $\mathfrak{A}\mathfrak{B}$ = ·59391; and, by Davies's *Tables*, Northampton 3 per Cent., we have $\underset{1}{\mathfrak{A}\mathfrak{B}}$ = ·13701 ∴ $\mathfrak{A}\mathfrak{B} - \underset{1}{\mathfrak{A}\mathfrak{B}}$ = ·4569; and, if the sum advanced be 1,000, the equivalent sum to be paid on the death of B if A survive, will be $\frac{1000}{\cdot 4569} = 2188\cdot663$, which is also the sum to be assured on A against B, the single premium for which will be 2188·663 × ·13701 = 299·869. We have consequently 2188·663 payable at the death of the joint lives, the present value of which is,

2188·663 × ·59391 = 1299·869
Deduct premium of insurance = 299·869

Leaves £1000· the sum advanced.

The same method of treating all isolated cases of annuities and reversions, whether absolute or contingent, immediate or deferred, should be used whenever it is desired to arrive at the *market value* of such securities.

Thus in the case of a simple reversion the formula will be $1 - d(1 + \mathrm{A})$; and although values so obtained appear very small at the early ages of life, they will be found to approximate to the ordinary tabular values at the more advanced ages.

Taking the sum in reversion as £1, and using Davies's Equitable Experience, interest 5 per cent., it will be seen that

At age 50 the market value is ·32005, and the ordinary value ·40944
„ 60 „ „ ·45326 „ „ „ ·51060
„ 70 „ „ ·60050 „ „ „ ·63077
„ 80 „ „ ·75218 „ „ „ ·76360

TABLE II.—*Showing the value of £1, payable on the death of B, if A survive; allowing the purchaser a given rate of interest on the sum advanced, besides the premium necessary to secure his capital by an insurance on A against B, according to the Northampton 3 per Cent. Table.*

Age of B.	Age of A.	Interest, 5 per cent.		Interest, 6 per cent.	
		Value of £1 on B against A.	*Amount per £ to be insured on A against B.	Value of £1 on B against A.	*Amount per £ to be insured on A against B.
80	50	·6348	1·575	·5912	1·691
	45	·6508	1·536	·6065	1·649
	40	·6628	1·509	·6181	1·618
	35	·6738	1·484	·6289	1·590
	30	·6807	1·469	·6356	1·573
	25	·6856	1·459	·6402	1·562
	20	·6903	1·449	·6446	1·551
75	45	·5430	1·842	·4862	2·057
	40	·5586	1·790	·5010	1·996
	35	·5728	1·746	·5148	1·942
	30	·5822	1·718	·5238	1·909
	25	·5884	1·700	·5296	1·888
	20	·5941	1·683	·5349	1·869
70	40	·4595	2·176	·3901	2·563
	35	·4752	2·104	·4050	2·469
	30	·4870	2·053	·4161	2·403
	25	·4946	2·022	·4231	2·363
	20	·5011	1·996	·4290	2·331
65	35	·3803	2·629	·2982	3·353
	30	·3939	2·539	·3111	3·214
	25	·4029	2·482	·3191	3·134
	20	·4100	2·439	·3254	3·073
60	30	·3048	3·281	·2106	4·748
	25	·3148	3·177	·2194	4·558
	20	·3226	3·100	·2260	4·425
55	25	·2348	4·259	·1291	7·746
	20	·2429	4·117	·1357	7·369
50	20	·1668	5·995	·0495	20·202

* These values are the reciprocals of the numbers in the previous columns, and are also the reversions of which £1 is the present value.

I am, Sir,

Yours truly,

ROBERT TUCKER.

Lombard Street, 21st October, 1854.

THE INTEREST IN ASSURANCES EFFECTED BY ONE PERSON UPON THE LIFE OF ANOTHER.

To the Editor of the Assurance Magazine.

SIR,—The letter of your correspondent "Verus," in the last Number of the *Magazine*, induces me to trouble you again upon this subject; especially as, if the views there expressed are entertained by any number of your readers, it is evident that so much difference of opinion exists on this question as to render further discussion desirable.

"Verus" expresses his belief that "the Act 14 Geo. III., cap. 48, has been and is a valuable protection to life assurance"; but the reasons assigned for this opinion seem hardly to warrant such a conclusion. He admits that the question of interest is very rarely raised when a claim is payable, which he looks upon as a proof of the "wholesome influence of the Act," whereas all that it really proves is that one of the main provisions is habitually disregarded; and speaks of the facility and confidence with which an Office can fall back upon the *want of interest* in a case where it may be necessary to prove *fraud or swindling*. Now, bearing in mind that the undoubted object of this Act is to suppress, not fraudulent, but speculative assurances, and that fraud is never mentioned in it from one end to the other, it seems rather equivocal evidence of its successful working, that it accomplishes a purpose never contemplated at the time of its enactment. But surely fraud and gambling are two different things: if the law as it stands is insufficient to suppress the former, by all means let it be made more stringent; but do not let an Assurance Company defeat a swindler by assuming the character of moralist for that single occasion. I am not aware that the Act now under consideration has ever been made available for such a purpose; indeed, so far as I know, the only instance on record of a claim having been successfully resisted on this ground, is in the case of "Godsall *v.* Boldero," where no imputation of fraud was even insinuated.

"Verus" further asserts, that a comparative immunity from gambling assurances must exist, because the question of interest is so seldom agitated. But this, again, is only begging the question: it is notorious, that when once a policy has been issued, the interest of the assured is thenceforth, by general consent, ignored altogether; because it is so universally admitted to have no bearing upon the contract, that any Office objecting to pay a claim for such a reason would at once lose ground in public estimation.

We have abundant evidence that speculation in life policies prevails to a great extent in open defiance of the law. They are advertised for sale almost daily; auctioneers have what they term "periodical sales," in which this class of property is regularly included; Reversionary Societies announce in their prospectuses their willingness to purchase life assurances—one of them, in its recently published report, boasting of its success in this branch of its business; and many private individuals are known to be constantly in the habit of buying policies. I myself met with an instance, in the case of a gentleman recently deceased, where no less than fifty-nine policies, on the lives of various persons, formed part of the assets of his estate. And all these transactions are entered into upon the faith that the Offices will unhesitatingly pay the claims, in accordance with their invariable *practice*, but in utter disregard of the *law*, whose declared object is to prohibit such dealings

altogether, and not, as "Verus" seems to suppose, to protect Offices against the cases in which "any thing fraudulent or incorrect is mixed up with the policy."

Again: "Verus" remarks that the case I had "referred to as an example of an illegal insurance is strictly within the meaning of that term"; and that a person wishing to provide for a relative after his own death, can have no difficulty, upon the score of interest, in effecting a policy in the name of the latter in any Office. To this the simple answer is, that since the publication of his letter I have made some further inquiries upon this point, and can now assert, as a matter of fact, that leading Companies, transacting an extensive business, *do* refuse to entertain such proposals, informing the applicant that he must effect the assurance in his own name. This is a fair illustration of the varying practice that prevails in regard to this law.

Further: "Verus" contends that assuring the life of a debtor may be a bad speculation, but is a legitimate transaction. It may be a bad or good speculation, according as the event may prove; but once admit that it is a *speculation*, and there is an end of the argument. Where, then, is the efficiency of this law that professes to put a stop to speculative assurances altogether?

As regards Irish assurances, I believe that I was justified in the assertion that neither assurers nor assured were, generally speaking, aware that the Gambling Act did not apply to Ireland; it was certainly a very common application from that part of the kingdom to have the interest admitted on the policy. But be this as it may, it is I think undeniable that in former years some of the older Companies attached great importance to the question of interest in Irish assurances; and if this be true, the existing state of the law in that country certainly cannot be charged with the undue mortality that is known to have prevailed among Irish lives.

I believe that I have now noticed all the arguments adduced by "Verus," and confess that I see no reason to depart from any of the statements put forth in my former letter. Much might have been added, of the ignorance of the principles of life assurance evinced both in the framing of this Act by the legislature, and in its interpretation by the courts of law; the erroneous idea having apparently been entertained, that a life assurance policy is a contract of indemnity, resembling a fire or marine insurance. But this objection it can be necessary only to mention, the subject having already been treated by Mr. De Morgan, with his usual clearness and force, in a work (*Essay on Probabilities*, page 244) familiar to everyone who has made the theory of life assurance his study.

It can hardly be denied that a law must be in a very unsatisfactory state, when of its two main provisions one is altogether unheeded, and the other very partially acted upon. The so-called Gambling Act is in this predicament; and therefore, should circumstances again bring it under consideration, the only alternatives seem to be—either that it should be rendered more stringent, or abandoned altogether. In arriving at the conclusion that the latter is the more desirable course, it is certainly not from any favour for speculative assurances, but quite the reverse. I am not disposed to agree with the views of "Verus" as regards policies on the lives of debtors; on the contrary, I would look with some degree of suspicion upon all proposals where the pecuniary interests of the parties concerned would be best served by the failure of the life proposed, and I think that the motive for

the assurance might in every case be a legitimate subject of inquiry in the first instance. But just as (to cite one out of many similar cases that might be mentioned), although no one defends exorbitant rates of interest, yet the usury laws did not prevent their exaction, and have been swept away; so, not desiring speculation in life assurance, I doubt the wisdom of legislative interference in such a matter, because I doubt its power to attain the object it has in view. I would leave it entirely free, to be settled between the contracting parties; if a speculative assurance is offered, *caveat emptor*. I firmly believe that such freedom of action would not encourage or increase any "mischievous kind of gaming," while it would effectually abolish the anomalies and inconsistencies of the present state of things.

I am, Sir,

Your most obedient Servant,

Eagle Insurance Office,
1st November, 1854.

ARTHUR H. BAILEY.

NOTE.—It will be interesting to our readers to know that the leading case of Godsall *v.* Boldero (9 East, 72), referred to in the above communication—in which it was decided that a life policy by one person on the life of another was a mere contract of indemnity, and that therefore when a creditor insured the life of his debtor the policy became void upon payment of the debt, whatever the number of premiums the Office might have received—has been overruled by the decision of the Court of Exchequer Chamber, sitting as a Court of Error, in the case of Dalby *v.* the India and London Life Assurance Company (xviii. *Jurist*, 1024. *Weekly Reporter*, 16 Dec., 1854). We may hereafter give the report of this case at length; but for the present it will be sufficient to state that it decides that the contract is not one of indemnity—that the Gambling Act (14 Geo. III., cap. 48) was not declaratory of the common law, and that the requirements of that statute are satisfied by the existence of an insurable interest at the time that the policy is granted—and that the policy is not affected by the cesser of the interest at the time of the death. (*See* Bunyon on *Life Assurance*, pp. 6—26, where the view taken by the Court of Error is contended for.—ED. *A. M.*)

WORKS RECENTLY PUBLISHED.

Friendly Societies' Sickness and Mortality. Mr. Alexander Glen Finlaison's Report.' Second Part. Ordered by the House of Commons to be printed, 12th August, 1854.

Census of Great Britain, 1851. Population Tables II. Ages, Civil Condition, Occupations, and Birthplace, of the People; with the Numbers and Ages of the Blind, the Deaf and Dumb, and the Inmates of Workhouses, Prisons, Lunatic Asylums, and Hospitals. Presented to both Houses of Parliament, by command of Her Majesty, 1854.

Nouvelles Tables pour les Calculs d'Intérêts simples et composés, d'Amortissement, d'Annuités, de Primes, &c. Par P. A. Violeine, Membre de l'Ordre impérial de la Légion d'Honneur, Chef de Bureau au Ministère des Finances, &c. Paris. 1854.

REPORTS OF ASSURANCE COMPANIES.

National Life Assurance Society.—Report of the Receipts and Payments for the Year ending 31st December, 1853.

	£.	s.	d.	£.	s.	d.
Balance in hand 1st January, 1853				4,075	7	2
Premiums:—						
On assurances renewed	33,424	15	0			
Commuted on policies 56, 195, 415, 975	725	8	5			
On policies issued during the year 1853	4,122	18	0			
Commuted premiums on ditto	752	4	9			
Additional for error in age, policy 1,012	67	4	2			
Extra for various foreign risks	705	3	10			
				39,797	14	2
Sale of £18,800, New 3½ per Cents.				18,912	0	0
Ditto 26,000, Consols				25,932	10	0
Loans paid off—						
Mortgages	10,242	17	2			
On policies of the Society	1,470	0	0			
On ditto, with collateral security	250	0	0			
				11,962	17	2
Interest on mortgages, and dividends on stock				10,358	10	4
Received for interest and premiums in advance				3,069	1	8
„ for policy stamps, up to 10th October				183	15	0
„ for bond stamps				6	18	9
Commission on reinsurances				23	13	5
Loan on security of Consols				15,200	0	0
				£129,522	2	8

	£.	s.	d.	£.	s.	d.
Amount allowed to the members in reduction of premiums this year				9,856	18	10
Purchase of £1,000 per annum, Long Annuities				5,312	10	0
„ £565 „ Annuities for terms of Years				3,142	16	3
„ £3,000, 6 per cent. Canada Government Debentures				3,457	10	0
„ £5,000, 5 ditto ditto				5,156	5	0
„ £10,000, Southampton Dock Bonds				10,000	0	0
„ £5,000, Stockton and Darlington Railway Bonds				5,006	5	0
Loans on mortgage	£32,700	0	0			
„ on policies of the Society	6,760	7	4			
Ditto, with collateral security	9,930	0	0			
Temporary loan	3,000	0	0			
				52,390	7	4
Paid claims on policies:—Outstanding, 1st January, 1853	£8,250	0	0			
Claims arising in 1853	18,223	14	7			
				26,473	14	7
Purchase of policies				539	3	5
Agencies				329	0	7
Fees to directors, auditors, and medical officers				936	5	0
Office salaries				1,470	0	0
Painting premises				174	14	5
Messenger, postages, printing, advertising, stationery, &c.				516	9	5
Annuity to late Secretary				300	0	0
Law charges				44	12	6
Rates and taxes				137	5	11
Income and property tax				362	17	4
Paid for policy and bond stamps				128	1	9
Premiums for reinsurances				522	5	10
Allowances for premiums and extra premiums overpaid				733	19	6
Balance in hand, 31st December, 1853	£2,155	10	2			
Interest due, but not yet paid	375	9	10			
				2,531	0	0
				£129,522	2	8

Statement of Property belonging to the Society, and the Income arising therefrom.

	Property. £.	s.	d.	Income. £.	s.	d.
New 3¼ per Cent. Stock	37,300	0	0	1,212	5	0
Consols	52,132	6	9	1,563	19	9
Government Annuities, £1,565 per annum	8,455	6	3	380	9	5
Five per cent. Canada Government Debentures	5,000	0	0	250	0	0
Six per cent. ditto ditto	8,000	0	0	480	0	0
Loans secured by mortgages, bonds, and debentures	183,476	8	6	7,386	19	4
Annual income from the above investments				11,273	13	6
Cash balances, as above	2,155	10	2			
Policy stamps on hand	159	4	6			
Freehold premises in King William Street	7,100	0	0			
Amount of annual premiums				37,315	1	4
				£48,588	14	10

A General Statement of the Affairs of the National Life Assurance Society, estimated on the 31st December, 1853, as up to 25th March, 1854.

	£.	s.	d.	£.	s.	d.
Present value of £1,057,959 assured on lives of members, less reinsurances				545,929	18	10
Present value of non-members' policies, less reinsurances				6,586	14	5
Claims admitted, but not yet paid				8,791	16	2
Reserve for possible claims between 1st January and 25th March, 1854				15,000	0	0
Reserve for salaries, fees, and outstanding accounts				1,400	0	0
Loan on security of Consols				15,200	0	0
				£592,908	9	5
Capital:—	£.	s.	d.	£.	s.	d.
New 3¼ per Cents, £37,300	36,181	0	0			
Consols, £68,132. 6s. 9d.	64,725	14	5			
Long Annuities, £1,000 annuity	5,312	10	0			
Annuities for Terms, £565 annuity	3,142	16	3			
Five per cent. Canada Government Debentures, £5,000	5,000	0	0			
Six per Cent. ditto, £8,000	8,800	0	0			
Loans secured by mortgages, bonds, and debentures	183,476	8	6			
Premises in King William Street	7,100	0	0			
Policy stamps on hand	159	4	6			
Cash balances	2,155	10	2			
				316,053	3	10
Various sums payable to the Society before 25th March, 1854				5,000	0	0
Present value of £8,303. 12s. 1d. annual premiums, which members not yet entitled to reduction will pay in full				21,841	8	3
Present value of £33,922. 4s. 10d., amount of annual premiums on members' policies, less reinsurances	£425,089	12	0			
Less value of amount to be paid in full, as above	21,841	8	3			
Present value of amount, subject to reduction	403,248	3	9			
Thirty-eight per cent. reduction thereon	153,234	6	5			
				250,013	17	4
				£592,908	9	5

Palladium Life Assurance Society.—Capital and Assurance Account, December 31, 1858.

Dr.	£.	s.	d.
Paid-up capital and additions	74,962	12	3
Reserve fund	12,377	15	5
Provision for outstanding risk on assurances	286,876	8	7
Dividends due	3,268	0	4
	£377,484	16	7

Cr.			£.	s.	d.
Mortgage investments			254,295	0	1
Government securities			30,144	16	4
Annuity investments			14,580	12	7
Advances on policies and bonds			39,355	0	0
India transfer loan, and Palladium stock			13,455	13	10
Estimated value of house and fixtures			4,530	0	0
Amount at debit of agents	£1,985	10 2			
Premiums otherwise outstanding	1,970	7 4			
			3,955	17	6
Cash at London and Westminster Bank, on deposit, and in current account			17,167	16	8
			£377,484	16	7

Transactions of the year 1858.

Dr. *Receipts.*	£.	s.	d.
Balance of assets from previous year	373,625	18	7
Premiums	44,214	12	0
Commission on reassurances	197	10	10
Interest	16,411	17	11
Proceeds of reassurances cancelled	135	0	0
Transfer fines	11	5	0
	£434,596	4	4

Cr. *Payments.*	£.	s.	d.
Claims settled in the course of the year	42,184	2	1
Returns on surrender of policies	1,131	8	10
Premiums on reassurances	3,726	0	11
Charges, including agencies and medical reports	5,208	16	8
Commission	1,535	19	3
Proprietors' dividends	3,325	0	0
Balance (assets) invested as above	377,484	16	7
	£434,596	4	4

Number of new policies issued in the year, 139. Amount assured, £128,849. New premiums, £4,159. 11s. 8d.

			£.	s.	d.
Amount under assurance 31st December, 1853			1,349,479	14	9
Ditto 1852, as in last report	£1,280,198	19 7			
Bonus added on completion of valuation	53,243	14 5			
			1,333,442	14	0
Increase at close of 1853			£16,037	0	9

	£.	s.	d.
Balance of assets, 1853	377,484	16	7
Balance of assets, 1852	373,625	18	7
Increase in 1853	£3,858	18	0

Provident Clerks' Mutual Life Assurance Association (Benevolent Fund).—District Meeting at Liverpool, May, 1852.—From the deputy chairman's address it appeared that they had 101 assurers in the town of Liverpool, to the amount of £29,850, and eight subscribers to the benevolent fund. Since their last meeting, in 1849, they had only added 32 policies to the number which then existed. In the assurance department the Society, up to the 31st March, 1852, had issued 3,168 policies, amounting to £614,390, some of which, of course, had dropped. The amount invested in stock and mortgages was upwards of £42,000, and the amount of annual premiums on existing assurances upwards of £16,000.

The Provident Clerks' Mutual Life Assurance Association and Benevolent Fund.—Since the establishment of the Association, to the 31st December, 1852, 3,471 life policies had been issued, assuring £673,140; and there remained in force at the same period 2,852 policies, covering assurances to the amount of £556,436.

The state of the Society on the 31st December was as follows, viz.:—

	£.
Liabilities for the—	
Values of 2,756 policies under Table A 1	33,728
„ 201 policies under other tables	9,295
Value of declared bonuses (1848)	420
Making the total liabilities	£43,443
Assets as per account	50,565
Balance, net profit for division	7,122
One tenth assigned to the benevolent fund	712
Remainder, applicable for bonuses to members	£6,410*

If the division be confined to assurances of three or more years' standing, effected under Table 1, the sum required to give a bonus of 12*s.* per cent. per annum would be £5,370. 2*s.* 3*d.*, and the proportionate reserve to be made for assurances effected during the last three years would be £973. 5*s.* 7*d.*, making together £6,343. 7*s.* 10*d.*; and it was therefore recommended that a bonus, at the rate of 12*s.* per cent. per annum, should be declared on all policies entitled to participate therein.

Statement of Assets and Liabilities, 31*st December,* 1852.

Assets.	£.	*s.*	*d.*
£35,500 stock, 3¼ per Cents, cost	33,932	4	1
Amount lent on mortgage	11,050	0	0
Lease of house, furniture and fixtures	572	18	5
Cash at Bank of England	2,041	7	8
Petty cash on hand	0	10	7
Balance due by agents	1,072	12	6
Policy stamps on hand	61	0	0
Loans to members on security of their policies	520	19	0
Balance due by benevolent fund	75	17	4
Renewal premiums due in December, 1852	1,047	0	0
Quarterly and half yearly premiums	1,425	6	1
Interest accruing on £35,500 stock, £11,050 mortgages, and £520. 19*s.* loans to members, up to 31st December, 1852	420	3	5
Premiums due on policies on the half credit system	45	3	0
	£52,265	2	1

* Equivalent to a reversionary bonus of over £13,800.

Liabilities.	£.	s.	d.
Values of 2,756 policies, assuring £534,871 under Table A 1 . .	33,727	8	11
„ 74 policies, assuring £12,248 under Table A 2 (Endowment Assurances)	2,293	11	8
Values of 51 policies, assuring £914 under Tables B 1 and B 2 (Deferred Annuities, premiums returnable and not returnable) . . .	2,233	0	0
Values of 37 policies, assuring £3,980 under Tables C 1 and C 2 (Endowments, premiums returnable and not returnable)	862	0	0
Value of £424 per annum, Annuity Table D	3,688	7	8
Values of 23 policies under other tables, assuring £9,348 . . .	217	16	1
Value of reversionary bonuses accepted at the last valuation (£807. 10s.)	420	0	0
Claims admitted in 1852, but not due till 1853	1,700	0	0
Profit for division	7,122	17	9
	£52,265	2	1

Scottish Amicable Life Assurance Society (*Established* 1826).—*Report by the Ordinary Directors to the Annual Meeting*, 20*th March*, 1851.—The amount of capital sums assured during 1850 has been £356,367, by 944 policies, and corresponding premiums £12,273. 0*s*. 8*d*.—the net increase to the total capital sums assured, deducting for policies emerged, forfeited, surrendered, and not taken up, being £272,354. 9*s*., by 809 policies, and corresponding net increase of premiums £8,764. 15*s*. Besides, there has been received, of single premiums for assurances and annuities, £4,962. 12*s*. 9*d*.

The rate of mortality during the year has not been so much as one per cent.; but from the falling in of some heavy policies, the per centage of claims considerably exceeds the per centage of deaths amongst the members. Among the many who have been assured through the Office in London, not one death has occurred since its opening, 18 months ago.

Local boards have been established in Dublin and Aberdeen.

Amount of assurances in force, £1,371,700. Present annual income, £55,500. Accumulated fund, £270,500.

Scottish Amicable Life Assurance Society.—Report by the Ordinary Directors to the Annual Meeting, 18*th March*, 1852.—The new business for the year 1851 has very considerably exceeded that of 1850.

New business in	No. of Policies.	Sums assured. £.	s.	d.	Corresponding premiums. £.	s.	d.
1850 ..	944 ..	356,367	0	0 ..	12,273	0	8
1851 ..	1,079 ..	444,170	17	0 ..	16,345	7	2

The net addition to the total business, after deducting for all policies emerged, forfeited, surrendered, and not taken up, for the two years, being—

Net addition.	No. of Policies.	Sums assured. £.	s.	d.	Corresponding premiums. £.	s.	d.
1850 ..	809 ..	272,354	9	0 ..	8,764	15	0
1851 ..	882 ..	368,051	4	0 ..	13,103	8	9

The premiums above stated are exclusive of the single premiums for assurances and the single and annual premiums for annuities.

The amount of claims is positively less than it has been for many years, notwithstanding the augmented business and advancing age of the earlier members. The per centage of claims is only about 14*s*. per £100, or about £1 out of £143. The next periodical investigation into the affairs of the Society takes place at the close of next year.

Amount of assurances in force, £1,739,750. Present annual income, £70,000. Accumulated fund, £313,100.

The Standard Life Assurance Company (1825).—*Business for the Year ended 15th Nov.*, 1853.

Number of proposals for assurance made to the directors			1059
Number of assurance policies issued, exclusive of annuity and other transactions			875
	£.	s.	d.
Sums proposed for assurance	555,544	7	0
Sums in policies issued	455,248	17	1
Premiums on new policies, exclusive of single payments	14,886	9	3
Annual revenue at 15th November, 1853	205,035	6	2
Claims by death, exclusive of bonus additions	84,445	17	0
Addition to accumulated fund during the year	125,761	1	10

Results of the Business during the last Nine Years.

Number of proposals for assurances made to the directors			8,351
Number of assurance policies issued, exclusive of annuities and other transactions			6,608
	£.	s.	d.
Sums proposed for assurance	5,000,498	12	4
Sums in policies issued	3,961,215	17	3
Claims by death paid, exclusive of bonus additions	491,210	14	4

Victoria Life Assurance Company (1838).—*Extract from the Fourteenth Annual Report of the Directors, submitted to the General Meeting of Proprietors on the 10th March*, 1853.—The receipts of the past year were £36,205. 14*s.*, and the disbursements £19,260. 6*s.* 8*d.*, leaving a balance in favour of the Company of £16,945. 7*s.* 4*d.*, which has been carried to the general or assurance fund. New assurances were effected under 164 policies, amounting to nearly £108,000, adding in premiums over £3,700 a year to the income of the Company. The claims by death arise from the failure of 15 lives under 18 policies, and amount to £11,237. 9*s.* 11*d.*, including bonuses. The number of policies lapsed during the past year, by death, purchase, or otherwise, amounts to 123, assuring £73,246, and have produced in premium nearly £10,000, within £1,200 of the amount paid for claims by death. The assets of the Company now amount to over £150,000, and are profitably invested, the much larger proportion in mortgages, reversions, life interests, debenture bonds, and consols. The number of policies in force at the end of the year were 1,155, assuring £763,642, the income being over £35,000 a year. Up to the end of 1852, 2,545 policies had been issued, assuring over one million six hundred thousand pounds.

Victoria Life Assurance Company.—Fifteenth Annual and Second Septennial Report of the Directors, 30th March, 1854.—During the past year, 142 policies have been issued, assuring £107,022, and adding nearly £4,000 to the income of the Company. The total receipts were £35,395, and the payments £14,968, (including £6,953, claims by death,) leaving £20,427 to be carried to the general or assurance fund—a larger sum than has hitherto been carried to that fund in any one year. A most careful investigation of the assets and liabilities of the Company has been made by the actuary; each policy has been separately valued; the future profits of the business have in no way been anticipated; and, after making a proper reserve to protect the interests of those assurers who have come in during the last three years, but who will not on this occasion participate in the division, and allowing fully for all contingencies, the result is as follows:—

	£.	s.	d.
The assets are valued at	176,861	7	2
The liabilities at	144,376	13	4
Leaving a surplus for division of . .	£32,484	13	10

more particularly set forth in the following table:—

Liabilities.	£.	s.	d.
Value of £815,171, assured under 1,190 policies, including reserve for annuities and accounts unpaid, extra risks, and other contingencies	100,844	9	4
Proprietors' fund	34,109	7	6
Claims unpaid	4,275	13	6
Value of £518 per annum, annuities granted	5,147	3	0
	144,376	13	4
Balance—surplus for division	32,484	13	10
	£176,861	7	2

Assets.	£.	s.	d.
Three per Cent. Consols	24,876	1	6
On mortgage	29,750	0	0
On life interests and reversions	21,301	14	3
„ railway and other debentures	35,463	18	9
„ the Company's policies	3,841	0	0
„ bonds with policies of assurance	38,261	13	2
„ deposit (London and County Bank)	5,000	0	0
Due by agents	897	13	4
Cash at bankers	6,723	15	11
Annuities and reversions purchased, lease of premises, profit rent, &c. .	3,745	10	3
Premiums on credit, and premiums and interest payable but not due .	7,000	0	0
	£176,861	7	2

Of this surplus, the directors advise that £32,266 should be divided in terms of the deed of settlement: viz., four fifths to the assured entitled to participate, and one fifth to the shareholders. This will enable the directors to declare a bonus of 17*s.* 6*d.* a share to the proprietors, which they recommend should be thus appropriated—by the addition of 7*s.* 6*d.* a share to the paid-up capital (making, with the former bonus of 12*s.* 6*d.* a share, £1 a share added to the original capital, equal to £25 per cent. thereon), and the remaining 10*s.* a share the directors propose should be paid to the proprietors in cash. The reversionary bonuses to the assured will average 53 per cent. on the amount of premiums paid, and 11½ per cent. on the sums assured by them. The present value of this bonus may be received in cash, or may be applied to the reduction of future premiums. It may, perhaps, be interesting to mention, that since the establishment of the Company 2,689 policies have been issued, assuring £1,714,524; that 1,499, assuring £903,812, have lapsed from one cause or another; and that 1,190 policies, assuring £810,712, remain on the books. £260,016 has been received in premium, and £44,634 in interest. The claims by death have arisen from the failure of 133 lives, assured under 169 policies, assuring £87,867; while the premiums received on these, on purchased and other lapsed policies, have amounted to £89,135. The directors proposed a division of profits in future every five years, instead of every seven years, and the power of having a less number of directors and auditors than the deed of settlement now provides for.

Examples of bonus declared 1854:—

Age when Assured.	Sum Assured.	Reversionary Bonus now added.	Sum now Assured.	Amount of Premiums paid.	Per Centage of Bonus on Premiums paid.	Per Centage of Bonus on Sum Assured.
	£.	£.	£.	£.		
29	500	75	575	120	62·5	15·
35	1,000	162	1,162	283	57·24	16·2
43	1,000	179	1,179	365	49·	17·9
51	1,000	207	1,207	477	43·4	20·7
58	1,000	253	1,253	644	39·3	25·3

INSTITUTE OF ACTUARIES.

As on the last occasion, we publish the questions for the Second Year's Examination, which we understand has, on the whole, been very satisfactorily met. The third year's examination will take place for the first time in December, 1855.

SECOND YEAR'S EXAMINATION, 1854.

1. Explain the difference between common and Naperian logarithms, and why it is desirable to retain both systems.

2. Given $\log. n = M\left\{(n-1)-\frac{1}{2}(n-1)^2+\frac{1}{3}(n-1)^3-, \&c.\right\}$ where M is the modulus of the system whose base is a, to find $\log. \frac{1+n}{1-n}$.

3. Find the values of $\log. (1+n)$, $\log. (1-n)$, and $\log. \frac{1+n}{1-n}$, when $n=1$.

4. In a table of logarithms it is observed that the difference between the logarithms of contiguous numbers diminishes as the numbers themselves increase. Explain this.

5. Show how to find the number corresponding to a logarithm found only partially in the tables.

6. If n balls, a, b, c, d, &c., be thrown promiscuously into a bag,—and two balls be drawn out, show that the probability that these will be a and b is $\frac{2}{n.(n-1)}$.

7. In an ordinary lottery, show that, *à priori*, the chances are the same, whether a person is to draw first or last or intermediately.

8. Find the probability that some two at least out of three lives will be alive at the end of the nth year.

9. Show that the probability of a single life failing in any assigned year, as the xth from the present time, is $p_{x-1}-p_x$.

10. Two lives, A and B, being proposed, find the probability that A will die before B.

11. Find the amount in n years of an annuity certain of £a at i per cent. compound interest.

12. Express the value of an annuity on a given life, and convert the formula into one which gives the value of an annuity at any age in terms of the next higher age.

13. Find the value for n years of an annuity certain commencing with £1 and increasing by £1 each year.

14. If the first payment of an annuity certain be £a, and the future payments be increased by £p each year, find the value of such annuity for n years.

15. Find according to the columnar method the value of an annuity on a single life, commencing with £1 and increasing by £1 each year until death.

16. If the first payment of a life annuity be £a, and the following payments be decreased by £p annually, find the value of the annuity.

17. State algebraically the difference between the amount of £1 in 1 and 2 years respectively, when $\frac{1}{m}$th of the interest is converted into principal m times in each year, instead of the whole interest once a year.

18. Find the present value of £1 at the end of n years, $\frac{1}{m}$th of the interest being converted into principal m times in each year.

19. What is the present value of £10 payable two and a half years hence, on M. D'Alembert's hypothesis, compound interest at 5 per cent.?

20. State briefly the practical objections to M. D'Alembert's reasoning on this subject.

21. Explain the relative advantages attending the D and N column system of construction, and the common method of forming tables of life annuities.

22. Explain the difference between Barrett's formula and Davies's, for finding the value of an annuity on a single life; and show how a table is constructed according to Barrett's method.

23. Show how a table of the probabilities of survivorship between two lives may be readily constructed.

24. Show how to construct a table of single and annual premiums for the assurance of £1 on a single life, according to the columnar method.

25. Find the value of an annuity on the life of A after the death of B; also the value of the annuity during the life of the survivor.

26. Find $\mathfrak{AB}_{1}$, the present value of £1 to be received at the end of the year in which A dies, provided he die while B is living.

27. The value required in the preceding question being unknown, find the annual premium for the contingency therein mentioned.

28. Express the value of an annuity on n lives, to continue so long as any one of them exists.

29. Determine the value of an annuity on the life of A, to commence at the death of B and C.

30. Given $\mathfrak{ABC}_{1.111}$, the present value of £1 payable at decease of the survivor of two lives A and B, provided that C die first or last of the three; find the equivalent annual premium to be paid during the continuance of the risk.

ORIGINAL TABLES.

CONTRIBUTED BY MR. CHARLES GABRIEL SHAW.

Preparatory Table for finding the Values of Annuities on THREE *Joint Lives of Equal Ages. Carlisle 3 per Cent.*

Common Age.	D.	N.	Common Age.	D.	N.
0	1000000	6589175·46	52	16810·39	140546·326
1	588068·5	6001106·96	53	15587·74	124958·586
2	443707·7	5557399·26	54	14412·35	110546·236
3	352215·2	5205184·06	55	13295·24	97250·9962
4	304489·9	4900694·16	56	12226·32	85024·6762
5	270873·1	4629821·06	57	11206·38	73818·2962
6	249187·1	4380633·96	58	10212·06	63606·2362
7	233123·5	4147510·46	59	9211·921	54394·3152
8	220413·5	3927096·96	60	8206·243	46188·0722
9	209798·0	3717298·96	61	7193·290	38994·7822
10	200597·4	3516701·56	62	6260·540	32734·2422
11	192143·7	3324557·86	63	5421·271	27312·9712
12	183862·6	3140695·26	64	4682·213	22630·7582
13	175843·1	2964852·16	65	4024·744	18606·0142
14	168081·0	2796771·16	66	3445·395	15160·6192
15	160495·6	2636275·56	67	2936·404	12224·2152
16	152945·1	2483330·46	68	2487·843	9736·37227
17	145522·2	2337808·26	69	2094·190	7642·18227
18	138373·2	2199435·06	70	1748·120	5894·06227
19	131556·3	2067878·76	71	1447·593	4446·46927
20	125056·9	1942821·86	72	1171·620	3274·84927
21	118860·8	1823961·06	73	920·4860	2354·36327
22	113010·9	1710950·16	74	700·1758	1654·18747
23	107433·2	1603516·96	75	511·9799	1142·20757
24	102115·6	1501401·36	76	367·7977	774·409877
25	97046·53	1404354·83	77	257·7464	516·665477
26	92167·59	1312187·24	78	177·9423	338·721177
27	87519·66	1224667·58	79	122·2747	216·446477
28	83005·76	1141661·82	80	81·33943	135·107047
29	78503·28	1063158·54	81	53·50098	81·6060679
30	73991·65	989166·896	82	33·75686	47·8492079
31	69681·23	919485·666	83	20·79576	27·0534479
32	65601·37	853884·296	84	12·36063	14·6928179
33	61774·61	792109·686	85	7·143586	7·54923198
34	58184·98	733924·706	86	3·890429	3·65880298
35	54786·92	679137·786	87	1·981696	1·67710698
36	51571·22	627566·566	88	·9263786	·750728386
37	48500·80	579065·766	89	·4270937	·323634686
38	45571·29	533494·476	90	·2002235	·123411186
39	42778·28	490716·196	91	·07859246	·0448187261
40	40069·99	450646·206	92	·02780735	·0170113761
41	37404·77	413241·486	93	·01007674	·00693463613
42	34835·14	378406·296	94	·003976320	·00295831613
43	32383·14	346023·156	95	·001628651	·00132966513
44	30084·51	315938·646	96	·0007125417	·000617123438
45	27930·70	288007·946	97	·0003315944	·000285529038
46	25930·24	262077·706	98	·0001514735	·000134055538
47	24072·51	238005·196	99	·00007133346	·0000627220785
48	22362·32	215642·876	100	·00003793198	·0000247900985
49	20815·96	194826·916	101	·00001732744	·00000746265854
50	19391·37	175435·546	102	·000006130750	·00000133190854
51	18078·83	157356·716	103	·000001285678	·00000004623054

Preparatory Table for finding the Values of Annuities on FOUR *Joint Lives of Equal Ages. Carlisle 3 per Cent.*

Common Age.	D.	N.	Common Age.	D.	N.
0	1000000	4260804·40	52	7188·121	51260·7845
1	497564·7	3763239·70	53	6563·992	44696·7925
2	345160·2	3418079·50	54	5971·035	38725·7575
3	256201·4	3161878·10	55	5415·149	33310·6085
4	213082·0	2948796·10	56	4890·527	28420·0815
5	184112·4	2764683·70	57	4397·386	24022·6955
6	166357·3	2598326·40	58	3923·474	20099·2215
7	153721·7	2444604·70	59	3453·549	16645·6725
8	144062·3	2300542·40	60	2989·535	13656·1375
9	136221·8	2164320·60	61	2532·757	11123·3805
10	129585·9	2034734·70	62	2125·454	8997·92653
11	123567·6	1911167·10	63	1771·671	7226·25553
12	117672·1	1793495·00	64	1471·620	5754·63553
13	111976·9	1681518·10	65	1214·668	4539·96753
14	106479·3	1575038·80	66	997·0972	3542·87033
15	101112·3	1473926·50	67	813·6775	2729·19283
16	95758·93	1378167·57	68	658·7809	2070·41193
17	90500·24	1287667·33	69	528·7829	1541·62903
18	85459·27	1202208·06	70	419·7236	1121·90543
19	80683·51	1121524·55	71	329·6168	792·288637
20	76159·62	1045364·93	72	251·0782	541·210437
21	71875·09	973489·843	73	183·8211	357·389337
22	67863·02	905626·823	74	128·9023	228·487037
23	64062·39	841564·433	75	85·75663	142·730407
24	60462·66	781101·773	76	55·72136	87·0090471
25	57053·66	724048·113	77	35·02774	51·9813071
26	53789·00	670259·113	78	21·58440	30·3969071
27	50700·13	619558·983	79	13·21790	17·1790071
28	47711·71	571847·273	80	7·751647	9·42736010
29	44731·16	527116·113	81	4·478032	4·94932810
30	41746·09	485370·023	82	2·447372	2·50195610
31	38916·97	446453·053	83	1·295575	1·20638110
32	36264·44	410188·613	84	·6538774	·552503707
33	33803·07	376385·543	85	·3178896	·234614107
34	31518·80	344866·743	86	·1427788	·0918353076
35	29376·79	315489·953	87	·05865819	·0331771176
36	27368·85	288121·103	88	·02149199	·0116851276
37	25467·77	262653·333	89	·007730397	·00395473068
38	23669·73	238983·603	90	·002843173	·00111155768
39	21970·93	217012·673	91	·0008252209	·000286336782
40	20335·51	196677·163	92	·0002085552	·0000777815822
41	18736·04	177941·123	93	·00005441438	·0000233672022
42	17208·56	160732·563	94	·00001590528	·00000746192223
43	15767·35	144965·213	95	·000004885953	·00000257596923
44	14434·54	130530·673	96	·000001638846	·000000937123239
45	13202·85	117327·823	97	·0000005968699	·000000340253339
46	12075·72	105252·103	98	·0000002120628	·000000128190539
47	11044·46	94207·6435	99	·00000007846685	·0000000497236897
48	10110·00	84097·6435	100	·00000003413877	·0000000155849197
49	9279·757	74817·8865	101	·00000001212922	·00000000345569975
51	8526·386	66291·5005	102	·000000003065375	·000000000390324754
52	7842·595	58448·9055	103	·0000000003857017	·000000000004623054

Preparatory Table for finding the Values of Annuities on FIVE *Joint Lives of Equal Ages. Carlisle 3 per Cent.*

Common Age.	D.	N.	Common Age.	D.	N.
0	1000000	2822392·08	52	3073·640	19183·0387
1	420989·6	2401402·48	53	2764·097	16418·9417
2	268500·2	2132902·28	54	2473·800	13945·1417
3	186360·9	1946541·38	55	2205·591	11739·5507
4	149114·7	1797426·68	56	1956·211	9783·33974
5	125141·2	1672285·48	57	1725·534	8057·80574
6	111060·1	1561225·38	58	1507·399	6550·40674
7	101364·1	1459861·28	59	1294·735	5255·67174
8	94159·08	1365702·20	60	1089·088	4166·58374
9	88448·81	1277253·39	61	891·7839	3274·79984
10	83712·50	1193540·89	62	721·5915	2553·20834
11	79466·32	1114074·57	63	578·9820	1974·22634
12	75310·13	1038764·44	64	462·5300	1511·69634
13	71306·90	967457·541	65	366·5867	1145·10964
14	67454·65	900002·891	66	288·5599	856·549742
15	63700·70	836302·191	67	225·4700	631·079742
16	59954·66	776347·531	68	174·4452	456·634542
17	56282·10	720065·431	69	133·5177	323·116842
18	52779·65	667285·781	70	100·7756	222·341242
19	49483·20	617802·581	71	75·05376	147·287482
20	46381·20	571421·381	72	53·80606	93·4814220
21	43462·87	527958·511	73	36·70907	56·7723520
22	40751·75	487206·761	74	23·73093	33·0414220
23	38200·40	449006·361	75	14·36423	18·6771920
24	35799·93	413206·431	76	8·441785	10·2354070
25	33541·85	379664·581	77	4·760270	5·47513702
26	31391·26	348273·321	78	2·618188	2·85694902
27	29370·59	318902·731	79	1·428855	1·42809402
28	27424·69	291478·041	80	·7387319	·689362126
29	25487·81	265990·231	81	·3748113	·314550826
30	23553·10	242437·131	82	·1774345	·137116326
31	21735·13	220702·001	83	·08071434	·0564019868
32	20046·98	200655·021	84	·03459012	·0218118668
33	18497·03	182157·991	85	·01414609	·00766577683
34	17073·73	165084·261	86	·005239980	·00242579683
35	15751·84	149332·421	87	·001736283	·000689513838
36	14524·64	134807·781	88	·0004986142	·000190899638
37	13373·13	121434·651	89	·0001399202	·0000509794384
38	12294·05	109140·601	90	·00004037307	·0000106063684
39	11284·27	97856·3317	91	·000008664820	·00000194154845
40	10320·27	87536·0617	92	·000001564164	·000000377384456
41	9384·886	78151·1757	93	·0000002938377	·0000000835467565
42	8501·025	69650·1507	94	·00000006362112	·0000000199256365
43	7677·123	61973·0277	95	·00000001465786	·000000052677651
44	6925·693	55047·3347	96	·000000003769345	·000000014984315
45	6240·985	48806·3497	97	·000000001074366	·000000004240655 12
46	5623·658	43182·6917	98	·0000000002968878	·000000001271777 12
47	5067·195	38115·4967	99	·0000000000831349	·000000000408642227
48	4570·733	33544·7637	100	·0000000000372491	·000000000101393127
49	4136·916	29407·8477	101	·000000000008490452	·000000000016488607
50	3749·052	25658·7957	102	·000000000001532688	·000000000001161727
51	3402·117	22256·6787	103	·0000000000001157104	·0000000000000004623

Common Age.	D.	N.	Common Age.	D.	N.
0	1000000·	1911430·15	52	1314·288	7307·60333
1	356199·3	1555230·85	53	1163·960	6143·64333
2	208866·3	1346364·55	54	1024·895	5118·74833
3	135558·9	1210805·65	55	898·3371	4220·41123
4	104359·5	1106455·15	56	782·4845	3437·92673
5	85058·49	1021396·66	57	677·0995	2760·82723
6	74143·74	947252·929	58	579·1427	2181·68453
7	66839·46	880413·469	59	485·3963	1696·28823
8	61542·36	818871·109	60	396·7546	1299·53363
9	57429·80	761441·309	61	313·9971	985·536534
10	54078·27	707363·039	62	244·9803	740·556234
11	51104·79	656258·249	63	189·2113	551·344934
12	48198·49	608059·759	64	145·3732	405·971734
13	45408·24	562651·519	65	110·6359	295·335834
14	42732·52	519918·999	66	83·50923	211·826604
15	40131·44	479787·559	67	62·47774	149·348864
16	37537·62	442249·939	68	46·19308	103·155784
17	35001·84	407248·099	69	33·71322	69·4425642
18	32596·72	374651·379	70	24·19624	45·2463242
19	30348·05	344303·329	71	17·08973	28·1565942
20	28246·16	316057·169	72	11·53064	16·6259542
21	26281·99	289775·179	73	7·330802	9·29515222
22	24471·42	265303·759	74	4·368864	4·92628822
23	22778·90	242524·859	75	2·406009	2·52027922
24	21197·14	221327·719	76	1·278930	1·24134922
25	19719·25	201608·469	77	·6469207	·594428529
26	18319·94	183288·529	78	·3175862	·276842329
27	17014·38	166274·149	79	·1544592	·122383129
28	15763·72	150510·429	80	·07040116	·0519819692
29	14522·96	135987·469	81	·03137168	·0206102892
30	13288·68	122698·789	82	·01286399	·00774629927
31	12139·07	110559·719	83	·005028502	·00271779727
32	11081·97	99477·7493	84	·001829817	·000887980270
33	10121·58	89356·1693	85	·0006295008	·000258479470
34	9248·840	80107·3293	86	·0001923072	·0000661722705
35	8446·136	71661·1933	87	·00005139396	·0000147783105
36	7708·228	63952·9653	88	·00001156785	·00000321046056
37	7022·227	56930·7383	89	·000002532555	·00000067790556
38	6385·531	50545·2073	90	·0000005732974	·00000010460816
39	5795·601	44749·6063	91	·00000009098060	·00000001362756
40	5237·537	39512·0693	92	·00000001173123	·00000000189633
41	4700·889	34811·1803	93	·000000001586724	·00000000030960
42	4199·507	30611·6733	94	·0000000002544845	·00000000005512
43	3737·991	26873·6823	95	·00000000004397358	·00000000001114
44	3322·948	23550·7343	96	·000000000008669493	·00000000000247
45	2950·113	20600·6213	97	·000000000001933859	·00000000000054
46	2618·938	17981·6833	98	·0000000000004156429	·00000000000012
47	2324·830	15656·8533	99	·0000000000000949485	·00000000000003
48	2066·429	13590·4243	100	·0000000000000276524	·00000000000000
49	1844·237	11746·1873	101	·0000000000000005943316	·00000000000000
50	1648·458	10097·7293	102	·0000000000000000766343	·00000000000000
51	1475·838	8621·89133	103	·0000000000000000847132	·00000000000000

Value of £1 *per annum during the joint Continuance of* THREE, FOUR, FIVE, *and* SIX *Lives, of Equal Ages. Carlisle 3 per Cent.*

Common Age.	Three Lives.	Four Lives.	Five Lives.	Six Lives.	Common Age.	Three Lives.	Four Lives.	Five Lives.	Six Lives.
0	6·589	4·261	2·822	1·911	52	8·363	7·131	6·241	5·560
1	10·205	7·563	5·704	4·366	53	8·016	6·809	5·941	5·278
2	12·525	9·903	7·944	6·446	54	7·670	6·486	5·637	4·994
3	14·778	12·341	10·445	8·932	55	7·315	6·151	5·323	4·698
4	16·095	13·838	12·054	10·603	56	6·954	5·811	5·001	4·394
5	17·092	15·016	13·363	12·008	57	6·588	5·463	4·670	4·077
6	17·580	15·619	14·057	12·776	58	6·228	5·123	4·346	3·767
7	17·791	15·903	14·402	13·172	59	5·905	4·820	4·059	3·495
8	17·817	15·969	14·504	13·306	60	5·628	4·568	3·826	3·275
9	17·718	15·888	14·440	13·259	61	5·421	4·392	3·672	3·139
10	17·531	15·702	14·258	13·080	62	5·229	4·233	3·538	3·023
11	17·303	15·467	14·019	12·841	63	5·038	4·079	3·410	2·914
12	17·082	15·242	13·793	12·616	64	4·833	3·910	3·267	2·793
13	16·861	15·017	13·568	12·391	65	4·623	3·738	3·124	2·669
14	16·639	14·791	13·342	12·167	66	4·400	3·553	2·968	2·537
15	16·426	14·577	13·129	11·955	67	4·163	3·354	2·799	2·390
16	16·237	14·392	12·949	11·782	68	3·914	3·143	2·618	2·233
17	16·065	14·228	12·794	11·635	69	3·649	2·915	2·420	2·060
18	15·896	14·068	12·643	11·494	70	3·372	2·673	2·206	1·870
19	15·718	13·900	12·485	11·345	71	3·072	2·404	1·962	1·648
20	15·535	13·726	12·320	11·189	72	2·795	2·156	1·737	1·442
21	15·345	13·544	12·147	11·026	73	2·558	1·944	1·547	1·268
22	15·139	13·345	11·956	10·841	74	2·363	1·773	1·392	1·128
23	14·926	13·137	11·754	10·647	75	2·231	1·664	1·300	1·047
24	14·703	12·919	11·542	10·441	76	2·106	1·561	1·212	·9706
25	14·471	12·691	11·319	10·224	77	2·005	1·484	1·150	·9189
26	14·237	12·461	11·095	10·005	78	1·904	1·408	1·091	·8717
27	13·993	12·220	10·858	9·773	79	1·770	1·300	·9994	·7923
28	13·754	11·985	10·628	9·548	80	1·661	1·216	·9332	·7384
29	13·543	11·784	10·436	9·364	81	1·526	1·105	·8392	·6570
30	13·369	11·627	10·293	9·233	82	1·417	1·022	·7728	·6022
31	13·196	11·472	10·154	9·108	83	1·301	·9312	·6988	·5405
32	13·016	11·311	10·009	8·977	84	1·189	·8450	·6306	·4853
33	12·823	11·135	9·848	8·828	85	1·057	·7380	·5419	·4106
34	12·614	10·942	9·669	8·661	86	·9405	·6432	·4629	·3441
35	12·396	10·739	9·480	8·485	87	·8463	·5656	·3971	·2875
36	12·169	10·527	9·281	8·297	88	·8104	·5437	·3829	·2775
37	11·939	10·313	9·081	8·107	89	·7577	·5116	·3643	·2677
38	11·707	10·096	8·877	7·916	90	·6164	·3910	·2627	·1825
39	11·471	9·877	8·672	7·721	91	·5703	·3470	·2241	·1498
40	11·246	9·672	8·482	7·544	92	·6118	·3730	·2413	·1616
41	11·048	9·497	8·327	7·405	93	·6882	·4294	·2843	·1951
42	10·863	9·340	8·193	7·289	94	·7440	·4691	·3132	·2166
43	10·685	9·194	8·072	7·189	95	·8164	·5272	·3594	·2536
44	10·502	9·043	7·948	7·087	96	·8661	·5718	·3976	·2859
45	10·311	8·886	7·820	6·983	97	·8611	·5702	·3947	·2818
46	10·107	8·716	7·679	6·866	98	·8850	·6045	·4284	·3112
47	9·887	8·530	7·522	6·735	99	·8793	·6337	·4734	·3623
48	9·643	8·318	7·339	6·577	100	·6535	·4565	·3300	·2439
49	9·359	8·062	7·109	6·369	101	·4307	·2849	·1942	·1348
50	9·047	7·775	6·844	6·126	102	·2173	·1273	·0758	·0454
51	8·704	7·453	6·542	5·842	103	·0360	·0120	·0040	·0013

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INSTITUTE OF ACTUARIES.

On the Improvement of Life Contingency Calculation. By Edwin James Farren, Esq., *one of the Vice Presidents of the Institute of Actuaries.*

[Read before the Institute 8th January, 1855, and ordered by the Council to be printed.]

THE prevailing system of life contingency calculation is one not of variable but of invariable quantities. At the very threshold the admission of two such important assumptions is asked for, as that the rate of mortality is always invariable at the same age, whether old or young, and that the rate of interest is equally invariable for all periods, whether long or short. Upon these admissions of invariability a system is formed for assessing the relative values of different cases, thereby necessarily in every instance indicating an invariable answer; and with such indications the system rests content. Whether such assessments, however logically fair in connection with agreed postulates of invariability, are themselves eventually justified by the same invariability of actual result as was *à priori* assumed, has not hitherto been commonly brought within the general scope of the actuary's studies. Directly, however, he is called on to take upon himself the practical responsibility of upholding this theory of invariability, he is somewhat surprised to find that, good as the mere logic of his studies may have been, it is by no means an easy task to connect such logic with the nature of the events he may see passing around him. In place of a definite rate of mortality at each age, he may find a perfect series

of such rates—in place of one uniform rate of interest, he may find one portion of the funds yielding no interest whatever, while other portions may be lent upon classes of security, fruitful indeed in interest, but not as equally fruitful in obvious expedients for realization of the principal, should such be desirable. The conviction is thus forced upon him at a very early period of his career, that, as a practical man, he must either henceforth shut his eyes altogether to the prevailing theory of the subject, or else open his eyes much wider than as a student he was taught to expect would be necessary in the application of rules professedly based upon invariable elements.

I am aware it may be said, that although the rates both of mortality and interest, as used by actuaries under the prevailing system, are certainly specific rates, yet that they are to be considered as averages, and therefore typical, as well in theory as in practice, of a diversity of rates. But I would ask, has the subject ever been so treated? Can we find, for instance, in the treatises of those who are termed by common consent, and therefore rightly termed, our standard writers, not merely a single chapter, but even a single page, on the proper calculation and employment of average results, as applicable to insurance transactions? So long then as this omission exists, so long I think we may without presumption assert that the prevailing system of life contingency calculation is susceptible of considerable improvement. But in alluding to this omission, I particularly wish to guard myself from the inference that I thereby desire to stigmatize the labours of such men as Price, Morgan, Baily, and Milne, or of our late excellent cotemporary David Jones, as not worthy of the favourable reception they have received. On the contrary, I believe (and it is one of the objects of the present paper to illustrate such belief) that the style of treatment adopted by these pioneers of our literature constituted the only style that was really applicable to the nature of insurance, as then publicly understood. Indeed, in all branches of natural philosophy we find a similar assumption of invariability at an early period of their history, as preparatory to the more comprehensive study of deviations required by modern science. One of these branches presents us with so close a parallelism to the line of progress I wish to illustrate as advisable in insurance calculations, that I cannot consider it wholly a digression to allude to it even in some detail.

In the early history of navigation, we find it taken almost as the basis of the science that the compass needle pointed in a fixed

direction, and that such direction was due north. The utility of so simple an assumption in early days can scarcely be overrated; the more especially as it was by this simplicity then rendering itself acceptable that the real properties of the magnet have been reserved for more elaborate study in our own time. Indeed, had variability in place of invariability been originally taken for its prevailing attribute, the important uses of the mariner's compass would probably have been lost to modern science, inasmuch as our simplicity-loving ancestors might have considered such indications too uncertain and complex either to be useful to themselves or to their posterity. In further illustration of this, we find that for a long time the simplicity of such a theory of invariability completely overrode the nature of the actual facts. Gradually, however, the proverbial stubbornness of facts developed itself in this as in other sciences, and brought about the admission that the compass needle did not really and in fact point due north. There was as yet, notwithstanding, no absolute surrender of the principle of fixity of direction itself; such would have been too sudden a change either to have been expected or even desired. A variety of adjustments were accordingly advocated, each claiming some favourite point as that of the real fixity. The correct theory, nevertheless (started, I believe, by Gunter, and now admitted by all conversant with the subject, because the only theory that can interpret all the facts), is, that the direction of the compass needle is variable even in the same localities, and must be so apprehended by mariners, if safety based upon truth, and not merely upon simplicity of theory, be their aim. It is, then, by the known variability of the compass needle, and not by a pseudo invariability, that the triumphs of modern over ancient navigation have been achieved.

The use I seek to make of this as a parallelism in illustration of the subject before us is, I presume, sufficiently obvious. I consider the formerly universal adoption of the Northampton Table and 3 per cent. as typified by the assumption that the compass needle pointed fixedly due north, and the various petty controversies for other fixed points as equally typified by the various pros and cons for the Carlisle and other tables. Further, that the true theory, in this as in the former subject, is one strictly of variation, both as to mortality and interest, and that it must be so accepted as the only guide to safe practice, if we would avoid those rocks and shoals which a purblind adherence to a fixed in place of a variable course might unpreparedly develope. Indeed, the distinction between the proper treatment of variable and invariable ele-

ments is precisely the distinction that characterizes the vocation of an actuary as compared with that of an accountant. Thus the actuary who should take probabilities, because fairly assessed now, as necessary certainties hereafter, would be virtually an accountant, because he allows no range for the possibly conflicting evidences of the future. The accountant, on the other hand, who endeavours to put estimates upon fluctuating things to come, is virtually striving to be an actuary; for he cannot but allow that no estimate of to-day, unless professedly subject to variation, can pretend to also fulfil the condition of being an equally good estimate for a changeable to-morrow. In some Offices, I believe, this distinction between fixed and variable estimates is already sufficiently carried out—in the first case, by the actual amount of assets at one period, as compared with the actual amount at another, being illustrated by DR. and CR. after the manner of accountants; and in the second, by the difference between the amount of the life valuation at one period and another being substantiated, actuary-wise, by taking into consideration the accrued contingencies of the past as compared with the range of contingencies to be provided against for the future. The actuaries, then, of what I shall venture to call the old school, were essentially accountants in the modern sense, for it was only with fixed quantities they professed to deal, as is sufficiently proved by their assuming a fixity when they found it not.

The results of such a system have been exactly those to be expected. Where exorbitancy secretly existed, as in the rates required by the Insurance Societies, there the errors of a fixed and affected precision eventually came to light, in the shape of bonuses added to the sums assured, in varying amounts from time to time, in strange contrast with the declared formality of the original fixedness of calculation. Where no such exorbitancy of charge was allowable or even possible, and the fixed calculations had to stand or fall by their own merits, there the dangers of professing to deal with variable quantities as if they were absolutely invariable were unfortunately not so easily neutralized. To the numerous Friendly and Annuity Societies of the last century throughout the country, a reliance on fixed tables and on such tables alone has at once proved a delusion and a snare; for it tempted them to appropriate the temporary surplus of a day, in the vain expectation that the fixed nature of the tabular values assigned to the future would necessarily be sufficient guarantee for the fulfilment of impending engagements.

The nature, then, of the improvement I seek to introduce into

life contingency calculation is to openly take as our guide not merely a calculus of averages, but of their fluctuations; and to thereby declaredly characterize our methods, not as composing a system of specific and precise *results*, whatever it may be of *prices*, but of results expected to vary between limits of assigned ranges of probability. By such a declaration it would at once become manifest that our expected gain by computation would not be to find even averages themselves invariable, but that their fluctuations, being considerably less, would therefore be more readily dealt with than the fluctuations of the elements of which they may be composed. The phrases therefore of the prevailing system implying "a true table," or "a true rate of interest," would under such a calculus have to give way to average tables, with their probable limits and the per centages of their expected deviations.

What experienced actuary, for instance, can read without feeling the truth of the following reflection, extracted from the article on "Probability" in the *Encyclopædia Metropolitana*:—

> "Not being well able to decide upon the relative importance of small details, calculators on this subject (life contingencies) have hitherto judiciously presented their results such as they ought to be if the tables were mathematically exact, and to the nearest farthing. But more extended views on the subject of probabilities, and on the nature of observations in general, would have caused the time which has been wasted in carrying out annuities to many more decimals than the data are good for, to be employed in apportioning the risks of fluctuation by estimation of the mean risks of the tables."

Or the following, from the equally excellent article on "Probability" in the *Encyclopædia Britannica*:—

> "We may remark that, although English writers have almost without exception confined themselves to the explanation of the methods of computing annuity tables and of determining from them the values of sums depending on life contingencies, the aid which this branch of economy derives from the general theory of probabilities is by no means confined to the consideration of such elementary questions. The number of observations necessary to inspire confidence in the tables, the extent to which risks may be safely undertaken, the comparative weights of different sets of observations, and the probable limits of departure from the average results of previous observations in a given number of future instances, are all questions of the utmost importance, which come within the scope of the calculus, and cannot in fact be justly appreciated by any other means."

In the concluding part of the latter extract we have, indeed, the real explanation of the formal and what I may venture to call the "wooden" cast that has been given to the subject by our standard writers already referred to: for we are to remember that,

though complete masters of their art as then understood, yet that they were all teachers or disciples of a school and of a day when the differential and integral calculus was but little employed by English writers on any branch of science. The omission of such processes has now, however, become the exception and not the rule. Thus, for instance, if we look into any of our modern treatises on mechanics, engineering, or navigation, all of them essentially practical subjects, we find every aid that the calculus can give or has a chance of giving sedulously pressed into the service. By these means the great discovery of Newton and Leibnitz is brought home—vicariously indeed, but still effectually—to the uses of the humblest mechanic, engineer, or mariner, whenever he has to avail himself of what can be done for him, by way of previous calculation, in guiding him to the simplest and most trustworthy results. Indeed, the modern improvements of the *Nautical Almanack* alone form at once a sufficient and striking illustration of what benefit can be achieved by the calculus in devising the best forms for practical computation. Whatever therefore may have been the opinion of our elder school of writers, I think the time has now come for our students when, as in other subjects, the more searching investigations of the calculus should also be brought to openly and commonly bear upon that of life contingencies.

Indeed, without this or some other extra aid, how is it possible for us to intelligibly explain to a modern public those differences of results in various Offices, which, when judged by a hypothetical standard of invariability, appear rather to proclaim the failure of all methods whatever, than to justify the indications of any particular one? So long as this diversity remains unexplained by having no proper limits assigned to it, so long assuredly may any amount of diversity appear justifiable to boards of management, and actuaries continue to be exposed to the risk of having their opinions only treated with respect when not obstructive of other money arrangements. That the calculus, especially considered as a calculus of averages, contains within itself the means of dealing with and explaining these diversities, has been too often asserted both by continental and native writers on probability, to be strengthened by mere reassertion on my part. But as I am not aware that any very ready example, in a professional sense, has been given of the sort of assistance to be derived by actuaries from this calculus, when treated as a calculus of averages, I shall beg leave to conclude this paper by offering at least one such illustration, hoping it may prove an incentive to other actuaries to look

further into the subject than perhaps they have hitherto done. Before, however, giving such an example, I should wish to state that I have purposely selected such an one as will show that I by no means pretend, as a practical man, that a more general study of the differential and integral calculus by actuaries would materially alter the external appearance of insurance results and rates, as at present accepted by the public. On the contrary, I believe that no actuarial theorizing would or ought to induce the public to be otherwise than mainly led by their own experience of the past, already somewhat extensive, and every day becoming more and more patent to themselves. But there is considerable difference, in a professional point of view, between venturing on general assumptions, however plausible, and the cautious adoption of approximations based upon elaborate investigations. Were then the calculus capable of no more than pointing out to us convenient approximations, and referring us to its own processes for its justification of them, it would still, I think, be an ally obviously well worthy of the actuary's seeking. It is to illustrate the calculus in this character that has decided the kind of example I have selected.

Example.—A hundred pounds has to be put out at compound interest for twenty years, at rates indefinitely fluctuating between 3 and 4 per cent. per annum. What is the general average of all the possible sums, even to infinity, to which the hundred pounds may be thus made to amount?

Putting this into the form of a definite integral, we have

$$\frac{1}{m^{n-1}}\int_a^{a+1}(b+cx)^n dx=\frac{1}{m^{n-1}}\,\frac{(b+c(a+1))^{n+1}-(b+ca)^{n+1}}{c(n+1)};$$

which, when $m=100$, $a=3$, $b=100$, $c=1$, and $n=20$, as in the case before us, becomes $\frac{1}{100^{19}}\left(\frac{104^{21}-103^{21}}{21}\right)=199{\cdot}2731$, which is the general average amount required.

Having thus determined what would appear as the more recondite question of the average *amount* of a sum at fluctuating rates of interest, it may be well, in order to show the ductility of the calculus when studied as an extensive system of averages, to also determine by its means the more simple question of what is the average rate of *interest* between 3 and 4 per cent., so obviously determinable by other means as $3\frac{1}{2}$? This extra illustration, however simple, is considered advisable; because there may be many minds, even in our own profession, so framed, that it is only by

treating well known examples, having obvious solutions by the current methods, that the reliability of any new method of solution is considered admissible by them in more difficult cases. To determine, then, the average rate of interest between 3 and 4 per cent. by means of the calculus, we have to consider the definite integral of

$$\int_a^{a+1} x dx = \frac{(a+1)^2 - a^2}{2};$$

which, when $a=3$, becomes $\frac{4^2-3^2}{2} = 3\frac{1}{2}$; in exact equality with the obvious result of sheer mental arithmetic. I have already hinted that the first example is purposely chosen as one susceptible of an easy approximation, and such has just been portrayed: for if £100 be invested at $3\frac{1}{2}$ per cent. per annum for twenty years throughout, it will amount to £198·9789, a close approximation to the general average, or £199·2731, as determined by the calculus. But should we therefore be justified in saying that the £100 must necessarily be considered as having to be invested at $3\frac{1}{2}$ per cent. per annum throughout? Decidedly not; and the less so, because all the supposed advantages of such a misstatement are more readily obtained by adhering to the scientific truth, and saying that the proposed calculation, being really one of indefinite fluctuation, has been accordingly so dealt with, and the general average ascertained to be £199·2731; without, however, guaranteeing either that or any other as the precise result, that experience alone can determine. It is moreover manifest, that $3\frac{1}{2}$ as a rate of interest could not be connected as such with the average amount at the end of the term, for it is as obvious by common arithmetic as it is by the calculus that the accumulations between $3\frac{1}{2}$ and 4, considered as fixed quantities, would more than counterbalance those between 3 and $3\frac{1}{2}$: and hence, whether $3\frac{1}{2}$ would have to be considered as affording a good or bad approximation is not matter for assumption, but for demonstration; and it is precisely these demonstrations that are beyond the reach of the common methods.

The instances I have given will, I think, sufficiently portray, so far as isolated instances can do, the nature of the improvement I am advocating in life contingency calculations. It will be seen that, though I seek to deprive the prevailing system of its pretensions to an invariability that does not really belong to it, yet that at the same time I propose a similar equivalent, by the adoption of the calculus and its limits, to that which has already

been accepted, in place of a similarly false invariability, in other branches of natural philosophy. It is true, indeed, that before being exactly adapted to our wants as actuaries, the calculus must be moulded into one of averages; but this is a transformation so legitimate, that I consider no better method of studying the calculus exists even for the more general mathematical student.

In concluding this paper, I am perfectly well aware it is considered by many as dangerous, in an official and commercial sense, for any actuary to show he has been studying other books and productions than directories, prospectuses, and advertisements; but I trust that a better spirit is beginning to prevail, and that, within the walls of this Institute at least, any advocacy for the improvement of the theory of our subject will be immediately seen as also implying a desire to improve its practical aspects. Speaking for myself, I have long considered that the wants of the public are daily forcing upon actuaries the investigation of subjects which the incompleteness of the prevailing theory renders it too powerless to sufficiently grapple with; and it is the hope of exciting attention to this view of the matter that has induced me to offer the present paper. Considerable difficulty, as may be imagined from its tenor, has been felt in keeping it within due bounds; for, had examples of general limits been chosen, the subject in this form appears to be at present so little understood in its practical bearings, that at least the range of a lengthened essay, if not of a volume, might have been required to treat the matter with that fulness of illustration which the importance of it demands. It may therefore be allowable for me to attempt to reinforce the object of so circumscribed a paper by a general declaration on my part that, after having devoted considerable attention, and indeed some years, to the subject, I feel confident the proposed change from an invariable to a variable calculus as the basis of our calculations will be beneficial in every respect. We shall thereby be able to wholly dismiss the ancient doctrine of chances, with its fixed equalities of paper cards, wooden dice, and similar mechanical illustrations, and rely upon the more modern doctrine of probability, as the science of observation based upon experience. The actuarial adaptations of this doctrine, aided by the calculus, will assuredly ultimately bring a class of problems involving averages and their fluctuations within reach of our solutions which at present are merely statistically guessed at, even by the most experienced actuary, the most cautious finance minister, or the most learned political economist. To improve our own science, more-

over, is virtually a step towards the improvement of others, and thereby the better helps us to substantiate the claims of our studies to those designations of learned and liberal so duly prized by other professions.

POSTSCRIPT.—The writer is glad to avail himself of the interval between the reading and printing of his paper to state, that he does not by such paper claim for his views—as might perhaps hastily, without this disclaimer, be inferred—the merit of perfect originality as regards the proposed improvement of life contingency calculation. Lacroix has long since glanced at the differential and integral calculus as essentially a calculus of averages, and the calculus itself has already been often employed in connection with life contingencies in England, as by De Moivre, Waring, Young, Gompertz, Lubbock, De Morgan, Galloway, Edmonds, and indeed by the present writer himself, in his last publication on Life Contingency Tables. It is necessary further to remark, in the same spirit, that even the terms "true table" and "true rate of interest," though commonly used in the prevailing system, have also been frequently associated with the notion of a margin for fluctuations, or accompanied with the qualification that it is only by neglecting variations that the epithet of "true" becomes allowable, and that, if it be proposed to include such variations, that modification of the ordinary language should ensue. Reference may be made to Mr. Jellicoe's paper, in Vol I. p. 172 of the present *Journal*, for instances of this. It is, then, rather to excite renewed attention to the subject of variability, than to propose it as wholly new, that has been the writer's real aim; and he has accordingly treated the matter in the preceding paper in that mixed style of pleading and demonstration, as appearing to him the most suitable for such a purpose in its more general form. As however it may aid the illustrations already given in the paper itself, if a tabular form be presented, and may also tend to better satisfy many minds to whom tabular forms are more acceptable than even the most earnest disquisitions upon principles, such a table is now appended. It may be taken as a temporary specimen of the proposed improved manner of dealing with such subjects—without, however, the writer's wishing such table to be understood as having the exact form, even in his own opinion, that may ultimately be best adapted for the purpose under consideration.

Table of the Average Amounts of £100 at Fluctuating Rates of Interest.

Average Amounts of £100 *at Compound Interest from* 1 *to* 100 *years, at rates fluctuating between the limits of* 0 *and* 6 *per cent. per annum.*

N.B. The maximum rate of interest is taken at 6, rather than at 5, per cent.; because 5 per cent., if payable by half yearly, quarterly, monthly, or smaller instalments, can be made to exceed 5 per cent. per annum.

Term of Years.	Average Amount.	Term of Years.	Average. Amount	Term of Years.	Average Amount.	Term of Years.	Average Amount.
1	103·0000	26	235·9473	51	631·3104	76	1901·0467
2	106·1200	27	244·7433	52	658·4496	77	1990·5568
3	109·3654	28	253·9803	53	686·8833	78	2084·5473
4	112·7419	29	263·5273	54	716·6764	79	2183·2499
5	116·2553	30	273·5538	55	747·8971	80	2286·9085
6	119·9120	31	284·0306	56	780·6170	81	2395·7800
7	123·7183	32	294·9793	57	814·9117	82	2510·1350
8	127·6813	33	306·4428	58	850·8605	83	2630·2580
9	131·8080	34	318·3851	59	888·5470	84	2756·4491
10	136·1058	35	330·8913	60	928·0588	85	2889·0240
11	140·5829	36	343·9679	61	969·4884	86	3028·3153
12	145·2472	37	357·6426	62	1012·9330	87	3174·6731
13	150·1076	38	371·9448	63	1058·4948	88	3328·4664
14	155·1731	39	386·9049	64	1106·2813	89	3490·0835
15	160·4533	40	402·5553	65	1156·4058	90	3659·9337
16	165·9581	41	418·9299	66	1208·9873	91	3838·4479
17	171·6981	42	436·0641	67	1264·1512	92	4026·0800
18	177·6842	43	453·9955	68	1322·0292	93	4223·3082
19	183·9280	44	472·7634	69	1382·7602	94	4430·6361
20	190·4416	45	492·4090	70	1446·4903	95	4648·5943
21	197·2377	46	512·9758	71	1513·3730	96	4877·7418
22	204·3297	47	534·5094	72	1583·5703	97	5118·6675
23	211·7316	48	557·0580	73	1657·2523	98	5371·9917
24	219·4580	49	580·6718	74	1734·5982	99	5638·3681
25	227·5245	50	605·4040	75	1815·7968	100	5918·4853

Example.—The average amount of all the amounts possible, even to infinity, to which £100 can be made to accumulate in twenty years, at rates of interest fluctuating between 0 and 6 per cent. per annum, is £190·4416; as may be seen set forth in the table opposite to the term of 20 years.

When it is remembered that money is more likely to remain unproductive, or at 0 per cent., for short than for long periods, it is obvious that the relative effect of unproductiveness must be more operative when considering brief than enlarged cycles of finance. The preceding table, by its averages, properly represents this effect among others; and shows that while on the one hand the average amount at the end of the first period or year from the original times of the deposits may be taken as sufficiently defined by the common mean rate of interest between the limits, or in the present

case by 3 per cent., yet that, on the other hand, the period of a century is allied by its average amount, in connection with the same limits, to the amount productive by a uniform rate throughout of about 4 per cent. (4·165). To deal with such wide limits as from 0 to 6, and with such durations as a century, is obviously to strain the calculus to its utmost; but even in this extreme state it will be seen to keep closely attendant upon the incidents of practical insurance, for it certainly appears consistent with even popular justice that, as a matter of calculation, those who may remain longest insured should be also rated as those to be relatively assigned the higher ratios in the general appropriation of accumulations of interest.

The table has been virtually calculated by the aid of the same definite integral as that given in the paper, viz.—

$$\frac{1}{m^{n-1}}\int_a^{a+g}(b+cx)^n dx=\frac{1}{m^{n-1}}\,\frac{(b+c(a+g))^{n+1}-(b+ca)^{n+1}}{gc(n+1)}\,;$$

in which n varies by units from 1 to 100; a varies indefinitely to and from $a+g$, or from 0 to 6; $b=100$; $m=100$; $c=1$; and $g=6$. Or, putting such expressions into the form of a rule, we have the following extremely simple one whenever 0 is the lower limit:—

Subtract £100 *from its amount, when improved for the whole term and one year beyond, at the maximum rate of interest considered as uniform as by the common tables. Divide the remainder by the product of such maximum rate and the number of years including the year beyond, and the quotient will be the average amount required.*

Example for Twenty Years.—£100, put out at 6 per cent. per annum uniform interest for twenty years and one year beyond, will amount by the ordinary tables (Smart's) to £339·95636; from which if the £100 be subtracted, the remainder is £239·95636; which, divided by 6 times 20 and the year beyond, or 126, gives a quotient of £190·4416, which is the average amount required between the assigned limits in conformity with the result as given by the table in question.

The reasons for the trustworthiness of the rule can of course only be explained by aid of the calculus itself, or by some allied process of reasoning which it would be here out of place to dilate upon.

On the Calculation of Annuities, and on some Questions in the Theory of Chances. By J. W. Lubbock, Esq., B.A.*

[Extracted, by permission of the Author, from the *Transactions of the Cambridge Philosophical Society*.]

1. The object of the following investigation is to show how the probabilities of an individual living any given number of years are to be deduced from any table of mortality. All writers (with the exception of Laplace) have considered the probability of an individual dying at any age to be the number of deaths at that age recorded in the table, divided by the sum of the deaths recorded at all ages. This would be the case if the observations on which the table is founded were infinite; but the supposition differs the more widely from the truth the less extended are the observations, and cannot, I think, be admitted where the recorded deaths do not altogether exceed a few thousand, as is the case in the tables used in England. The number of deaths on which the Northampton Tables are founded is 4,689 (Price, vol. i. p. 357). The tables of Halley are founded upon the deaths which took place at Breslau, in Silesia, during five years, and which amounted to 5,869.

If a bag contain an infinite number of balls of different colours in unknown proportions, a few trials or drawings will not indicate the proportion in which they exist in the bag, or the simple probability of drawing a ball of any given colour; and not only the probability of drawing a ball of any given colour, calculated from a few observations, will be little to be depended on, but it will also differ the more from the ratio of the number of times a ball of the given colour has been drawn, divided by the number of the preceding trials, the fewer the latter have been.

Laplace (*Théor. Anal. des Probabilités*, p. 426) has investigated the method of determining the value of annuities. He there says—"Si l'on nomme y_o le nombre des individus de l'age A dans la table de mortalité dont on fait usage, et y_x le nombre des individus à l'age $A+x$, la probabilité de payer la rente à la fin de l'année $A+x$ sera $\frac{y_x}{y_o}$." This hypothesis coincides with that I have before alluded to, as adopted by all other writers. Laplace, however, means this as an approximation, for he has investigated differently the probability of an individual of the age A living to the age $A+a$ (p. 385 of the same work). He there considers two cases only possible; but as an individual may die at any instant

* Now Sir John William Lubbock, Bart., F.R.S., &c.

during life, I think it may be doubted whether this hypothesis of possibility should be adopted.

Captain John Graunt was the first, if I am not mistaken, who directed attention to questions connected with the duration of life. He published a book in 1661, entitled *Observations on the Bills of Mortality*, which contains many interesting details, although it is written in the quaint style which prevailed in those times. In this book, amongst other tables there is one showing in 229,250 deaths how each arose; and another showing of 100 births "how many die within six years, how many the next decad, and so for every decad till 76"—which is in fact a table of mortality, and is probably the first ever published.

After Captain Graunt, Sir W. Petty published his *Essays on Political Arithmetick*. Halley, however, was the first who calculated tables of annuities: he took the probabilities on which they depend, from a table of mortality founded on the deaths during five years at Breslau. Since his time a great number of writers have treated of these subjects, of whom a notice may be seen in the *Encyclopædia Britannica*, or in the *Report from the Committee on the Laws respecting Friendly Societies*, 1827 (p. 94). It is to be regretted that those who have published tables of mortality should generally not only have altered the radix or number of deaths upon which the table is constructed, but also the number of deaths recorded at different ages, in order to render the decrements uniform; this is the case particularly with the Northampton Tables, as published by Dr. Price (*see* Price on *Reversionary Payments*, vol. i. p. 358). For if observations were continued to a sufficient extent, they would probably show that some ages are more exposed to disease than others—that is, they would indicate the existence of climacterics, of which alterations such as these destroy all trace.

I annex four tables,* which I have calculated, with the assistance of Mr. Deacon, from the Tables of Mortality for Males and Females at Chester, given by Dr. Price (vol. ii. p. 392). The first two tables show the probability of an individual at any age living any given number of years, as well as the expectation of life at any age. The last two show the value of £1 to be received by an individual of any age after any number of years, and the value of an annuity. The difference between these values for a male and female is very great, and shows that tables which would be applicable for the one would not be for the other.

* *See* page 207, and the note there.

I have also subjoined a table comparing the values of annuities calculated from observations at Chester (according to the hypothesis of probability I have assumed), with some which have been calculated from observations at other places. Until lately, the Government of this country granted annuities, the price of which depended on the price of stock, which renders their tables complicated. I have given their values of a deferred annuity for five years, compared with those I have calculated from the observations at Chester: it will be seen that the former are much too high.

2. Suppose a bag to contain a number of balls of p different colours, and that, having drawn $m_1+m_2+m_3 \ldots\ldots+m_p$ balls, m_1 have been of the first colour, m_2 of the second colour, m_3 of the third colour, m_p of the p^{th} colour. If $x_1, x_2, x_3 \ldots x_p$ are the simple probabilities of drawing in one trial a ball of any given colour, the probability of the observed event is $x_1^{m_1} \times x_2^{m_2} \ldots\ldots \times x_p^{m_p}$, multiplied by the coefficient of $x_1^{m_1} x_2^{m_2} \ldots\ldots x_p^{m_p}$ in the development of $(x_1+x_2 \ldots +x_p)^{m_1+m_2 \ldots\ldots +m_p}$. The event being observed, the probability of this system of probabilities is $x_1^{m_1} \times x_2^{m_2} \ldots\ldots \times x_p^{m_p}$, divided by the sum of all possible values of this quantity.

The probability in $n_1+n_2 \ldots\ldots +n_p$ subsequent trials of having n_1 balls of the first colour, n_2 of the second, n_p of the p^{th}, is a fraction of which the numerator is the sum of all the values of $x_1^{m_1+n_1} \times x_2^{m_2+n_2} \ldots\ldots \times x_p^{m_p+n_p}$, and of which the denominator is the sum of all the values of $x_1^{m_1} \times x_2^{m_2} \ldots\ldots \times x_p^{m_p}$, multiplied by the coefficient of $x_1^{n_1} \times x_2^{n_2} \ldots\ldots \times x_p^{n_p}$ in the development of $(x_1+x_2+x_3 \ldots\ldots +x_p)^{n_1+n_2 \ldots\ldots +n_p}$.

Since $x_1+x_2 \ldots\ldots +x_p=1$, if x_1, x_2, &c. be all supposed to vary from 0 to 1, and all these values to be equally possible *à priori*, the numerator will be found by integrating the expression

$$x_1^{m_1+n_1} \times x_2^{m_2+n_2} \ldots (1-x_1-x_2-x_3 \ldots\ldots -x_{p-1})^{m_p+n_p} dx_1 \times dx_2 \ldots \times dx_{p-1}$$

first from $x_{p-1}=0$ to $x_{p-1}=1-x_1-x_2 \ldots\ldots -x_{p-2}$, then from $x_{p-2}=0$ to $x_{p-2}=1-x_1 \ldots\ldots -x_{p-3}$, and so on. The denominator will be found in the same way.

If the coefficient of $x_1^{m_1} \times x_2^{m_2} \ldots \times x_p^{m_p}$ in the development of $(x_1+x_2 \ldots\ldots +x_p)^{n_1+n_2 \ldots\ldots +n_p}$ be called C, these integrations give for the probability required

$$C \times \frac{(m_1+1)(m_1+2)(m_1+3)\ldots(m_1+n_1)(m_2+1)(m_2+2)\ldots(m_2+n_2)\ldots}{(m_1+m_2+m_3\ldots\ldots+m_p+p)(m_1+m_2+m_3\ldots\ldots+m_p+p+1)\ldots\ldots} \frac{(m_p+1)(m_p+2)\ldots\ldots(m_p+n_p)}{(m_1+m_2\ldots\ldots+p+n_1+n_2+n_3-1)};$$

or if the product $(m_p+1)(m_p+2)\ldots.(m_p+n_p)$ be denoted by $[m_p+1]^{n_p}$, which is the notation used by Lacroix (*Traité du Calcul Différentiel*, vol. iii. p. 121), the probability required is

$$C\times\frac{[m_1+1]^{n_1}[m_2+1]^{n_2}\ldots.[m_p+1]^{n_p}}{[m_1+m_2\ldots.m_p+p]^{n_1+n_2+n_3\ldots+n_p}}.$$

This probability is the same as if the simple probability of drawing a ball of the p^{th} colour were m_p+1, with the difference of notation.

When n_2, n_3, n_{p-1}, &c. $=0$, and $n_p=1$, this expression gives for the chance of drawing a ball of the p^{th} colour

$$\frac{m_p+1}{m_1+m_2\ldots.+m_p+p},$$

and the probability that the index of the colour drawn is between $n-1$ and $n+q+1$ is

$$\frac{m_n+m_{n+1}\ldots.m_{n+q}+q}{m_1+m_2\ldots.+m_p+p}.$$

If we suppose the law of the possibility of life to be such that p cases or ages are possible—*à priori*, m_1, m_2, &c. will be the number of recorded deaths in a table of mortality at those respective ages, and the chance of an individual living beyond the n^{th} age will be

$$\frac{m_n+m_{n+1}\ldots.m_p+p-n}{m_1+m_2\ldots.+m_p+p}.$$

$m_n+m_{n-1}+$&c.$+m_p$ is the number given by the table as living at the n^{th} year; therefore, on the hypothesis of this law of possibility, the chance of an individual living beyond the n^{th} year is a fraction of which the numerator is the number living at that age, $+p-n$, and the denominator is the whole population on which the table is founded, or the radix $+p$. The Tables I. and II. have been calculated from this formula, from observations at Chester given by Dr. Price (vol. ii. p. 107): p was taken equal to 101 for a child at birth—that is, the chances of a child living beyond a hundred years, and of its dying in each intermediate year, were supposed to vary from 0 to 1, all these values being equally probable, *à priori*. The value of any sum to be received after any number of years is equal to the sum itself, multiplied by the chance of the individual being alive to receive it: therefore these tables give the value of unity to be received after any number of years. Considering duration of life to be valuable in proportion to its

length, the value of the expectation of life to any individual is the sum of the chances of his living any number of years multiplied by the intervening time; so that if P_n be the chance of an individual living *exactly* n years, the value of his expectation of life is $\Sigma n P_n$, which is evidently equal to $\Sigma P'_n$, if P'_n be the chance of an individual *surviving* n years: therefore the value of the expectation of life of any individual is the sum of the numbers on the same line in Tables I. and II. The unity of expectation is here the expectation of an individual who is certain to live exactly one year. The Tables I. and II. give the values of contingencies depending on a single life, without discount; the Tables III. and IV. are the same values, discounted at the rate of three per cent. compound interest. These tables give the values of annuities about six per cent. higher than those calculated from the Northampton, and given by Dr. Price, vol. ii. p. 54. The only tables of annuities on female lives that I have met with are calculated from observations in Sweden, and are given by Dr. Price, vol. ii. p. 422; but they are calculated at four and five per cent. interest. It is not to be expected, however, that tables calculated from observations made in one country will serve in another, or even in different parts of the same country.*

The probability of having n_1 balls of the first colour in $n_1+\mathrm{N}$ trials, the colours of the other N balls being any whatever, is

$$\frac{\int x_1^{m_1+n_1}(1-x_1)^{\mathrm{N}}x_2^{m_2}x_3^{m_3}\ldots.(1-x_1-x_2\ldots.x_{p-1})^{m_p}\,dx_1dx_2\ldots.dx_{p-1}}{\int x_1^{m_1}x_2^{m_2}\ldots.(1-x_1-x_2\ldots.x_{p-1})^{m_p}\,dx_1dx_2\ldots.dx_{p-1}},$$

multiplied by the coefficient of x^{n_1} in the development of $(x_1+y)^{n_1+\mathrm{N}}$, the integrals being taken between the same limits as before.

These integrations give for the probability required

$$\mathrm{C}\times\frac{(m_1+1)(m_1+2)\ldots(m_1+n_1)(m_2+m_3+m_4\ldots+p-1)(m_2+m_3+m_4\ldots+p)\ldots.}{(m_1+m_2\ldots.+m_p+p)(m_1+m_2\ldots.+m_p+p+1)\ldots.}$$
$$\frac{(m_2+m_3+m_4\ldots.+p+\mathrm{N}-2)}{(m_1+m_2+m_3\ldots.+p+n_1+\mathrm{N}-1)},$$

C being equal to $\dfrac{(n_1+1)\ (n_1+2)\ \ldots.\ (n_1+\mathrm{N})}{1\ .\quad 2\ \ldots\ldots\ \mathrm{N}+1}$. Adopting the same notation as before, this probability is equal to

* Since writing the above, I find that Mr. Finlaison has given the values of annuities, distinguishing the sexes, in the *Report of the Committee on Friendly Societies*, 1825, p. 140.

$$C \times \frac{[m_1+1]^{n_1}[m_2+m_3 \ldots +m_p+p-1]^N}{[m_1+m_2+m_3 \ldots . m_p+p]^{n_1+N}}$$
$$= \frac{C[m_1+1]^{n_1}[m_2+m_3 \ldots . +m_p+p-1]^{m_1+1}}{[m_2+m_3 \ldots . +N+p-1]^{n_1+m_1+1}},$$

which probability, as before, is the same as if the simple probability of drawing a ball of the p^{th} colour were m_p+1.

If $m_2+m_3 \ldots . +m_p+p-2=M$, and if n_1 and N are in the same ratio as m_1 and M, the chance that the number of balls of the first colour in n_1+N trials is between the limits n_1 and $n_1 \pm z$, by the reductions given in the *Théorie Anal. des Probabilités*, p. 386, is

$$1-2\sqrt{\frac{(M+m_1)^3}{m_1M(N+n_1)(M+N+m_1+n_1)2\pi}}\int dz e^{-\frac{[(M+m_1)^3 z^2}{2m_1M(N+n_1)(M+N+m_1+n_1)}},$$

e being the number of which the hyperbolic logarithm is unity, and the integral being taken from $z=z$, to $z=$infinity.

The question of determining the probability that the losses and gains of an Insurance Company on any class of life are contained within certain limits, is precisely similar to this.

It will be seen from the formula $\frac{m_n+m_{n+1} \ldots . m_{n+q}+q}{m_1+m_2 \ldots . . +m_p+p}$ (p. 200, line 12), that if life were divided into an infinite number of ages or intervals (in which case p is infinite), the hypothesis of possibility remaining the same, the probability of an individual dying in any given interval would be the given interval divided by the whole duration of life, which coincides with that which is given by De Moivre's hypothesis. Thus if life were supposed to extend to a hundred years, the probability of an individual dying in any given year would be $\frac{1}{100}$, and any finite number of observations or recorded deaths would not influence the value of this probability. As diseases and other causes producing death are not equally distributed throughout life, the last hypothesis cannot be adopted.

In order to investigate accurately the probability of death at any age, it would be necessary to know the law of possibility. Let $\phi_p x_p$ be the probability of the possibility of x_p: then the probability in the former question of having n_1 balls of the first colour, n_2 of the second, &c., in $n_1+n_2 \ldots . +n_p$ trials, is

$$C \times \frac{\int x^{m_1+n_1}(\phi_1 x_1) x_2^{\ m_2+n_2}(\phi_2 x_2) \ldots (1-x_1-x_2 \ldots x_{p-1})^{m_p+n_p} dx_1 dx_2 \ldots dx_{p-1}}{\int x_1^{m_1}(\phi_1 x_1),\ x^{m_2}(\phi_2 x_2) \ldots . (1-x_1-x_2 \ldots x_{p-1})^{m_p} dx_1 dx_2 \ldots dx_{p-1}};$$

ϕ is a sign of function, and this function may be either continuous or discontinuous.

This expression must be integrated between the same limits as before.

The coefficients of the different powers of x_p in $\phi_p x_p$, or the constants in $\phi_p x_p$, will generally be functions of the index p. If the probability of life were known at a great many places, and if x_{p_1} were the value of x_p at q_1 places, x_{p_2} at q_2 places, &c., the law of possibility might be determined approximately by considering $\phi_p x_p$ as a parabolic curve, of which x_p is the abscissa, passing through the points, of which the ordinates are

$$\frac{q_1}{q_1+q_2+, \&c.}, \quad \frac{q_2}{q_1+q_2+, \&c.}.$$

3. In the preceding investigations, the results of the preceding trials are supposed to be known; it may be worth while to examine what the probability of any future event is when the results of the preceding trials are uncertain.

Let a bag contain any number of balls of two colours, white and black: suppose m trials have taken place; and let e_n be the probability that a white ball was drawn the n^{th} trial, f_n the probability that a black ball was drawn.

$$e_n + f_n = 1.$$

First let $e_1, e_2 \ldots e_n$ be all equal, and let x be the probability of drawing a white ball. If a white ball was drawn every time in the m trials which have taken place, the probability in n_1+n_2 future trials of having n_1 white balls and n_2 black balls is

$$\frac{(n_1+n_2)(n_1+n_2-1)\ldots(n_1+1)}{1.2\ldots\ldots n_n} \frac{\int x^{m+n_1}(1-x)^{n_2}dx}{\int x^m dx}.$$

But the probability that a white ball was drawn every time is e^m; therefore the probability of drawing a white ball n_1 times and a black ball n_2 times, on this hypothesis, multiplied by the probability of the hypothesis, is

$$\frac{(n_1+n_2)(n_1+n_2-1)\ldots(n_1+1)}{1.2\ldots\ldots n_2} e^m \frac{\int x^{m+n_1}(1-x)^{n_2}dx}{\int x^m dx};$$

and the probability of drawing n_1 white balls and n_2 black balls will be the sum of the probabilities on every hypothesis, multiplied respectively by the probability of the hypothesis, which is

$$\frac{(n_1+n_2)(n_1+n_2-1)\ldots(n_1+1)}{1.2\ldots\ldots\ldots n_2}\left\{e^m\frac{\int x^{m+n_1}(1-x)^{n_2}dx}{\int x^m dx}\right.$$

$$+me^{m-1}f\frac{\int x^{m+n_1-1}(1-x)^{n_2+1}dx}{\int x^{m-1}(1-x)dx}+\frac{m.m-1}{1.2}e^{m-2}f^2\frac{\int x^{m+n-2}(1-x)^{n_2+2}dx}{\int x^{m-2}(1-x^2)dx}+, \&c.$$

This integral being taken from $x=0$ to $x=1$, is

$$\frac{(n_1+n_2)(n_1+n_2-1)..(n_1+1)}{1.2\ldots\ldots\ldots n_2}\left\{\frac{n_2, n_2-1, n_2-2\ldots\ldots 1.m+1}{m+n_1+1.m+n_1+2\ldots m+n_1+n_2+1}e^m\right.$$

$$+me^{m-1}f.\ \frac{n_2+1.n_2.n_2-1\ldots.2}{m+n_1.m+n_1+1\ldots.m_1+n_1+n_2}m+1.m+, \&c.$$

$$=\frac{(n_1+n_2)(n_1+n_2-1)\ldots.(n_1+1)}{1.2\ldots\ldots\ldots\ldots n_2}\times\frac{1}{m+2.m+3\ldots m+n_1+n_2+1}$$

$$\left\{n_2.n_2-1.n_2-2\ldots m+n_1.m+n_1-1\ldots m+1.e^m+n_2+1\ldots\right.$$

$$2.m+n_1-1\ldots m+1.m.me^{m-1}f+, \&c.$$

This series is equal to $\frac{d^{n_1+n_2}.y^{n_2}x^{n_1}(ex+fy)^m}{dx^{n_1}dy^{n_2}}$, when x and y are made equal to 1, and this is equal to $1.2.3\ldots.n_1.1.2.3\ldots.n_2\times$ coefficient of $h^{n_1}k^{n_2}$, in the development of

$$(1+h)^{n_1}(1+k)^{n_2}(1+eh+fk)^m$$

$$(1+eh+fk)^m=1+m(eh+fk)+\frac{m.m-1}{1.2}(eh+fk)^2$$

$$+\frac{m.m-1.m-2}{1.2.3}(eh+fk)^3+, \&c.$$

$$(1+h)^{n_1}(1+k)^{n_2}=h^{n_1}k^{n_2}+n_1h^{n_1-1}k^{n_2}+\frac{n_1.n_1-1}{1.2}h^{n_1-2}k^{n_2}$$

$$+\frac{n.n-1.n-2}{1.2.3}h^{n_1-3}k^{n_2}+, \&c.$$

$$+n_2h^{n_1}k^{n_2-1}+n.n_2h^{n_1-1}k^{n_2-1}+\frac{n_2.n_1.n_1-1}{1.1.2}h^{n_1-2}k^{n_2-1}+, \&c.$$

$$+\frac{n_2.n_2-1}{1.2}h^{n_1}k^{n_2-2}+\frac{n_1.n_2.n_2-1}{1.2}h^{n_1-1}k^{n_2-2}+, \&c.$$

$$+\frac{n_2.n_2-1.n_2-2}{1.2.3}h^{n_1}k^{n_2-3}+, \&c.$$

Coefficient of $h^{n_1}k^{n_2}=1+m(n_1e+n_2f)$

$$+\frac{m.m-1}{1.2}\left(\frac{n_1.n_1-1}{1.2}e^2+2n_1n_2ef+\frac{n_2.n_2-1}{1.2}f^2\right)+, \&c.$$

The probability required is

$$\frac{1.2.3.\ldots\ldots n_1+n_2}{m+2.m+3\ldots.m+n_1+n_2+1}$$

$$\left\{1+m(n_1e+n_2f)+\frac{m.m-1}{1.2}\left\{\frac{n_1.n_1-1}{1.2}e^2+2n_1n_2ef+\frac{n_2.n_2-1}{1.2}f_2\right\}+, \&c.\right.$$

If there are p different colours, and if m trials have taken place, and $e_{q,p}$ is the chance that a ball of the p^{th} colour was drawn the q^{th} trial, the probability of drawing n_1 balls of the first colour, n_2 of the second, n_p of the p^{th}, in $n_1+n_2\ldots.+n_p$ future trials, may be found in the same way. Let

$$e_{1,1}+e_{1,2}+e_{1,3}+\&c.\ldots\ldots e_{1,m}=S_1,\ e_1,$$
$$e_{1,1},\ e_{1,2}+e_{1,3},\ e_{1,4}\ldots\ldots\ldots\&c.=S_2,\ e_1,$$

(the sum of the products of e_1 two and two together,)

$$e_{1,1},\ e_{2,2}+e_{1,2},\ e_{2,3}+,\&c. \qquad =S_1e_1,\ S_1e_2,^*$$

and so on; then it may be shown that this probability is equal to

$$\frac{1.2.3..n_1+n_2+n_3.\ldots.+n_p}{m+p.\ldots.m+n_1+n_2\ldots.+n_p+p-1}(1+Se_1)^{n_1}(1+Se_2)^{n_2}\ldots.(1+Se_p)^{n_p},$$

$1+(Se_1)$, $1+(Se_2)$, &c. being expanded by the binomial theorem, and the indices of S written at the foot.

The method which was used for summing the series in the last page is of very general application, and depends, in fact, on this principle, that the generating function of the sum of any series is the sum of the generating functions of each of the terms of the series.

If in the last formula n_2, n_3, &c.$=0$, and if there be only two events possible, and $n_1=1$, the probability required is $\frac{1+Se_1}{m+2}$. In order to apply this, suppose an individual to have asserted m events to have taken place, of which the simple probabilities are equal, and equal to p; and suppose it required to find the probability of his telling the truth in another case, where the simple probability of the event he asserts to have taken place is not known. Let x be the veracity of the individual, the probability of his telling the truth on this hypothesis is $\frac{px}{px+(1-x)(1-p)}$; and the probability of his telling the truth is the sum of the probabilities of his telling the truth on each hypothesis, divided by the number of the hypotheses.

* This is a method of notation which obtains, but it is not meant to imply that $S_1e_1S_1e_2=S_1,e_1\times S_1,e_2$.

Suppose x to vary from 0 to 1, and all these values of x to be equally probable *à priori*, then the probability of his having told the truth and the event having taken place is $\int \frac{pxdx}{px+(1-p)(1-x)}$, taken from $x=0$ to $x=1$, which integral is

$$\frac{p}{2p-1}\left\{1-\frac{(1-p)}{2p-1}\text{ hyp. log. }\frac{p}{1-p}\right\}.$$

If $p=\frac{9}{10}$, this probability is ·81601. Generally, if $p>\frac{1}{2}$, the assertion that the event has taken place (on this hypothesis of veracity) rather diminishes the probability that the event has taken place; if $p=\frac{1}{2}$, the assertion does not alter the probability; if $p<\frac{1}{2}$, the assertion rather increases it.

If $p=\frac{9}{10}$, $e=$·81601, let $m=10$; then $\frac{1+Se}{m+2}=\frac{9\cdot1601}{12}$, which is the probability that the individual will tell the truth in another case. If the individual had told ten truths, the chance of his telling the truth in another case would have been $\frac{11}{12}$.

All values of x between 0 and 1 were supposed equally possible: if they are not, let ϕx be the probability of the possibility of any value of x; then the probability of an individual telling the truth will be $\frac{\int \phi x dx}{px+(1-x)\ (1-p)}$ divided by $\int \phi x dx$, these integrals being taken from $x=0$ to $x=1$.

Table formed from the Burials in All Saints' Parish, Northampton, from 1735 *to* 1780. (See *page* 198, *line* 26.)

Age.	Actual Number of Burials.	Reduced to radix 11650.	
		By the Observations.	As altered by Dr. Price.
Under .. 2	1,529	3798½	4367
Between 2 and 5	362	899½	1034
„ 5 „ 10	201	499⅜	574
„ 10 „ 20	189	469⅝	543
„ 20 „ 30	373	926⅛	747
„ 30 „ 40	329	817⅜	750
„ 40 „ 50	365	906¾	778
„ 50 „ 60	384	954	819
„ 60 „ 70	378	939½	806
„ 70 „ 80	358	889⅜	763
„ 80 „ 90	199	494⅜	423
„ 90 „ 100	22	54⅝	46
TOTAL . . .	4,689	11650	11650

TABLE I.—*Males.*

Age.	Expectation of Life, by the Author's method.	Expectation of Life, by usual method.
0	29·75345	28·13
1	36·69540	35·76
2	40·21306	39·42
3	42·54615	41·97
4	43·83928	43·33
5	44·09357	43·20
10	42·75204	41·92
15	38·95786	38·05
20	35·82561	34·86
25	32·99367	32·00
30	30·27118	29·25
35	27·04332	25·97
40	24·04172	22·92
45	21·34876	20·20
50	18·81444	17·64
55	16·34857	15·14
60	13·63392	12·36
65	12·05917	10·79
70	9·41263	8·05
75	8·43636	7·00
80	6·99009	5·43
85	5·90384	4·25
90	4·32000	2·50
95	2·14285	1·00

TABLE II.—*Females.*

Age.	Expectation of Life, by the Author's method.	Expectation of Life, by usual method.
0	34·55535	33·27
1	40·04475	39·54
2	43·65276	43·25
3	45·87700	45·68
4	47·23533	47·11
5	47·99860	47·44
10	45·69310	45·17
15	41·96030	41·36
20	38·76308	38·10
25	35·49329	34·78
30	32·79565	32·27
35	30·00384	29·26
40	27·13292	26·37
45	24·29072	23·50
50	21·43212	20·62
55	18·35900	17·52
60	15·09954	14·20
65	12·83834	11·94
70	9·78378	8·81
75	8·12794	7·14
80	6·30434	5·20
85	5·97402	4·81
90	4·55263	3·46
95	2·07692	1·71

TABLE III.—*Males.*

Age.	Value of Annuity by the Author's method.	Value of Annuity by usual method.
0	13·96256	
1	17·35468	
2	19·17322	
3	20·38907	
4	21·15972	
5	21·42118	21·283
10	21·55443	21·512
15	20·38198	20·283
20	19·42818	19·285
25	18·55566	18·399
30	17·67138	17·492
35	16·36473	16·134
40	15·05567	14·667
45	13·81590	13·493
50	12·59164	12·316
55	11·28375	10·866
60	9·63491	9·140
65	8·77709	8·220
70	6·94786	6·260
75	6·17140	5·291
80	5·11141	
85	4·30505	

TABLE IV.—*Females.*

Age.	Value of Annuity by the Author's method.	Value of Annuity by usual method.
0	15·75290	
1	18·42550	
2	20·14850	
3	21·32367	
4	22·11858	
5	22·64306	22·624
10	22·41169	22·439
15	21·25267	21·235
20	20·36435	20·323
25	19·33571	19·265
30	18·65687	18·583
35	17·62920	17·534
40	16·56779	16·366
45	15·42043	15·282
50	14·14872	13·983
55	12·58232	12·376
60	10·67266	10·410
65	9·37655	9·080
70	7·26287	6·889
75	6·02689	5·338
80	4·65536	
85	4·34792	

NOTE.—We do not reprint all the tables originally given in this paper. They are of value now only as exhibiting the results of the methods laid down by the Author, and for that purpose those here quoted will suffice.—ED. *A. M.*

On the rate of Sickness and Mortality amongst the Members of Friendly Societies in France. By SAMUEL BROWN, F.S.S., *Actuary of the Guardian Assurance Company.*

[Read before the Institute of Actuaries, 8th January, 1855, and ordered by the Council to be printed.]

IT is not without some degree of pride, that we recall the fact that England has taken the lead in fostering and extending those social institutions which appear destined to carry out the beneficent design of procuring the greatest possible amount of happiness for the greatest possible number. She has not been afraid to encourage a spirit of self reliance in the mass of the people, and to allow her working classes to associate freely in the effort to equalize the uncertainties of life, so that those who may have a little better fortune than the average may assist those who have a little less. Now and then, indications may have been observed of fear amongst the ruling powers lest this free association should be used for political purposes, and doctrines dangerous to the Government or the good order of society be thereby enabled to circulate too easily amongst classes who may have real wrongs to redress or fancied rights to assert; but, in the long run, good sense has prevailed, and Government has fortunately perceived that Friendly Societies for mutual aid in sickness or want, and other associations for bettering the condition of the working classes, gave them a direct interest in the preservation of the public peace, and formed by their very principles the antidote to the fears they had excited. It would be gratifying, therefore, to perceive the rapidity with which they have spread in this country, could we at the same time feel that they have been conducted as well as they have been designed, and, in their pecuniary point of view, not fallen short of the expectations they have raised. The contrary has been lamentably the case. The greater part of our Friendly Societies or Sickness Clubs were for many years in a deplorable state of insolvency, inflicting widespread misery just at the time when their aid was most important, and failing to fulfil their obligations to their members at a period of life when, from incapacity to work, they were obliged to claim the relief for which they had been so long laboriously saving. There can be no doubt that the great cause of this wretched condition of Societies which might have been so useful was the want of data for a correct estimation of the risks, the ignorance of the managers of the rapid growth of their liabilities with the age of the members, the smallness of the

numbers in each club (which probably arose from the desire which so many of the members had to be office-bearers), and the consequent inordinate expenses when compared with the income. These evils were found to prevail even in the best regulated. Other causes rapidly hastened the dissolution of the worst—such as the meeting in public houses, the owners of which, being treasurers of the funds, saw them diminish without fear whilst they passed into their own pockets in the way of business; costly processions, with banners and music, losing the day's wages from which the sickness of a future day was to be provided for; and various other equally absurd practices, opposed to the very spirit and object of these institutions. Notwithstanding these serious drawbacks, the benefits derived from the system have been so great, these unions are so adapted to the wants of the working classes, and have such an inherent vitality of good in them, that the evils complained of have been gradually removed, or are in the process of it, and the defects of information are every year being supplied. The first item, the want of data, is by far the most important; and several distinguished writers have given themselves with zeal to the collection and publishing of facts, or theories which, in the absence of facts, they endeavoured to draw from other reasonings. In the early history of this branch of study, Price and Morgan did good service; and in later periods Ansell, Edmonds, Neison, Ratcliffe, and last of all Finlaison, have proved the energy and spirit with which this difficult subject has been taken up in this country, both by the Government and the people. Mr. Edmonds, in his interesting paper in the *Lancet* (which is also inserted in the last Number of the *Assurance Magazine*), enumerates the extent of the observations of sickness and mortality, according to age, already recorded:—The Highland Society's report, 85,000 years of life; Mr. Ansell's, 25,000; his own, in the *Lancet* of April 1839, 30,000; Mr. Neison, for the five years ending with 1840, 1,000,000; Mr. Ratcliffe, the Manchester Unity of Odd Fellows, 600,000, during the three years ending with 1848; and, lastly, Mr. Finlaison's, Jun., in the Government Report just issued, about 800,000 years of life, extending over the five years ending with 1850.

By the side of these large and important collections for this country, the observations in France, to which I now beg to call your attention, appear very trifling. They are nevertheless important, as being the first fruits of a direct inquiry made with the laudable purpose of encouraging in France the increase of institutions which it is admitted owe their greatest development there to the example of England. Both in this and in the kindred sub-

ject of life assurance, our neighbours, now so nobly emulous with us in the glories of war, are commencing a still more honourable rivalry in the arts and enterprises of peace, and in those social improvements which confer more than glory, in diffusing happiness amongst the people.

From 1801 to 1847 the Société de Secours Mutuels of Paris received great encouragement and aid from the Société Philanthropique; but, as the system spread, it was found to distract attention too much from the other important objects of that Society, and in 1849 a number of enlightened men, standing high in government, in banking, or in commerce, formed a new union, under the title of "Comité pour la Propagation des Sociétés de Prévoyance," in order to extend the working of these useful institutions. In the inquiries made by this committee into the state of the Sociétés de Secours Mutuels, it was found that many were in the same deplorable state as they were formerly discovered to be in England, and principally for the same reason—the want of the data for calculating the risks. Besides, therefore, collecting the documents and statistics of these Societies, the committee sent out forms for a new collection of facts as to the sickness and mortality experienced amongst their members, and were so far successful in the results that in 1851 they determined to print a *Mémoire sur l'Histoire et l'Organization des Sociétés de Secours Mutuels*," and a table of sickness deduced from the observations they had obtained. The publication was entrusted to M. M. G. Hubbard, their secretary, who acknowledges the obligations he owed in the formation of the tables to the mathematical talents of M. Olinde Rodrigues, who had already written on the *Caisses de Retraite*. This treatise was published in July, 1852.

It would take too much of your time to go generally into the subject of the Societies of which this useful work gives a sketch of the history and organization. I propose only, on this occasion, to show the method employed in obtaining the facts relating to sickness and mortality, and give a brief comparison of the results with the English tables.

The principal tables of mortality in use in France are—

1. That by Deparcieux, which was deduced from the registers of the French Tontines of 1689, 1706, 1709, and 1734; but which is considered, from the nature of the observations, to be more especially applicable to the higher classes.

2. Dupré de Saint Maur and Buffon published a table based upon the deaths recorded in three parishes in the city of Paris, and twelve parishes in the environs; but though prepared with

scrupulous care, it is too full of anomalies to have come into general use, although M. de Saint Cyran, in 1779, corrected its irregularities.

3. In 1806, Duvillard, in his *Analyse de l'Influence de Petite-Vérole sur la Mortalité*, formed a table of observations from a great number of facts collected in various parts of France before the Revolution, which he conceived to represent very closely the actual law of mortality in that country; but so many changes in the population have occurred for the better since that period, that if it ever represented the real state of the question, it has for some time been deemed to show too high a rate of mortality, and ceased to be regarded with much authority.

It may be remarked, that the table of Duvillard bears much the same analogy to that of Deparcieux which the Northampton Table does to the Carlisle, and has consequently been used in a similar manner; the former having been adopted to a great extent for *life assurance*, and the latter for *life annuities*. Out of 1,000 persons at age 25, the survivors would be—by Duvillard, 353, and by Deparcieux 510; by the Northampton 343, and the Carlisle 514.

In recent periods (about 1838) a very extensive series of observations have been completed by M. de Montferrand, which, I believe, are corrected in a great measure from the population returns given in the *Annuaires du Bureau des Longitudes*. They make a distinction between the mortality of males and females, and have lately excited much attention.

In the *Annales d'Hygiène* appear some excellent papers, by M. Benoiston de Châteauneuf and M. Villermé, on the comparative mortality of the rich and poor; and, with this exception, the tables hitherto published are all defective in the special application for the purpose in view, and throw little light on the subject of the mortality of the labouring classes, taken by themselves, and none at all on the important inquiry as to the rate of sickness amongst them.

In 1809, M. Mourgue, having devised a plan for the relief of the working classes, in connection with a savings fund and provision for old age, directed his inquiries to the number of sick persons received in the hospitals of Paris, compared with the total labouring population; and the result was seven days of sickness for each individual, as the mean of five years. At a later period M. Gérando concluded that, for each inhabitant of Paris, the average sickness was eight or nine days per annum.

In 1830, M. Villermé, being requested by the Société Philanthropique de Paris to make a report on the subject to the delegates

of the Friendly Societies of Paris, first drew attention to the returns of the Highland Society, and suggested that this table might be adapted to the wants of the French Societies by correcting the table according to the difference of the rate of mortality in England and France. It will be observed, that this was at once recognizing the law, which most subsequent observations have confirmed, of the constant ratio which prevails between the laws of sickness and mortality. In 1844 this hint was acted upon by M. Debouteville, in calculating the rates to be charged by the French Societies, graduated according to ages; and after comparing, for the first thirty years of the ages taken, the sickness of the Highland Society's returns with the Carlisle Table, he multiplied the numbers so obtained by $\frac{4}{5}^{ths}$, being the ratio of the mortality of France to that of England, as deduced and published by M. Quetelet. The result was a mean of $18\frac{1}{2}$ days annually between the ages 26 and 70, which would be reduced to 12·77 if restricted to the periods 21 to 65 years of age.

The mean for each five years of age, by the above table, is

Ages.	Days of Sickness.	Ages.	Days of Sickness.
20 to 25	5·9	45 to 50	12·2
25 „ 30	6·7	50 „ 55	14·5
30 „ 35	8·0	55 „ 60	18·3
35 „ 40	9·5	60 „ 65	28·7
40 „ 45	10·7	65 „ 70	69·8

The sum of the total days of sickness, 923·5, is 60 more than that of Mr. Ansell's, and 130 less than that of Mr. Neison. However defective this table may be in authority, from the mode in which it was formed, it deserves attention as the first attempt practically to apply the knowledge already acquired in another country, and regulate the sickness rates in France by a law deduced from the mortality.

To obtain, however, a more accurate collection of facts, the committee we have described sent, in the month of March, 1850, a circular to all the delegates of the Societies of Paris and of the departments whose address they could obtain, requesting them to have the goodness to fill up two forms enclosed with all the facts relating to each Society from its commencement. The circular contained also instructions, and a list of questions to be answered and returned to the committee, with the statutes and rules of the Society. The first form was almost exactly the same as that required from the English Societies by the Government Act of 1829; although, instead of restricting the returns for five years,

the committee left the Societies to fill in for as many years as they pleased. They requested also the money payments for the days of "sickness" and "infirmity," and the date when the members quitted the Society, by becoming entitled to a pension, by withdrawal, or dismission.

As the Societies do not generally admit members until they have satisfied the law of conscription, and as almost all grant pensions commencing at 65 years of age, the observations may naturally be expected to be limited between the ages 21 and 65, although a few may be found to extend to 70. Having collected these facts, it was thought sufficient to combine them in periods of five years—ages 21 to 25, 26 to 31, &c.

The 25 Societies (out of 150) which responded to the appeal—some giving their experience for even 30 years—furnished a total of observations amounting to 44,069 years of life, during which they have paid 453,222 francs (£18,129) for 257,478 days of "sickness," and 46,755 francs (£1,870) for 102,979 days of "infirmity." During that time they have admitted 3,319 new members, and have lost by deaths 590, and by withdrawals 1,898.

There is great difficulty in ascertaining at present the number of the Sociétés de Secours Mutuels in France; but judging from the accounts rendered by the Government of their investments in the savings' banks, which neither include the whole of those publicly established nor those privately attached to particular workshops, the subject appears to be still quite in its infancy. M. Hubbard gives a table showing the number of the Societies so ascertained in each department of France, and the total investments are not equal to much more than two thirds of one year's income of the Manchester Unity of Odd Fellows in England. In a population of 35,401,761, he enumerates only 2,056 Societies, with investments of £228,809; but allowing for those unknown, he estimates that there may be about 2,500 Societies, with an average of 160 members each, giving a total of 400,000 members and an annual income of £288,000. It appears that not only are many of them in the same deplorable condition as the English clubs, but the same causes contribute to their dissolution. Thus some divide their whole funds at the end of every year. The saint's day of the Society is sometimes held by others in doubtful honour at a cabaret. The aubergiste lets his rooms to the members, and, becoming the delegate, takes care that the funds are to a great extent expended with himself. Occasionally, even, there is a fine for anyone who does not spend 50 cents in beer. This system is now wearing out; but no wonder it led to frequent failures formerly. In Paris

alone, out of 205 Societies established before 1831, 66 had failed by 1840, and 37 of the remainder had not a capital of £4 for each member; and in Rouen, of 35 Societies founded since 1808, 22 had already disappeared in 1843.

To return to the observations newly collected, the following table will show the general summary of the 44,069 years of life, classified in quinquennial periods of age. In deducing therefrom the tables for use, the number entering and the number withdrawn have been taken at one half, as being supposed to continue in each case only through half the year of observation. The number "unknown" have not been classified in proportion, it being considered that some of these Societies contained an unusual proportion of old lives.

TABLE I.—*Showing, in Quinquennial Periods of Age, the Returns of Sickness and Mortality in 25 Sociétés de Secours Mutuels of France.*

(1) Ages.	(2) Admissions.	(3) Years Observed.	(4) Days of Sickness.	(5) Payments for ditto.	(6) Deaths.	(7) Withdrawals.	(8) Days of Infirmity.	(9) Payments for ditto.
				£.				£.
20 & under	62	115	290	14·14	3	14		
20 to 25	540	1,755	7,201	470·04	12	160	500	4·13
25 „ 30	741	4,085	19,090	1329·81	27	267	1,643	23·67
30 „ 35	904	6,650	28,329	1986·36	57	354	2,781	47·17
35 „ 40	758	8,366	40,832	2907·77	64	357	4,564	78·94
40 „ 45	190	7,365	44,438	3169·98	64	230	14,151	236·50
45 „ 50	11	5,593	35,466	2517·02	86	88	11,169	193·18
50 „ 55	2	3,702	23,507	1570·02	60	55	9,883	164·06
55 „ 60	..	2,427	21,290	1479·46	49	44	6,875	124·74
60 „ 65	..	1,409	15,116	994·13	41	78	11,592	209·44
65 „ 70	..	544	5,677	418·92	29	20	8,751	191·27
70 „ 75	..	147	1,774	128·45	19	11	7,570	91·16
Unknown	111	1,911	14,468	1142·77	79	220	23,500	505·97
	3,319	44,069	257,478	18128·87	590	1,898	102,979	1870·23

M. Hubbard notices the error which has occurred in all English tables, except the recent ones of Finlaison, of confounding claims for superannuation with claims for sickness. Mr. Henry Tompkins, in his very excellent essay on the Sickness and Mortality experienced in Friendly Societies, read before this Institute and published in the 17th Number of the *Assurance Magazine*, points out the consequences of this error very forcibly, and shows that both by Ratcliffe's and Neison's tables the effect above the age of 60 is to double the cases of sickness as compared with Finlaison's tables; which is also noticed in Mr. Edmonds' paper in the last Number of the *Assurance Magazine*. Columns 8 and 9 in the above table,

headed "days of infirmity" and "payments for ditto," are intended to allow of this correction being made—the word "infirmity" having the same meaning as "superannuation," though more expressive of the truth, as some such cases are found at comparatively young ages. Taking, then, the proportion of payments for a measure, M. Hubbard deduces from the days of "sickness" the number of days which would correspond with the same amount of payment for "sickness" as is made for "infirmity." Thus the average payment for each day of sickness, on the total observations, is 1·76 francs, and for infirmity 0·45 francs, the former being to the latter as 1 to ·258: about four days of "infirmity," therefore, may be counted as one of sickness. He consequently adds $102{,}979 \times {\cdot}258 = 26{,}568$ days' "infirmity," to the days of "sickness," making the total of observations $257{,}478 + 26{,}568 = 284{,}046$ days; and, deducting from the total years of life half the number who entered and withdrew, he finds $\frac{284{,}046}{41460{\cdot}5} = 6{\cdot}85$, average number of days of sickness per year.

It should be observed, that as this average results from money payments made by Societies, it would not exactly represent the average of sickness in a community, as in most cases some days of sickness would elapse before relief would be actually granted. M. Hubbard thinks that so much as five days should be added for every case of sickness observed: it seems to us rather a large proportion.

After the correction made as above suggested, the following table gives the summary of the results in quinquennial periods of age:—

TABLE II.—*Corrected Tables of the rates of Sickness and Mortality in the French Societies.*

Ages.	Years of Observations.	Days of Sickness and Infirmity.	Rate of Sickness.	Deaths.	Rate of Mortality per cent.
20 & under	77	290	3·76	3	3·89
20 to 25	1,405	7,265	5·17	12	·85
25 „ 30	3,581	19,430	5·42	27	·75
30 „ 35	6,021	28,991	4·81	57	·94
35 „ 40	7,809	41,936	5·37	64	·82
40 „ 45	7,155	47,763	6·67	64	·89
45 „ 50	5,544	38,191	6·88	86	1·55
50 „ 55	3,674	25,958	7·07	60	1·63
55 „ 60	2,405	23,105	9·61	49	2·00
60 „ 65	1,370	18,304	13·36	41	2·99
65 „ 70	534	8,267	15·48	29	5·43
70 „ 75	1,415	3,031	21·34	19	13·42
Unknown	..	20,883	11·97	79	4·53
All ages	41,460	..	6·85	590	1·42

In order to complete these observations for the use of the English reader, I subjoin a table combining the facts of two which M. Hubbard gives for each age, having deduced the results from the original observations by third differences, and the mortality after 70 years being continued by the table of Deparcieux.

TABLE III.—*Showing the Decrements of Life, the Rate of Mortality and the Mean Days of Sickness, and the Mean Duration of Life, at each Age, by the experience of the French Sociétés de Secours Mutuels.*

Age.	Living.	Deaths.	Mortality per cent.	Mean days of Sickness for each Life.	Mean duration of life.	Age.	Living.	Deaths.	Mortality per cent.	Mean days of Sickness for each Life.	Mean duration of life.
21	10,000	90	·90	4·072	41·83	58	6,656	120	1·80	9·500	15·70
22	9,910	83	·84	4·604	41·21	59	6,536	141	2·16	10·748	14·98
23	9,827	79	·80	5·000	40·55	60	6,395	155	2·42	11·724	14·30
24	9,748	76	·78	5·276	39·88	61	6,240	163	2·62	12·476	13·64
25	9,672	74	·77	5·448	39·19	62	6,077	170	2·80	13·052	12·99
26	9,598	74	·77	5·532	38·49	63	5,907	177	3·00	13·500	12·35
27	9,524	77	·78	5·544	37·77	64	5,730	195	3·35	14·172	11·72
28	9,447	76	·80	5·500	37·08	65	5,535	212	3·76	14·736	11·12
29	9,371	81	·86	5·296	36·38	66	5,323	229	4·23	15·164	10·54
30	9,290	83	·89	5·148	35·69	67	5,094	247	4·77	15·428	9·99
31	9,207	83	·90	5·052	35·01	68	4,847	267	5·40	15·500	9·47
32	9,124	83	·91	5·004	34·32	69	4,580	300	6·41	16·188	8·99
33	9,041	81	·90	5·000	33·63	70	4,280	283	6·62	17·084	8·59
34	8,960	79	·88	5·012	32·93	71	3,997	275	6·87	18·236	8·17
35	8,881	76	·86	5·076	32·22	72	3,722	275	7·38	19·692	7·73
36	8,805	74	·84	5·184	31·49	73	3,447	274	7·96	21·500	7·31
37	8,731	71	·81	5·328	30·76	74	3,173	275	8·65	..	6·90
38	8,660	69	·80	5·500	29·99	75	2,898	261	9·00	..	6·50
39	8,591	60	·70	5·740	29·24	76	2,637	261	9·89	..	6·10
40	8,531	58	·68	5·960	28·44	77	2,376	261	10·98	..	5·71
41	8,473	61	·72	6·160	27·63	78	2,115	247	11·68	..	5·36
42	8,412	66	·79	6·340	26·82	79	1,868	247	13·23	..	5·00
43	8,346	75	·90	6·500	26·04	80	1,621	233	14·40	..	4·69
44	8,271	94	1·14	6·784	25·27	81	1,388	220	15·84	..	4·39
45	8,177	108	1·32	6·952	24·56	82	1,168	192	16·47	..	4·12
46	8,069	117	1·45	7·028	23·88	83	976	165	16·90	..	3·84
47	7,952	122	1·54	7·036	23·22	84	811	151	18·64	..	3·52
48	7,830	125	1·60	7·000	22·57	85	660	137	20·83	..	3·21
49	7,705	125	1·62	6·752	21·93	86	523	124	23·68	..	2·92
50	7,580	124	1·63	6·636	21·29	87	399	96	24·13	..	2·67
51	7,456	121	1·62	6·644	20·63	88	303	83	27·27	..	2·36
52	7,335	118	1·61	6·768	19·96	89	220	69	31·26	..	2·06
53	7,217	115	1·60	7·000	19·28	90	151	55	36·36	..	1·77
54	7,102	111	1·57	7·212	18·58	91	96	41	42·85	..	1·50
55	6,991	110	1·57	7·596	17·87	92	55	27	50·00	..	1·25
56	6,881	111	1·61	8·124	17·15	93	28	14	50·00	..	1·00
57	6,770	114	1·69	8·768	16·43	94	14	14	100·00		

With regard to the table of mortality here exhibited, it will be found to approach nearer to the Carlisle and Deparcieux than to the Northampton or Duvillard, but, no doubt, for the same reason

which has affected the results in the experience of Life Assurance Companies—that is, the exclusion from the Societies of lives which are manifestly unhealthy at the age of entry, as well as the admission of a class who, by the payments they are enabled to make of their surplus earnings, evidently show that they are not suffering those extremes of indigence and misery which would act on a table including the general population of a country.

The comparison with other tables will be best shown by the following short summary of the mean duration of life, by the principal tables used in France, Belgium, and England :—

TABLE IV.—*Showing a comparison of the Mean Duration of Life by different Tables.*

Age.	FRANCE.			BELGIUM.	ENGLAND.			
	French Sociétés de Secours Mutuels.	Deparcieux.	De Montferrand (Males).	Quetelet.	Carlisle.	Farr.	17 Life Assurance Comps.	Friendly Societies (Neison).
30	35·69	34·06	34·00	31·66	34·34	33·13	33·17	36·60
40	28·44	27·48	27·00	25·70	27·61	26·56	26·06	29·33
50	21·29	20·38	19·91	19·79	21·11	20·02	19·41	22·19
60	14·30	14·25	13·25	13·58	14·34	13·59	13·47	15·69

From the comparison it appears that a rather greater mortality was experienced in the French Friendly Societies, at and under the age of 40, than by Neison's table, and rather less at the advanced ages. It is remarkable that the mortality in the French Sociétés de Secours Mutuels appears to be less than by the table of Deparcieux, which corresponds very nearly with the Carlisle Table, and that the same favourable circumstances may be observed in Mr. Neison's table of the mortality in the Friendly Societies of England.

In regard to sickness, a considerable difference will be observed between the French and English tables; but this arises in a great measure from the correction which M. Hubbard has thought it incumbent on him to make as to the difference of rates for superannuation allowance. In order to compare them together it is necessary to take the facts as they originally stood, one day of the latter counting in the same manner as one day of the former. In the application of the table to real use the correction may be just; but it would evidently lead to false inferences in comparison with the English tables, in which such correction had not been made. We have then the fair comparison as follows :—

TABLE V.—*Showing the Rate of Sickness in Days in each Quinquennial Period.*

Ages.	Hubbard (French).	Ansell.	Neison.	Finlaison.
20 to 25	5·48	5·5	6·0	6·86
25 „ 30	5·79	5·9	6·3	6·91
30 „ 35	5·17	6·4	6·6	6·92
35 „ 40	5·83	7·3	7·6	7·82
40 „ 45	8·19	8·8	9·5	8·83
45 „ 50	8·41	10·8	12·3	10·53
50 „ 55	9·09	14·	16·7	12·97
55 „ 60	11·70	19·7	23·8	16·45
60 „ 65	19·49	31·8	42·3	23·65
65 „ 70	27·02	62·7	79·7	36·05
70 „ 75	66·03			58·26

The total number of the days of sickness up to age 70 is, by Ansell, 863; Neison, 1,053; Finlaison, 685; and Hubbard, only 403.

I will only conclude with the observation, that none of the French tables serve to indicate the mortality or sickness caused by different occupations. In the present state of these studies in France we could not expect to find the elaborate subdivision which we notice in Ratcliffe or Neison, or that useful combination of trades into classes of "light labour exercised in the open air" or "in shelter," and "heavy labour exercised in the open air" or "in shelter," by which Mr. Finlaison's recent tables are distinguished, nor even that distinction of various provinces in which particular classes of labour more generally prevail; but there exist some observations made in the hospitals of Paris in 1850, in which an attempt was made to deduce the duration of sickness, and the number of sick for each death, according to the occupation of the patient. These, like similar observations made in the English hospitals, are of little use without knowing the number of persons engaged in each trade, and whether the due proportion of each entered the hospitals; but it affords some curious results as to the intensity and duration of sickness according to occupation in France, and I therefore subjoin the table.

If the like observations could be made for all the hospitals of Paris, and a careful enumeration effected of the number of persons engaged in each trade or occupation within the limits to which the benefits of admission to the hospitals would extend, the facts would be useful for many important applications.

TABLE VI.—*Showing, from observations in the Hospitals of Paris for the year* 1850, *the Duration of Sickness and the Mortality in different Trades.*

Occupations.	Observations.			Duration of each case of Sickness.	Number of Sick for each Death.
	Cases of Sickness.	Days of Sickness.	Deaths.		
Coachmen	303	5·398	21	17·81	14·42
Stonecutters	227	5·771	17	25·42	13·35
Masons	1,000	23·381	69	23·38	14·49
Tilers	216	4·045	21	18·72	10·28
Zinc workers	59	1·623	6	27·51	9·83
Plumbers	73	1·848	7	25·31	10·42
Marble workers	102	2·425	9	23·77	11·33
House painters	905	19·894	66	21·98	13·71
Paviors	113	2·608	9	23·07	12·55
Scavengers	29	754	2	26·00	14·50
Woolcombers	80	1·503	9	18·78	8·88
Spinners	136	2·961	8	21·76	17·00
Weavers	221	5·699	10	25·78	22·10
Dyers	162	3·735	15	23·05	10·80
Tailors	927	21·424	73	23·11	12·69
Tanners	106	1·373	9	12·95	11·77
Bakers	584	12·473	29	21·35	20·13
Braziers	57	1·296	5	22·73	11·40
Whitesmiths	146	4·276	11	29·28	13·27
Jewellers	300	6·081	24	20·27	12·50
Gilders	107	2·174	5	20·31	21·40
Varnishers	79	1·515	4	19·18	19·75
Colour makers	39	809	3	20·74	13·00
Turners	458	9·085	25	19·96	18·32
Smelters	327	6·794	22	20·77	14·86
Polishers	126	2·461	10	19·53	12·60
Workers in metal	123	2·065	7	16·70	17·57
FEMALES.					
Winders	172	3·678	15	21·38	11·46
Woolcombers	93	2·006	5	21·56	18·60
Spinners	116	2·703	15	23·30	7·73
Washerwomen	1,668	35·802	112	21·46	14·82
Confectioners	4,268	105·924	388	24·81	11·00
Polishers	134	2·863	7	21·37	19·14
Burnishers	159	3·428	12	21·55	16·41
Gilders	46	1·037	2	22·54	23·00
Varnishers	59	1·321	6	22·39	9·83

In a remarkable work on the benevolent institutions of France, M. de Watteville, in 1851, stated the ratio of deaths in the hospitals of Paris to be 1 in 11, both for males and females. This was for the year 1847, which from the dearness of provisions and the consequent misery in the poorer classes must be considered exceptional. He estimates also the the average duration of sick cases in the hospital as 24 days for males and 25 for females. In the preceding table no class of females exceeds the latter number; but

in the males, the former is exceeded by the stonecutters, the zinc workers and plumbers, the scavengers, the weavers, and whitesmiths. The average mortality amongst female cases seems by the table to be greater amongst females than males, the former being one death for 11·94 patients, and the latter one in 13·92; but this may arise from females not having recourse to the aid of the hospital till the last extremity.

The researches made by the Society we began by alluding to in France, and the publication of M. Hubbard's excellent treatise, will no doubt awaken the same interest which those of the Government and of individuals have excited in England. We cannot but heartily wish them success. Whilst the results of these inquiries serve the cause of science, they tend to alleviate the miseries to which society is subject, and raise at once the character of the people in the scale of civilization and humanity.

PARIS.—*Guide de l'Assureur et de l'Assuré en matière d'Assurances Maritimes.*—This very useful work, by M. le Capitaine Gabriel Lafond, the able manager of the principal Maritime Assurance Company of France (l'Union des Ports), and one of the Foreign Correspondents of the Institute of Actuaries, is about to be republished, with much additional information and notes, and the usages of marine assurance and forms of policies in all foreign countries. It is of quite a different character from any work on marine assurance in this country, which, with the exception of Lee's *Manual for Shipmasters*, principally relate to the laws and the changes in the laws of shipping. We hope very shortly to be able to make an analysis of this book for such of our readers as take an interest in these matters.

We have also much gratification in recording Capitaine Lafond's offers of service to the members of the Institute, as expressed in the following note:—

"*Note à lire dans la première Séance de l'Institut des Actuaries de Londres.*—G. Lafond, Directeur-Ménager de l'Union des Ports, Correspondant de l'Institut à Paris, offre de bien bon cœur à Messieurs les Membres de l'Institut de recevoir dans ses bureaux les lettres, journaux, et paquets qui pourront leur être adressés lorsquils viendront visiter l'Exposition Universelle. Une chambre séparée, avec almanach des adresses, encre, papier et plumes, sera à la disposition de tous les Membres, pour faire leur correspondance. Ils pourront laisser dans des cartons leurs papiers. La bibliothèque de M. G. Lafond, de 3,000 volumes au moins, leur sera ouverte. Un bureau de poste étant dans la maison, qui est située Place de la Bourse, No. 4, et par conséquent au centre de Paris, leur facilitera toutes leurs moindres besoins. Il va sans dire que M. G. Lafond offre son local et ses services gratuitement, pour être agréable à ses collégues et amis de l'Institut."


THE

An Inquiry into the Marine Insurance of Hamburg. *By* WILHELM LAZARUS, *Esq., Foreign Correspondent of the Institute of Actuaries.*

WHEN I began to direct my particular attention to the balance-sheets of the Hamburg Marine Insurance Societies, I hoped that a minute examination would lead to conclusions of general interest, and enable us to judge of the risks and state of navigation in general. I hoped that the collection of data and the comparison of the different balance-sheets would not offer great difficulties, and that my examinations might embrace a long period. My hopes have not been fulfilled. I perceived very soon that the work would be very troublesome, and that it would be difficult to form a complete collection of balance-sheets. There were none at the Public Commercial Library, and it was only by the aid of private friends that I at length succeeded in forming a complete collection of the balance-sheets since 1847. Thus I was obliged to limit my comparisons to the years 1847-53; and I think this is the principal reason why the law which governs the events cannot be deduced, and why no general conclusions can be drawn.

Nevertheless, I have finished this labour. The results are different from what I expected when I commenced. The facts which ought to form the foundation of my conclusions have become the principal, and it is only since 1847 that I can give a true representation of the Hamburg marine insurance business.

The large sums yearly insured in Hamburg, the number of Societies engaged in this branch, the capital employed, will justify, I hope, at least, this work—the more so, as the data given by others are very insufficient, either from being inexact, or from omitting some of the particulars. I shall mention these data, in order to compare them with the results which I have found.

The number of Marine Insurance Societies in Hamburg was 18 in 1836, rose to 24 in 1846, fell to 22 in 1849, and again rose to 23 in 1853. Amongst these there are three Societies which take fire insurances also: the Patriotische Assecuranz Compagnie, the Neue Funfte Assecuranz Compagnie, and the See und Feuer Assecuranz Compagnie. The latter renounced fire insurance at the end of 1852, and now undertakes only marine insurances, under the name of the Neptunus Insurance Compagnie. The fire insurances of these three Societies are included in the balances, and I had first to extract this branch.

Table A.—*Fire Insurance*, 1843-1853.

Year.	(A) Sum Insured.	(B) Reserved Sum.	(C) Income of Premium.	(D) Income of Interest.	(E) Total Amount.	(F) Loss.	(G) Brokerage, &c.	(H) Reserves for current risk and unsettled claims.	(J) Total.	(K) Profit.	(L) Dividend.
					Patriotische Assecuranz Compagnie.						
	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.
1843	30,863,165	100,000	117,263 4	4,000	221,263 4	13,014 10	5,248 10	125,000	143,263 4	78,000	
1844	28,341,005	125,000	104,248 14	5,000	234,248 14	4,097 4	4,151 10	160,000	168,248 14	66,000	20,000
1845	27,541,880	160,000	88,038	6,000	254,038	60,474 2	4,563 14	160,000	225,038	29,000	20,000
1846	22,610,570	160,000	70,575 3	6,000	236,575 3	14,270	3,305 3	200,000	217,575 3	19,000	20,000
1847	23,844,815	200,000	73,688 13	10,000	283,688 13	..	3,688 13	250,000	253,688 13	30,000	30,000
1848	21,360,545	250,000	74,086 4	10,000	334,086 4	32,930	4,156 4	270,000	307,086 4	27,000	20,000
1849	26,043,125	270,000	71,550 4	10,000	351,550 4	..	4,550 4	280,000	284,550 4	67,000	40,000
1850	26,404,985	280,000	76,550 14	10,000	366,550 14	43,880 8	4,670 6	300,000	348,550 14	18,000	18,000
1851	22,016,085	300,000	67,333 8	12,000	379,333 8	8,291 14	4,041 10	300,000	312,333 8	67,000	40,000
1852	21,733,450	300,000	77,018 7	12,000	389,018 7	8,455 11	4,562 12	300,000	313,018 7	76,000	40,000
1853	21,520,765	300,000	62,997 7	12,000	374,997 7	6,820 3	4,177 4	300,000	310,997 7	64,000	60,000
					Neue Funfte Assecuranz Compagnie.						
1843	34,117,840	..	177,944 3	2,000	179,944 3	53,984 1	9,557 8	28,402 10	91,944 3	88,000	
1844	28,541,825	28,402 10	132,539 11	4,500	165,442 5	15,357 14	7,179 14	28,904 9	51,442 5	114,000	
1845	27,599,007	28,904 9	123,543	4,000	156,447 9	87,614 2	6,535 8	38,297 15	132,447 9	24,000	
1846	21,769,330	38,297 15	110,357 2	4,000	152,655 1	34,395 4½	6,636 7	39,623 5½	80,655 1	72,000	
1847	20,644,405	39,623 5½	92,353 15	4,000	135,977 4½	11,824 5	5,689 12½	28,463 3	45,977 4½	90,000	
1848	19,266,639	28,463 3	80,811 11	4,000	113,274 14	45,272 2	4,212 8	21,790 9	71,274 14	42,000	
1849	17,738,991	21,790 9	76,675 7	4,000	102,466	31,033 1	4,751 9	82,681 6	118,466	(loss) 16,000	
1850	19,320,733	82,681 6	78,664 14	4,000	165,346 4	19,198 13	4,172 13	85,974 10	109,346 4	56,000	
1851	10,239,865	86,993 7	70,770	4,000	161,763 7	5,209 10½	4,944 15½	132,608 13	142,763 7	19,000	
1852	14,300,895	132,608 13	63,206	6,000	201,814 13	18,182 7	4,495 9	139,136 13	161,814 13	40,000	
1853	13,602,113	139,136 13	64,748	6,000	209,884 13	13,450 5	4,046 15	124,387 9	141,884 13	68,000	
					See und Feuer Assecuranz Compagnie.						
1843	12,623,727	..	53,650 10	1,000	54,650 10	18,710 11	2,783 1	6,156 14	27,650 10	27,000	
1844	11,792,644	6,156 14	50,764 12½	2,000	58,921 10½	1,230 9	2,691 1½	10,500	14,421 10½	44,500	
1845	10,834,199	10,500	37,132 14	1,600	49,232 14	30,105 7	2,162 15	11,364 8	43,632 14	5,600	
1846	6,634,222	11,364 8	23,410 10	1,200	35,975 2	2,080 14	1,529 12	8,364 8	11,975 2	24,000	
1847	7,373,727	8,364 8	22,978 6	1,200	32,542 14	5,849 14	1,333 15	6,959 1	14,142 14	18,400	
1848	6,077,395	6,959 1	19,133 12	1,000	27,092 13	6,675 7½	1,563 12	7,853 9½	16,092 13	11,000	
1849	4,852,155	7,853 9½	17,029 15	800	25,683 8½	427 7	974 1	82 ½	1,483 8½	24,200	
1850	5,380,800	82 ½	16,699 6	800	17,581 6½	8,050 ½	1,331 6	..	9,381 6½	8,200	
1851	3,837,780	..	10,433 2	..	10,433 2	4,573 8	691 14	67 12	5,333 2	5,100	
1852	1,324,015	67 12	3,674 3	..	3,741 15	2,000	241 15	..	2,241 15	1,500	

The Table A contains the results of the fire insurance branch from 1843 to 1853. The columns A, B, C, F, H, are taken from the balance-sheets; the columns D and G are calculated. I thought it would be just to include therein a part of the interest received, because these Companies gain a portion of their interest by fire insurance premiums and reserves. Brokerage (Col. G) is taxed 5 to 7 per cent. of the receipt of premiums. Columns E, I, K, follow from the other; and column L, dividends, has been given, as a large proportion of them depends upon the results of fire insurance.

Having thus separated the fire insurance, we shall be at liberty to treat the three Companies before mentioned as if they were Marine Insurance Societies exclusively. On another occasion, I hope, I may treat separately the fire insurance branch; this examination refers only to marine insurance.

Year.	Number of Societies.	Number of Shares.	Subscribed Capital.	Paid-up Capital.
1836	18	3,335	Banco marcs.	Banco marcs.
1837	18	3,335		
1838	19	3,536		
1839	19	3,545		
1840	20	3,705		
1841	20	3,705		
1842	21	3,865		
1843	22	4,015	13,245,000	2,849,000
1844	23	4,215	13,845,000	2,969,000
1845	23	4,215	13,845,000	2,969,000
1846	24	4,475	14,625,000	3,125,000
1847	23	4,275	14,025,000	3,005,000
1848	22	4,095	13,485,000	2,897,000
1849	22	4,095	13,485,000	2,897,000
1850	22	4,095	13,485,000	2,897,000
1851	22	4,095	13,485,000	2,897,000
1852	22	4,095	13,485,000	2,897,000
1853	23	4,275	14,025,000	3,005,000

The dates from 1836 to 1842 are according to the publications of the Handelsstatistical Bureau, and from 1842 to 1846 according to the annual returns of Mr. Goverts. The number of the Societies and the capital engaged in this business have increased, and probably have not yet reached the maximum.

Before I proceed to analyze the data furnished by the balance-sheets, it will be necessary to explain the mode of forming these balances. Each of the Hamburg Marine Insurance Societies publishes every year, in April or May, a balance-sheet of the business done, and it must be acknowledged that it is given in a precise and clear form. The balance-sheet contains two parts; we may

call them gross balance and net balance. The gross balance refers to the last year (1st Jan. to 31st Dec.), and contains the amount insured, amount of premiums, amount of interest obtained, and a reserved sum from the net balance for unsettled claims of the past year, the payment of interest to the shareholders, for *ristorno* for reassured sums, for average and losses, expenses of management, brokerage, and a sum to be reserved for averages and losses unpaid, and for the current risk of which the amount and the premium paid is stated. The result of this balance (gross) is generally considered as the real one, and forms the foundation of the statements and conclusions concerning Hamburg marine insurance. I cannot agree with this; for in the next balance-sheet is published the actual portion of the reserved sums which was required to cover the actual losses and averages, and the small amount of reserve which remains to be carried over to the next balance, when nearly all the business of that year is settled. This balance, which I call the net one, and which shows the true result, precedes another gross balance of the following year in each statement. I have added to each gross balance the net balance relating to the same year, thus ending with the net result; whilst in most statements the net and gross balance of different years have been added together when this subject has been discussed.

From the statements of Mr. Goverts, including those of the private underwriters and the agencies of some foreign Companies established in Hamburg, I give the following table:—

Year.	Sum Insured.	Amount of Premiums.	Average Premium per cent.	Year.	Sum Insured.	Amount of Premiums.	Average Premium per cent.
	Banco marcs.	Banco marcs.			Banco marcs.	Banco marcs.	
1814	42 millions	1,489,000	3·5625	1834	189 millions	3,079,000	1·625
1815	87 „	3,058,000	3·5	1835	195 „	2,928,000	1·5
1816	124 „	3,576,000	2·875	1836	220 „	3,229,000	1·47
1817	151 „	3,862,000	2·5625	1837	215 „	3,419,000	1·594
1818	176 „	3,628,000	2·0625	1838	224 „	3,362,000	1·5
1819	129 „	2,231,000	1·875	1839	251 „	3,769,000	1·5
1820	151 „	2,544,000	1·6875	1840	267 „	4,000,000	1·5
1821	129 „	2,822,000	2·1875	1841	272 „	3,915,000	1·4375
1822	98 „	2,273,000	2·3125	1842	239 „	3,513,000	1·47
1823	100 „	3,059,000	3·0625	1843	265 „	3,729,000	1·406
1824	101 „	2,200,000	2·1875	1844	294 „	4,130,000	1·406
1825	124 „	2,562,000	2·0625	1845	331 „	4,969,000	1·5
1826	114 „	2,348,000	2·0625	1846	304 „	4,746,000	1·5625
1827	129 „	2,425,000	1·875	1847	361 „	5,530,000	1·531
1828	142 „	2,494,000	1·75	1848	250 „	5,152,000	2·0625
1829	160 „	2,533,000	1·5833	1849	281 „	4,569,000	1·625
1830	190 „	2,969,000	1·5625	1850	313 „	4,886,000	1·5625
1831	181 „	3,056,000	1·6875	1851	317 „	4,851,000	1·531
1832	203 „	2,917,000	1·4375	1852	331 „	5,072,000	1·531
1833	199 „	3,105,000	1·5625	1853	422 „	6,467,000	1·531

The sum insured shows a constant tendency to increase, in accordance with the general increase of commerce. From 42 millions in 1814, we come to 422 millions in 1853. At the same time it proves a mirror to the history of commerce and politics, and reflects all considerable events. Nevertheless there are some oscillations which cannot be explained in this way, and which seem to indicate a connection of the average premium with the sum insured, of which I shall speak more largely presently.

The average premium is not quite exact, because the amount of premium is reduced in the balance-sheet of some Companies by the *ristorno*, which is deducted from the amount of premium. The net sum is then stated as receipts, and the *ristorno* omitted in the expenditure. In such cases Mr. Goverts, in his publication, gives the net premium, which I think incorrect; for at all events a *ristorned* insurance represents a business—1, because the *ristorno* is not equal to the premium paid; 2, because brokerage is paid. The *sums insured* not being diminished by the *ristorned* sums, the average premium will appear lower than it really is. The table, nevertheless, is sufficient to show that a tendency to lessen the average premium prevails; and we should be glad if we were allowed to consider this as the consequence of improvements in science, and of a diminution of the dangers to navigation. Although I do not doubt that this progress of science and the increase of steam navigation have co-operated in a high degree to the abatement of the average premium, I think we must not consider the premium taken the true expression of the value of the risk.

Prices are regulated by the proportion of supply to demand, and so also the premiums of marine assurance. The influence of the competition of other ports can be proved by comparing the sums insured and the average premium, and it explains the fluctuations of the former. An increase of the average premium coincides with a decrease of the sum insured; and even in years in which no reason can be suggested which will explain both, may we not infer that a connection exists between the sum insured and the rising of the average premium? The assured always looks for the lowest premium; and if it is higher in Hamburg than in other places, he will go elsewhere, and the sum insured in Hamburg diminishes, whilst low premiums there occasion a rise in the amount insured. The following table will prove this interesting fact, and seems even to indicate a connection between the degrees of the fluctuations of the sum insured and the average premium—which supports my assumption. It is to be observed, that the

tendency to increase in the sums assured corresponds with the diminution in the average premium.

Year.	Average Premium per cent.	Sum Insured.
		Banco marcs.
1814 to 1820	Decrease from 3·5625 to 1·6875	Increase from 42 to 151 millions
1820—1823	Increase „ 1·6875 „ 3·0625	Decrease „ 151 „ 100 „
1823—1830	Decrease „ 3·0625 „ 1·5625	Increase „ 100 „ 190 „
1830—1831	Increase „ 1·5625 „ 1·6875	Decrease „ 190 „ 181 „
1831—1832	Decrease „ 1·6875 „ 1·4375	Increase „ 181 „ 203 „
1832—1834	Increase „ 1·4375 „ 1·625	Decrease „ 203 „ 189 „
1834—1836	Decrease „ 1·625 „ 1·47	Increase „ 189 „ 220 „
1836—1837	Increase „ 1·47 „ 1·594	Decrease „ 220 „ 215 „
1837—1841	Decrease „ 1·594 „ 1·4753	Increase „ 215 „ 272 „
1841—1842	Increase „ 1·4375 „ 1·47	Decrease „ 272 „ 239 „
1842—1844	Decrease „ 1·47 „ 1·406	Increase „ 239 „ 294 „
1844—1846	Increase „ 1·406 „ 1·5625	Decrease „ 294 „ 304 „
1846—1847	Decrease „ 1·5625 „ 1·531	Increase „ 304 „ 361 „
1847—1848	Increase „ 1·531 „ 2·0625	Decrease „ 361 „ 249 „
1848—1853	Decrease „ 2·0625 „ 1·531	Increase „ 249 „ 422 „

The year 1819 forms an exception which seems sufficiently explained by the commercial crisis.

			Banco marcs.
1818	Premium, 2·06	Sum Insured,	176,000,000
1819	„ 1·87	„	129,000,000
1820	„ 1·69	„	151,000,000

The exception of the year 1823 is very insignificant. The decrease of the sum insured in 1826, whilst the average premium remains constant, is in accordance with my assumption. The exception of 1845 shall be explained afterwards. A graphic table of this subject is annexed, and will clearly exhibit this connection.

I can state the sums insured with the Hamburg Companies since 1836: they are

Year.	Sum Insured.	Year.	Sum Insured.
	Banco marcs.		Banco marcs.
1836	179,621,800	1845	304,143,400
1837	195,667,000	1846	278,040,600
1838	219,163,600	1847	333,867,603
1839	246,281,400	1848	235,052,491
1840	260,696,300	1849	258,247,353
1841	266,375,200	1850	278,157,726
1842	232,181,400	1851	278,917,603
1843	248,977,800	1852	288,312,516
1844	270,894,700	1853	357,430,297

The consequences of the great fire in 1842—of the crisis, 1846—and of the revolution, 1848—are most striking.

Year.	Amount of Premium with the Hamburg Companies.	Average Premium per cent.
	Banco marcs.	
1847	5,123,155 5½	1·534
1848	4,901,677 9½	2·085
1849	4,270,540 5	1·654
1850	4,352,269 14	1·565
1851	4,347,343 2½	1·559
1852	4,458,363 3	1·546
1853	5,526,186 8½	1·546

In the publications of Mr. Goverts and of the Handelsstatistischen Bureau these numbers are given incorrectly, some *ristorno* being deducted.

Year.	Handelsstatistischen Bureau.		Mr. E. F. Goverts.	
	Amount of Premium.	Average per cent.	Amount of Premium.	Average per cent.
	Banco marcs.		Banco marcs.	
1836	2,486,170	1·38		
1837	3,048,839	1·56		
1838	3,222,625	1·47		
1839	3,570,953	1·45		
1840	3,776,653	1·45		
1841	3,746,648	1·41		
1842	3,270,711	1·40	3,472,840	1·48
1843	3,444,451	1·38	3,564,520	1·43
1844	3,726,411	1·38	3,848,220	1·42
1845	4,461,454	1·47	4,600,990	1·51
1846	4,174,543	1·50	4,413,790	1·5625
1847	4,939,245	1·48	5,122,700	1·53125
1848	4,778,420	2·02	4,901,680	2·08
1849	4,025,956	1·56	4,270,450	1·625
1850	4,175,606	1·50	4,352,170	1·5625
1851	4,171,531	1·50	4,347,250	1·53125
1852	4,286,628	1·49	4,458,840	1·53125
1853	5,528,724	1·55	5,526,040	1·53125

A comparison of the sums insured and the average premium will prove the before-mentioned connection.

Year.	Sum Insured.	Average Premium per cent.	Year.	Sum Insured.	Average Premium per cent.
	Banco marcs.			Banco marcs.	
1842	233,181,400	1·49	1848	235,052,500	2·08
1843	248,977,800	1·43	1849	258,247,400	1·65
1844	270,894,700	1·43	1850	278,157,700	1·56
1845	304,143,400	1·51	1851	278,917,600	1·56
1846	278,040,600	1·58	1852	288,312,500	1·54
1847	333,867,600	1·53	1853	357,430,300	1·55

The deviation of the year 1845, showing an increase of the sum insured and a simultaneous increase in the average premium, can be explained by the disastrous activity of the Assecuranz Compagnie of 1844, which failed in 1846. This Company insured in

	Banco marcs.		Banco marcs.		Per cent.
1844	22,838,900	Premium,	315,120	Average,	1·38
1845	25,825,800	„	469,200	„	1·82

The catastrophe which happened to this Company will justify the supposition that a considerable sum was insured in 1845 with this Company at high rates of premium—the average premium of this Company being 1·82 per cent., the average in general 1·51 per cent.—and for risks which would not have been underwritten at all in Hamburg had it not been for this Company; and thus the year 1845 would have had a smaller amount insured and a lower average premium. The average premium of 1845, the business of the Assecuranz Compagnie of 1844 excluded, is 1·48 per cent., and would be still lower if we could keep in view those policies underwritten by this Company at low rates of premium, being ordinary good risks which any Company would have accepted.

Reserves for unsettled losses carried forward from the last net balance :—

Year.	Banco marcs.
1847	195,637 9
1848	264,367 13½
1849	318,069 12
1850	232,613 7
1851	260,446 9½
1852	153,778 1
1853	163,957 13

With the view of considering each year separately, we cannot mention the sums relating to former years as receipts. The sums are small, and depend on so many factorials that interesting conclusions can hardly be obtained therefrom.

On the credit side there remains only the interest obtained from capital, premiums, and reserves. These sums depend on the current rate of interest, and on the amount of capital, premium, and reserved fund. The interest paid by the Companies to the shareholders is 4 per cent. of the paid-up capital.

Year.	Interest received.	Interest paid.	Surplus of receipts.	Surplus of payment.
	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.
1847	172,624 12	105,800	66,824 12	
1848	119,897 14	115,880	4,017 14	
1849	72,677 7	115,880	..	43,202 9
1850	120,255 5½	113,480	6,775 5½	
1851	126,561 6½	115,280	11,281 6½	
1852	147,817 3½	115,880	31,937 3½	
1853	146,812 14	113,240	33,572 14	
		Total....	154,409 7½	43,202 9

In 1847 the Patriotische Assecuranz Compagnie only paid 6,400 banco marcs instead of 20,800 banco marcs; in 1850 the See Assecuranz Compagnie of 1850 only paid 1,200 banco marcs —in 1851, 3,000 banco marcs—instead of 3,600 banco marcs. In 1853 the See Assecuranz Compagnie of 1846 paid no interest, instead of 3,600 banco marcs; the Hansa Assecuranz Compagnie, 1,560 banco marcs instead of 3,120 banco marcs; the Versicherungs Verein, 1,800 banco marcs instead of 3,600 banco marcs.

Year.	Receipt of Interest.			Payment of Interest.		
	For each 100 Thalers sum insured.	For each 100 Thalers total income.	For each 100 Thalers paid-up capital.	For each 100 Thalers sum insured.	For each 100 Thalers total income.	For each 100 Thalers paid-up capital.
1847	0·052	3·260	5·74	0·032	1·998	3·52
1848	0·051	2·388	4·14	0·049	2·308	4·
1849	0·028	1·673	2·51	0·045	2·668	4·
1850	0·043	2·689	4·15	0·041	2·537	3·92
1851	0·045	2·829	4·37	0·041	2·577	3·98
1852	0·051	3·209	5·10	0·040	2·516	4·
1853	0·041	2·588	4·89	0·032	1·996	3·77

Year.	Profit of Interest.		Loss of Interest.	
	For each 100 Thalers sum insured.	For each 100 Thalers total income.	For each 100 Thalers sum insured.	For each 100 Thalers total income.
1847	0·020	1·262		
1848	0·002	0·08		
1849	..	..	0·017	0·995
1850	0·002	0·152		
1851	0·004	0·252		
1852	0·011	0·693		
1853	0·009	0·592		

Year.	Total Income.	
	Amount.	For each 100 Thalers sum insured.
	Banco marcs.	
1847	5,295,780 1½	1·586
1848	5,021,575 7½	2·136
1849	4,343,217 12	1·682
1850	4,472,525 3½	1·608
1851	4,473,904 9	1·605
1852	4,606,180 6½	1·598
1853	5,672,999 1½	1·587

The income has risen constantly since 1849, notwithstanding the diminution of the average premium.

The payment of brokerage stated in the balance-sheets includes the brokerage for discounted bills, and for valuation of damaged goods, as well as for assurances effected. I have mentioned before that I have carried over a part of it to the fire insurance branch. The marine insurance brokerage paid by the underwriter is $\frac{1}{16}$ per cent. of the sum insured if the premium is less than 2 per cent., and $\frac{1}{4}$ per cent. if the premium is 2 per cent. or more.

Year.	Brokerage.	For each 100 Thalers sum insured.	For each 100 Thalers total income.
	Banco marcs.		
1847	260,814 6½	0·078	4·925
1848	209,799 7½	0·089	4·178
1849	202,629 7½	0·078	4·665
1850	219,756 7	0·079	4·914
1851	215,692 5½	0·077	4·821
1852	224,256 4½	0·078	4·869
1853	276,018 8½	0·077	4·865

The fluctuations in this table are insignificant. The maximum per centage on 100 thalers sum insured coincides with the minimum for 100 thalers total income, in 1848, which year shows the highest average premium. The maximum of payment for brokerage falls in 1853, the year with the greatest amount insured—the minimum, in 1849, the year with the smallest income.

The expenses of management seem dependent on the number of the Societies, and show only small fluctuations.

Year.	Expenses of Management.	For each 100 Thalers sum insured.	For each 100 Thalers total income.	Average for each Company.
	Banco marcs.			Banco marcs.
1847	238,086 2	0·071	4·496	10,352
1848	228,996 11	0·097	4·560	10,409
1849	234,431 11	0·091	5·398	10,656
1850	229,681 10	0·083	5·135	10,440
1851	229,336 15	0·082	5·126	10,424
1852	231,636 14	0·080	5·029	10,529
1853	245,310 12½	0·069	4·324	10,692

The conformity of the sums for *ristorno* and reassurance is very surprising; the proportions to the sums insured and to the income are constantly becoming smaller, but I cannot decide whether this is accidental or based in the nature of commerce.

Year.	Ristorno and Reassurance.	For each 100 Thalers sum insured.	For each 100 Thalers total income.
	Banco marcs.		
1847	182,140 8½	0·055	3·439
1848	230,267 12	0·098	4·586
1849	177,134 7½	0·069	4·078
1850	175,120 1	0·063	3·915
1851	174,149 12½	0·062	3·893
1852	170,210 8½	0·059	3·695
1853	192,985 6½	0·054	3·402

In the publications of the Handelsstatistischen Bureau of 1853, the premium is stated without the *ristorno* and reassurance; and, comparing this with the publications of Mr. Goverts, I find—

Year.	Ristorno and Reassurance.	For each 100 Thalers sum insured.	For each 100 Thalers total income.
	Banco marcs.		
1842	202,129	0·088	5·820
1843	120,769	0·050	3·388
1844	121,809	0·045	3·165
1845	139,536	0·046	3·033
1846	239,247	0·068	5·421

The losses from nonpayment of premiums are very insignificant, although the premiums are paid only twice a year, in January and July. The sums being so small, I have omitted this column, and included it in "damages" stated in the gross balance: they are,

Year.	Banco marcs.
1847	4,758 3½
1848	2,607 9½
1849	810 13
1850	796 2½
1851	1,544 2½
1852	781 4½
1853	663 5½

Before we proceed to the consideration of damages, we will add the different articles together.

	For each 100 Thalers sum insured.	For each 100 Thalers total income.
Brokerage	0·077 to 0·089	4½ to 5¼
Expenses of management	0·069 „ 0·091	4½ „ 5¾
Ristorno and reassurance	0·054 „ 0·098	3½ „ 4½
	0·200 to 0·278	12½ to 15¼

The total amount of damages in every year is composed of—

damages stated in the gross balance; loss of premiums remaining unpaid; damages stated in the net balance relating to the same year; amount to be reserved for unsettled claims, deducting therefrom the sum reserved for unsettled claims of former years.

Year.	Total amount of Damages.	For each 100 Thalers sum insured.	For each 100 Thalers total income.
	Banco marcs.		
1847	3,590,159 0½	1·075	67·798
1848	4,216,731 9	1·794	83·971
1849	3,109,982 2	1·204	71·606
1850	3,892,187 1½	1·399	87·025
1851	3,436,045 8	1·232	76·801
1852	4,460,696 11½	1·547	96·841
1853			

Great fluctuations are found here, as might have been expected, the more so as this period proved very unfavourable to marine insurance. But we can record as a fact for congratulation, that in no year was the income exceeded by the losses, and that even in 1852 the receipt of premium is only about 2,500 banco marcs less than the claims. I think no better testimony could be given to the solidity of our marine insurance business. It is very surprising that good years and bad ones alternate with great regularity; and adding two years—

	1847-48	1849-50	1851-52	
Damages ...	77·874	81·205	89·677	per cent. of income,

the average of the whole period is 1·326 per cent. of the insured sum, 82·709 per cent. of the receipt of premium. The difference of the maximum, 1·794 per cent., and the minimum, 1·075 per cent., is 0·719 per cent.; whilst the difference of the maximum of the average premium in this period, 2·085 per cent., and the minimum, 1·534 per cent., is only 0·551 per cent. These numbers do not allow of any law being deduced from them, and I think the laws must be too complicated to be observed in this short period. Nevertheless the subject is interesting enough to justify a closer examination. The statements of others with reference to the period before 1847 do not offer direct means for comparison, they being framed, as mentioned before, on other principles.

The numbers from which we have just calculated the total amount of claims are, separately—

Year.	I. Amount of claims stated in the gross balance.	II. Amount of claims stated in the net balance.	III. Reserved sum to be brought over for unsettled claims.	IV. Reserved sum for unsettled claims of former years.
	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.
1847	1,921,955 4½	1,599,473 7½	264,367 13½	195,637 9
1848	1,987,112 3	2,175,917 7½	318,069 12	264,367 13½
1849	1,809,152 15½	1,386,285 7½	232,613 7	318,069 12
1850	2,165,432 6½	1,698,921 8½	260,446 9½	232,613 7
1851	1,781,477 10½	1,761,236 6	153,778 1	260,446 9½
1852	2,501,090 0½	1,949,426 15	163,957 13	153,778 1
1853	2,371,884	..	..	163,957 13

The difference of the proportion between columns I. and II. shows the years in which the heaviest losses must have occurred—viz., in spring in 1848, 1851, and in autumn in 1847, 1849, 1850, 1852; besides which, I believe that the settlement of claims is quicker in modern times.

Including the column III. in II., and deducting one half of column IV. from I., one half from II., we shall find—

Year.	Payments stated in the gross balance, first year.	Payments stated in the net balance, second year.
	Banco marcs.	Banco marcs.
1847	1,824,136 8	1,766,022 8½
1848	1,854,928 4	2,361,803 5
1849	1,650,118 1½	1,459,864 0½
1850	2,049,125 11	1,843,061 6½
1851	1,651,254 6	1,784,791 2
1852	2,424,201	2,036,459 11½
Total.....	11,453,763 14½	11,202,038 2

Year.	Per cent. of claims. Claims paid.		Per cent. of insured sum. Claims paid.		Per cent. of total income. Claims paid.	
	First Year.	Second Year.	First Year.	Second Year.	First Year.	Second Year.
1847	50·81	49·19	0·546	0·529	34·44	33·35
1848	43·99	56·01	0·789	1·005	36·94	47·03
1849	53·05	46·95	0·639	0·565	37·99	33·61
1850	52·65	47·35	0·736	0·663	45·82	41·21
1851	48·05	51·95	0·592	0·640	36·91	39·89
1852	54·35	45·65	0·841	0·706	52·63	44·21
1853	..	..	0·641	..	40·36	

Two sums are reserved in the gross balance—one for unsettled claims, and the other for the current risk. Deducting as before, from the reserved sum for unsettled claims, one half of that reserve relating to former years, we shall find—

Year.	Reserved for claims.	Reserved for current risk.	Total Amount Reserved.	Surplus over payments of claims.
	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.
1847	1,328,521 14	1,133,340 10½	2,461,862 8½	695,840
1848	2,034,362 2½	949,281 2½	2,983,643 5	621,840
1849	1,249,078 6	985,085 10½	2,234,164 0½	774,300
1850	1,666,802	971,309 6½	2,588,111 6½	795,050
1851	1,494,405 7½	1,003,285 10½	2,497,691 2	712,900
1852	1,773,205 4	936,540 7½	2,703,745 11½	673,250
1853	1,867,238 15	1,196,299 6½	3,063,538 5½	

The same table, calculated for 100 thalers insured sum :—

Year.	Reserved for claims.	Reserved for current risk.	Total amount.	Surplus.
1847	0·398	0·339	0·637	0·208
1848	0·865	0·404	1·269	0·265
1849	0·484	0·381	0·865	0·300
1850	0·599	0·349	0·948	0·286
1851	0·536	0·360	0·896	0·256
1852	0·615	0·325	0·940	0·234
1853	0·522	0·335	0·857	

The same table, calculated for 100 thalers total income :—

Year.	Reserved for claims.	Reserved for current risk.	Total amount.	Surplus.
1847	25·09	21·40	46·49	13·14
1848	40·51	18·90	59·41	12·38
1849	28·76	22·68	51·54	17·83
1850	37·27	21·72	58·99	17·77
1851	33·40	22·43	55·83	15·93
1852	38·50	20·33	58·83	14·62
1853	32·91	21·09	54·00	

These tables prove more conformity and regularity than we might have expected.

Table of Current Risk at the period when the Gross Balance is formed (April or May of the succeeding year).

Year.	Amount of current risk.	Premium paid for the current risk.	Average Premium for the current risk.	Average Premium of the year.
	Banco marcs.	Banco marcs.	Banco marcs.	Banco marcs.
1847	9,992,685	359,736 7	3·59	1·534
1848	8,867,721	314,801 7	3·55	2·085
1849	9,336,018	297,115 3½	3·18	1·654
1850	9,758,695	328,851 10½	3·38	1·565
1851	9,120,978	304,870 8	3·34	1·559
1852	12,599,574	392,850 9	3·12	1·546
1853	12,826,672	417,775 2½	3·26	1·546

The average premium of the current risk is nearly constant, and seems independent of the average premium of the year. The current risk per cent. of sum insured was, in

1847	2·99	Forming $\frac{1}{34}$ to $\frac{1}{36}$ part.
1848	3·77	
1849	3·61	
1850	3·51	
1851	3·27	
1852	4·37	
1853	3·59	

The reserved sum for the current risk, compared with the amount of the latter, gives

Year.	Per cent.	
1847	11·34	The small reserve of 1852 will be explained by the heavy losses paid in this year (nearly half a million banco marcs more than in the preceding years), and by the large sum to be reserved for unsettled claims.
1848	10·71	
1849	10·55	
1850	9·98	
1851	11·	
1852	7·38	
1853	9·32	

This reserve per cent. of total income—

1847	21·40	About $\frac{1}{5}$ part.
1848	18·90	
1849	22·68	
1850	21·72	
1851	22·43	
1852	20·33	
1853	21·09	

The surplus of these two reserves is called "verbesserung" (bettering). In the net balance it fluctuates from 621,840 banco marcs in 1848, to 795,050 banco marcs in 1850; per cent. of sum insured, from 0·208 in 1847, to 0·300 in 1849; per cent. of total income, from 12·38 in 1848, to 17·83 in 1849. Each of these reserves separately would not have been sufficient.

Year.	Reserved for claims.	Paid for claims, including further reserve.	Excess of payments.
	Banco marcs.	Banco marcs.	Banco marcs.
1847	1,328,521 14	1,766,022 8½	437,500 10½
1848	2,034,362 2½	2,361,803 5	327,441 2½
1849	1,249,078 6	1,459,864 0½	210,785 10½
1850	1,666,802	1,843,061 6½	225,259 6½
1851	1,494,405 7½	1,784,791 2	290,386 11½
1852	1,773,205 4	2,036,495 11½	263,290 7½

It is interesting to observe that the excess of payments is constantly decreasing, and might have been covered by the mere premium for the current risks since 1849.

Year.	Payment of claims and further reserves stated in the gross balance.	Per cent. of sums insured.	Per cent. of total income.
	Banco marcs.		
1847	4,285,999 0½	1·183	80·93
1848	4,838,571 9	2·058	87·35
1849	3,884,282 2	1·504	89·53
1850	4,637,238 1½	1·684	104·81
1851	4,148,945 8	1·488	91·74
1852	5,127,946 11½	1·781	111·46
1853	5,516,638 13½	1·498	94·36

In the publications of the Handelsstatistischen Bureau there is a column, "payments of claims" since 1836.

Year.	Payment of claims.	Per cent. of sum insured.	Numbers according to my calculation.
	Banco marcs.		Banco marcs.
1836	2,104,902	1·172	
1837	2,508,557	1·282	
1838	2,561,757	1·169	
1839	2,230,608	1·140	
1840	3,062,503	1·151	
1841	3,053,917	1·146	
1842	2,704,385	1·162	
1843	3,355,419	1·347	
1844	3,515,090	1·283	
1845	5,252,431	1·727	
1846	3,553,899	1·278	
1847	3,517,408	1·054	3,590,159 0½
1848	4,355,370	1·853	4,216,731 9
1849	3,210,888	1·244	3,109,982 2
1850	4,054,017	1·457	3,892,187 1½
1851	3,455,827	1·239	3,436,045 8
1852	4,593,380	1·593	4,460,696 11½

It seems to me highly interesting to observe within what narrow limits the fluctuations of all claims, compared with the sum insured, are found, especially in the period from 1836 to 1846. Omitting the year 1845, the limits are 1·140 and 1·347; and the deviation of 1846 must be attributed to the See Assecuranz Compagnie of 1844, the history of which we have given before. The fluctuations are more considerable in the period 1847 to 1853—the year 1848 proves the worst; how much of this result must be ascribed to the Holstein war I am not prepared to say. Nor am I prepared to say whether the preceding table would justify the conclusion, that losses in general have increased in modern times; I am sorry that it seems to indicate that the losses of insurances effected in Hamburg have become heavier. It would be highly interesting to know whether this is caused by more frequent or by more heavy

losses. The materials for an answer to this question cannot be obtained from the balance-sheets.

We have now to consider the result of the whole business.

Year.	Profit.	Loss.
	Banco marcs.	Banco marcs.
1847	918,780	
1848	19,900	
1849	503,160	
1850	..	157,700
1851	303,400	
1852	..	596,500
Total.....	1,745,240	754,200

Year.	Per cent. of sum insured.		Per cent. of total income.	
	Profit.	Loss.	Profit.	Loss.
1847	0·2752	..	17·349	
1848	0·0085	..	0·396	
1849	0·1948	..	11·585	
1850	..	0·0567	..	3·526
1851	0·1088	..	6·781	
1852	..	0·2069	..	12·95

A table of the profit and loss since 1836, as given by the Handelsstatistischen Bureau, states—

Year.	Profit.	Loss.	For 100 Thalers insured sum.			
			Income.	Payment.	Profit.	Loss.
	Banco marcs.	Banco marcs.				
1836	112,252	..	1·38	1·32	0·06	
1837	250,915	..	1·56	1·43	0·13	
1838	349,626	..	1·47	1·32	0·15	
1839	1,044,559	..	1·45	1·03	0·41	
1840	376,994	..	1·45	1·30	0·15	
1841	362,721	..	1·41	1·27	0·14	
1842	231,455	..	1·40	1·30	0·10	
1843	..	299,700	1·38	1·50	..	0·12
1844	..	185,921*	1·38	1·44	..	0·06
1845	..	1,180,010	1·47	1·85	..	0·38
1846	240,323	..	1·50	1·41	0·09	
1847	1,018,303	..	1·48	1·17	0·31	
1848	..	20·531	2·02	2·03	..	0·01
1849	401,368	..	1·56	1·40	0·16	
1850	..	282,265	1·50	1·60	..	0·10
1851	314,182	..	1·50	1·38	0·12	
1852	..	706·891	1·49	1·73	..	0·24
1853	306,000	..	1·55	1·46	0·08	

* Including 405,000 banco marcs of the See Assecurans Compagnie of 1844.

I endeavoured in vain to find the key to these numbers, which I do not think to be quite correct; they demonstrate a profit of 2,343,380 banco marcs for marine insurance only.

It will be interesting to add a table showing the profits and losses of each year separately for those Companies which have published a favourable result, and for those which have proved unfortunate.

Year.	Number of Companies.	Profit.	Number of Companies.	Loss.
		Banco marcs.		Banco marcs.
1847	22	918,780	0	000,000
1848	15	309,900	7	290,000
1849	18	576,200	4	73,040
1850	8	98,500	14	256,200
1851	16	357,900	6	54,500
1852	6	107,000	16	703,500
	Total....	2,368,280	..	1,377,240

The dividends which have been paid to the shareholders are very considerable—they are

Year.	Companies for Marine Insurance only.	Companies for Marine Fire Insurance.	Total.
	Banco marcs.	Banco marcs.	Banco marcs.
1842	327,000	..	327,000
1843	226,000	..	226,000
1844	155,000	20,000	175,000
1845	18,000	20,000	38,000
1846	38,000	20,000	58,000
1847	202,000	30,000	232,000
1848	190,000	..	190,000
1849	212,500	40,000	252,500
1850	97,000	18,000	115,000
1851	215,000	90,000	305,000
1852	104,500	40,000	144,500
1853	207,500	60,000	267,500
Total....	1,992,500	338,000	2,330,500

The business of the different Companies is very different one from another—this we can see by comparing their average premiums; but more interesting conclusions cannot be deduced without having more minute particulars to examine than are given in the balance-sheets.

I end my essay requesting an indulgent judgment.

On the Methods pursued at the Present Day for estimating the Value of Contingent Reversionary Interests. By Robert Tucker, *Esq., one of the Vice Presidents of the Institute of Actuaries, and Actuary to the Pelican Life Insurance Company.*

[Read before the Institute the 26th February, 1855, and ordered by the Council to be printed.*]

THE subject proposed for discussion this evening is one of considerable importance to all who engage in the pursuits of an actuary. The old practice of valuing life contingencies by a pet table of mortality and at one fixed rate of interest has long since been found to work unprofitably; and parties seeking investments in securities of this description have been compelled, for their own safety and advantage, to adjust the valuation of the actuary before determining the price to be given for the particular property under consideration. If such a state of things is unsatisfactory to the public, it must be equally so to the actuary to find his calculations used only as a partial guide, instead of being, as in fact they ought to be, the index by which a proposed purchaser may know the rate of interest and the other advantages to be derived by obtaining the security in question at the price fixed by the actuary. In the ordinary transactions of a Life Assurance Office, where the fluctuation in the rate of mortality is protected by the admission of large numbers, it is necessary also to protect its pecuniary interests by limiting the amount to be assured on each life. Even with this precaution, it not unfrequently happens that particular years have proved less favourable than might have been expected—not so much from an increase in the number of deaths, as from the average amount of the sums insured by the policies so lapsed being greater than the average of the whole number. When, however, the average of a number of years is taken, and the amount insured upon any one risk is properly limited, these fluctuations become less apparent and generally disappear.

If it be necessary to protect an Office by paying due regard to the limit of its risks, how much more necessary is it to do this in dealing with life interests not involving the usual considerations of average! It should be the duty of the actuary, in valuing all life interests and reversions, to get rid of these contingencies in each case; or at all events to point out how they may be insured so that, after the premium for any particular risk is provided for, there may

* This paper was read by way of opening a discussion on the subject to which it refers.

remain to the capitalist—whether an individual or a Company it matters not—a fixed rate of interest upon the investment, as secure as in an ordinary mortgage.

The importance of these considerations has been pointed out by Mr. Jellicoe in more than one of the many papers he has read before the members of this Institute. It is to Mr. Jellicoe the profession is indebted for a clear exposition of the mode of dealing with isolated cases of contingent interests, and to him it properly belongs to open the discussion this evening. My apology for doing so must be the reason assigned to me by Mr. Jellicoe—that I am the latest writer on the subject, having furnished a letter "On the value of contingent reversionary interests," which appears in the last Number of the *Journal* of this Institute.

With this introduction, I proceed to notice some of the cases of contingent interests which occur most frequently in practice. They are—

1. A simple life interest, or an annuity on a single life.
2. A simple reversion, determinable on the death of a single life.
3. A contingent life interest, or an annuity on a single life commencing on the decease of another life.
4. A contingent reversion, determinable on one life surviving another.

A party purchasing or making an advance on a life interest should secure his capital by insuring the life of the annuitant. The value of a life annuity so secured is, as given by Mr. Griffith Davies, $\frac{1}{d+p} - 1$ where d is the discount of £1 for one year; and p is the Office premium per pound per annum.

Let the annuity be £100, age 40; the rate of interest required by a purchaser for the use of his money, 5 per cent.; and the premium charged for the insurance, the Northampton Three per Cent.: then $\left\{\frac{1}{d+p} - 1\right\} \times 100 = 11{\cdot}256 \times 100 =$ £1,125. 12*s*., the value of the annuity.

In addition to the sum advanced, it is usual to insure one year's annuity; therefore in this case $1125{\cdot}6 + 100 = 1225{\cdot}6$ will be the sum to be insured, the annual premium on which will be $1225{\cdot}6 \times {\cdot}033975 = 41{\cdot}639$.

Now	1125·6	being the sum paid for the annuity,
and	41·639	being the first year's premium,
	1167·239	will be the sum actually advanced, upon which 5 per cent. interest is to be allowed to the purchaser for the use of his money.

$1167{\cdot}239 \times {\cdot}05 = 58{\cdot}361$ = the annual interest,
and $41{\cdot}639$ = the annual premium;

and the sum of these = 100·000 = the annuity purchased.

Suppose the annuity, instead of being immediate, is deferred until the extinction of another life. Let A, aged 25 years *next* birthday, be entitled to an estate producing £1,000 per annum on the death of B, aged 65 years *last* birthday. What amount of annuity should he grant for what may remain of his life after the death of B, in consideration of an immediate advance of £1,000, the lender to receive 5 per cent. interest for the use of his money, besides the premium on the amount necessary to secure his outlay, according to the Northampton three per cent. rates? Here, in addition to the advance of £1,000, an annuity during the joint lives of A and B must be purchased or charged, equal in amount to the deferred annuity, in order to fulfil the conditions stipulated for of a clear 5 per cent. for the use of the money, freed from all contingencies.

Suppose the joint life annuity can be purchased according to the Carlisle Table, interest 3½ per cent. The value of the survivorship annuity being $\frac{1}{d+p} - 1 - \mathrm{AB}$, we have

$$\frac{1}{d+p} - 1 = 12{\cdot}958$$
$$\mathrm{AB} = \underline{8{\cdot}331}$$
$$4{\cdot}627 = \text{value of A's interest per £.}$$

The sum to be advanced to A being £1,000, the equivalent reversionary annuity to be granted by him will be

$$\frac{1000}{4{\cdot}627} = 216{\cdot}123.$$

Now $216{\cdot}123 \times 8{\cdot}331 = 1800{\cdot}521$ = price of annuity during the joint lives;
1000· = sum paid to A;
216·123 = one year's annuity;

making together 3016·644, the sum to be assured.

$3016{\cdot}644 \times {\cdot}02404 = 72{\cdot}52$ the annual premium.
1000·
1800·521

2873·041 = sum advanced, or presumed so to be.
·05

143·65205 = interest on do.
72·52 = annual premium.

216·172 = annuity as above, very nearly.

It occasionally happens, that instead of an immediate advance being required by the "tenant in reversion," it is only an annuity during the joint lives of himself and the "tenant in possession"—that is, he desires to increase his *present* at the expense of his *future* income. This does not differ in fact from the example last given: the cost of the joint life annuity is the only sum advanced to the expectant, the rest of the calculation being the same in both instances. But it may be urged that, if the Company making the advance also grant annuities as a branch of their business, they are charging on the one hand 5 per cent. to the borrower, and allowing in return only $3\frac{1}{2}$ per cent. This is true; but it should be borne in mind that the transactions are separate and distinct—that the Company on the one hand, as a Loan Office, advances the price of the annuity, and receives it on the other as an Annuity Office; and it seems but reasonable that a fair rate of interest should be allowed for the use of the money, and a fair per centage of profit charged on the annuity. If the negotiation were a mere family arrangement, then the relative values of the two annuities—that is, the survivorship and joint life—should alone enter into the calculation.

This method of dealing with contingent interests is, to my mind, much more satisfactory than charging a higher rate of interest and leaving the contingencies unprotected. Taking 5 per cent. as a fair and remunerative rate of interest, and charging a more moderate premium than that indicated by the Northampton Table, no borrower need complain of being overcharged; and no Office need be damaged by employing its capital so profitably, and at the same time adding to the number and amount of its insurances.

It is scarcely necessary to occupy the time of the meeting by going through an example of the two cases of reversionary sums before alluded to, $1-d\,(1+\mathrm{A})$, and $1-(p+d)\,(1+\mathrm{AB})$. The principle involved is the same in all; and they have been fully treated in the transactions of the Institute and under the head of "Correspondence" therein. The advantage of the plan recommended is to render a contingent life interest or reversion a marketable security, and to make it as available for the purpose of a loan as an ordinary mortgage, an immediate life annuity, or an absolute reversion: and this is done, as Mr. Jellicoe observes, by rendering certain that which is uncertain; treating each security as an investment not subject to any contingency whatever, and putting the holder of it into the receipt of a given rate of interest

so long as he retains it, and reproducing his capital at the termination of the contract.

There is one other subject which I desire to bring under the notice of the members; and although not (strictly speaking) within the limits proposed for discussion this evening, it is so immediately connected with it that a notice of the one would be incomplete without the other: it is, the mode of dealing with isolated cases of annuities certain. Like the ordinary tables of life annuities, tables of the values of annuities for terms of years should not be too implicitly relied upon as a guide to parties making occasional purchases in this description of security who have not the means of reinvesting their surplus annuity in the same way: the condition of the ordinary tables being, that the reinvestments are made at the same rate of interest as the purchase is supposed to yield. Now as this is obviously a matter of much uncertainty, and in isolated cases almost of impossibility, no correct idea can be formed of the actual rate of interest realized until the amount of surplus annuity is determined; that is to say, until it is known how much will be annually required to reproduce the capital at the end of the term, at some rate which may be safely assumed for the purpose.

In a paper read before the Institute in November, 1850, Mr. Hardy has given a table of the values of £1 per annum for a certain number of years, so as to pay a fixed rate of interest on the original purchase money, and to replace that purchase money at the expiration of the term, at a different rate of interest.

The difference between this table and the ordinary tables of annuities consists in the employment of two rates of interest instead of one. That which a purchaser realizes, or is supposed to realize, Mr. Hardy calls the *remunerative rate;* and that which is to reproduce the capital, the *accumulative rate.* In the ordinary table the remunerative and accumulative rates are the same.

The connection between an annuity for a term certain and a life annuity, and how one expression may be deduced from the other, has been clearly shown by Mr. Hardy and other writers. The same result may be arrived at thus:—

The value of an isolated life annuity is, as before explained, $s=\frac{1}{d+p}-1$. Here p, the annual premium, is payable in advance, and the sum insured includes this premium and the first year's interest. Now suppose the premium to be paid at the end of the year, and the annuity up to the day of death: then the sum to be

insured need only be the actual sum advanced; we shall have, therefore, denoting this annual premium by π,

$$\pi s + i s = a, \text{ or } s.(\pi + i) = a$$

$$\therefore \quad s = \frac{a}{\pi + i} = \frac{1}{\pi + i} \text{ when } a = 1.$$

This expression is equivalent to the value of an annuity for a term certain where π is the surplus annuity at the *accumulative rate*, and i is the annual interest at the remunerative rate.

By the aid of Corbaux's Tables of Compound Interest, and Barlow's Table of Reciprocals, the value of $\frac{1}{\pi + i}$ is readily obtained; π being given by Corbaux for every year, half year, or quarter, according as the ratio of interest is annually, half yearly, or quarterly. For example: Let three annuities be purchased, for the respective periods of 10, 20, and 30 years, to pay 5 per cent., with the surplus to be reinvested at 3 per cent. The several values of π are, according to Corbaux, ·08723, ·03721, and ·02102, $i = ·05$; the successive values of $\frac{1}{\pi + i}$ are $\frac{1}{·13723}$, $\frac{1}{·08721}$, $\frac{1}{·07102}$, respectively equal to 7·287, 11·466, and 14·081, which will be found to coincide with the values given in Mr. Hardy's table.

One of the most useful applications of this table is to test the comparative advantages of an investment in perpetual or in terminable annuities; and I propose to close these remarks by one example, taken from the prices of the Government securities as given in Wetenhall's list.

On the 14th of October last, immediately after the payment of the half yearly dividend, the Long Annuities had just $5\frac{1}{4}$ years to run; their market price was $4\frac{3}{8}$, and the corresponding price of Reduced 3 per Cents. due at the same time was 94.

	Per annum. £.	s.	d.
Now £100, invested in Long Annuities at $4\frac{3}{8}$, will produce	22	17	2
Deduct property tax at 14*d.* in the pound . .	1	6	8
leaves net	21	10	6
Required for sinking fund, at 3 per cent. . .	17	15	6
giving a net interest of . . .	£3	15	0
A similar sum, invested in Reduced, at 94, gives .	3	3	10
Deduct property tax	0	3	8
leaves a net interest of . . .	£3	0	2

On comparing the price of Long Annuities with the ordinary tables of annuities certain, it appears that such an investment would be equivalent to a 6 per cent. purchase, and that a gain of more than $2\frac{3}{4}$ per cent. interest would be derived from selling Reduced and buying Long Annuities; but when it is found that the property tax is levied on the whole annuity, and is equivalent to more than 40 per cent. on the interest yielded by Reduced instead of 6 per cent., and when the reproduction of capital has to be provided for at a rate of interest which may be safely relied upon, about 2 per cent. of the supposed advantage disappears, and this would be further reduced by the payment of income tax on the interest of the reinvestments.

Supposing there were no income tax, and the surplus annuity to be still reinvested at 3 per cent., then if

$$i=\cdot06\quad \frac{1}{\pi+i}=\frac{1}{\cdot1777+\cdot06}=\frac{1}{\cdot2377}=4\cdot207,$$

$$i=\cdot05\quad \frac{1}{\pi+i}=\frac{1}{\cdot1777+\cdot05}=\frac{1}{\cdot2277}=4\cdot392,$$

$$i=\cdot051\quad \frac{1}{\pi+i}=\frac{1}{\cdot1777+\cdot051}=\frac{1}{\cdot2287}=4\cdot373;$$

From which it appears, that an annuity of £1 for $5\frac{1}{4}$ years certain, which can be bought for the sum of £4. 7*s.* 6*d.*, including commission, would pay the purchaser (upon the supposition above stated) £5. 2*s.* per cent. for the use of his money, and enable him to reproduce his capital in a 3 per cent. stock at par.

On the Sickness and Mortality amongst the European and Native Troops of the Madras Army.

WE owe to the courtesy of W. H. Scales, Esq., a medical officer in the East India Company's service, Madras Presidency, the following series of tables on the sickness and mortality amongst the European and Native troops in the Madras army; and as, from the extension of life assurance in our Indian possessions, every information of this kind is of value, we present a summary of them to our readers. In the original tables the numbers "treated" and "died" to the "strength" are given for each year, as well as the sickness and mortality according to length of service; but for life assurance statistics the facts here recorded are the most important. We would especially draw attention to the returns classified according to ages.

Table exhibiting the Number of Admissions and Deaths from the most important Diseases amongst the European Troops of the Madras Army, from 1842 *to* 1850–51 *inclusive.*

Years.	Strength.	Cholera.		Fevers.		Liver.		Diarrhœa.		Dysentery.		Thoracic diseases.		Rheumatism.		Venereal diseases.		Dropsy.		Total from preceding diseases.		Grand total of all diseases.	
		Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.
1842–49.*	91901½	1350	672	25730	282	6536	269	10073	180	11436	806	3779	192	8598	47	21479	44	270	54	89251	2546	148268	3154
Per centage of treated to strength...		1·468		27·997		7·112		10·960		12·443		4·112		9·355		23·371		0·293		97·116		161·334	
Do. of deaths to do...		0·731		0·306		0·292		0·195		0·877		0·208		0·051		0·047		0·058		2·770		3·410	
Do. do. to treated ...		49·777		1·095		4·115		1·786		7·047		5·080		0·546		0·204		20·000		2·852		2·113	
1850–51.†	9136½	35	21	2932	14	626	28	659	9	704	45	971	45	932	4	2675	7	18	5	9552	178	14646	226
Per centage of treated to strength...		0·383		32·092		6·852		7·213		7·705		10·628		10·201		29·279		0·197		104·553		160·310	
Do. of deaths to do...		0·229		0·153		0·306		0·098		0·492		0·492		0·043		0·076		0·054		1·948		2·473	
Do. do. to treated ...		60·000		0·477		4·472		1·365		6·392		4·634		0·429		0·261		27·777		1·863		1·543	

* Excluding Cholera, the ratio of deaths to strength has been 2·679 per cent.

† Excluding Cholera, the ratio of deaths to strength has been 2·243 per cent.

Table exhibiting the Number of Admissions and Deaths from the most important Diseases amongst the Native Troops of the Madras Army, from 1842 *to* 1850–51 *inclusive.*

Years.	Strength.	Cholera.		Fevers.		Liver.		Diarrhœa.		Dysentery.		Thoracic diseases.		Rheumatism.		Venereal diseases.		Dropsy.		Total from preceding diseases.		Grand total of all diseases.	
		Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.	Treated.	Died.
1842–49.*	544933	10290	4356	156852	1646	796	91	13261	736	7252	488	4283	395	38240	393	17869	109	4536	673	253379	8887	407616	10820
Per centage of treated to strength...		1·888		28·783		0·146		2·433		1·330		0·785		7·017		3·279		0·832		46·497		74·801	
Do. deaths to do		0·799		0·302		0·016		0·135		0·089		0·072		0·072		0·020		0·123		1·630		1·985	
Do. do. to treated ...		42·332		1·049		11·432		5·550		6·729		9·222		1·027		0·609		14·836		3·507		2·654	
1850–51.†	50446¾	484	183	14544	127	95	3	1218	36	556	25	860	69	3798	35	1745	7	465	70	23765	555	39287	712
Per centage of treated to strength ...		0·959		28·829		0·188		2·414		1·102		1·704		7·528		3·459		0·921		47·107		77·876	
Do. deaths to do.....		0·362		0·251		0·005		0·071		0·049		0·136		0·069		0·013		0·138		1·100		1·411	
Do. do. to treated....		37·809		0·873		3·157		2·955		4·496		8·023		0·921		0·401		15·053		2·335		1·812	

* Excluding Cholera, the ratio of deaths to strength has been 1·182 per cent.

† Excluding Cholera, the ratio of deaths to strength has been 1·048 per cent.

Return exhibiting Strength, Admissions, and Deaths, of European Sick, from Four Diseases, according to Age, for Three Years.

Age.	Strength.	Fevers.				Dysentery, acute and chronic.				Hepatitis, acute and chronic.				Diseases of the Lungs.			
		Admitted.	Died.	Per centage of Sick to Strength.	Per centage of Deaths to Strength.	Admitted.	Died.	Per centage of Sick to Strength.	Per centage of Deaths to Strength.	Admitted.	Died.	Per centage of Sick to Strength.	Per centage of Deaths to Strength.	Admitted.	Died.	Per centage of Sick to Strength.	Per centage of Deaths to Strength.
20	3,864	963	8	24·922	0·207	225	11	5·822	0·284	154	5	3·985	0·129	295	5	7·634	0·129
25	8,733	2,478	14	28·375	0·160	737	43	8·439	0·492	453	23	5·187	0·263	814	18	9·320	0·206
30	9,108	2,875	14	31·565	0·153	834	64	9·156	0·702	672	23	7·378	0·252	992	34	10·891	0·373
35	4,626	1,145	6	24·751	0·129	320	27	6·917	0·583	320	10	6·917	0·216	375	13	8·106	0·281
40	1,928	480	5	24·896	0·259	124	12	6·431	0·622	139	8	7·209	0·414	170	8	8·817	0·414
45	480	71	1	14·791	0·208	24	4	5·000	0·833	29	..	6·041	..	44	2	9·166	0·416
50	112	14	..	12·500	..	1	..	0·892	..	..	..	..	..	2	1	1·785	0·892
Above 50	94	5	..	5·319	..	2	..	2·127	..	2	..	2·127	..	4	1	4·255	1·063
Total..	28,945	8,031	48	27·745	0·165	2,267	161	7·832	0·556	1,769	69	6·111	0·238	2,696	82	9·314	0·283

Table showing the comparative ratio of Punishments, &c., awarded to the following classes of Men.

Classes.	Strength.	Punished by		Tried by			Total Punishments and Trials.	Ratio per cent. to Strength.
		Regimental Captains.	Commanding Officers.	Regimental Courts Martial.	District Courts Martial.	General Courts Martial.		
Teetotalers	589	43	28	1	2	0	74	12·5
Temperate	6,801	1,320	2,536	135	51	10	4,052	59·5
Intemperate	1,534	502	1,727	323	60	10	2,622	170·9

General Return of European Sick, exhibited according to Age, for the Year 1850–51.

Years of Age	20		25		30		35		40		45		50		Above 50		Total.		Per centage of Admissions to Strength.	Per centage of Deaths to Strength.	Per centage of Deaths to Admissions.
Strength	824		2774		3431		2145		717		224		71		69		10,275				
	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.			
Fevers	272	2	923	5	1,047	3	432	2	158	1	22	1	4	0	5	0	2,863	14	27·863	0·136	0·488
Cholera	2	1	12	6	6	4	8	5	2	1	0	0	0	0	0	0	30	17	0·291	0·165	56·666
Dysentery, acute and chronic	29	1	233	8	243	20	109	8	33	2	9	2	0	0	2	0	658	41	6·403	0·399	6·231
Diarrhœa	50	0	202	1	207	4	91	0	52	3	7	0	7	0	6	3	622	11	6·053	0·107	1·768
Other Diseases of the Stomach and Bowels	80	0	251	3	265	5	129	0	56	1	11	0	4	0	6	0	802	9	7·805	0·087	1·122
Hepatitis, acute and chronic	50	3	174	10	195	9	118	4	33	0	10	0	0	0	1	0	581	26	5·654	0·253	4·475
Diseases of the Lungs	78	1	281	5	318	15	126	6	52	1	17	2	1	1	4	1	877	32	8·535	0·311	3·648
Do. Brain	26	0	127	5	150	6	68	5	48	2	11	0	3	0	4	2	437	20	4·253	0·194	4·576
Rheumatic Affections	60	0	296	0	332	2	159	2	90	0	20	0	2	0	4	0	963	4	9·372	0·038	0·415
Venereal do.	322	0	922	1	831	2	320	2	84	0	20	0	1	0	1	0	2,501	5	24·340	0·048	0·199
Dropsies	3	0	5	1	3	0	3	2	1	1	0	0	2	1	0	0	17	5	0·165	0·048	29·411
All other Diseases	353	3	1,207	5	1,204	12	613	4	240	6	90	1	22	1	24	1	3,753	33	36·525	0·321	0·879
Total	1,325	11	4,633	50	4,801	82	2,176	40	849	18	217	6	46	3	57	7	14,104	217	137·265	2·111	1·538
Per centage of Admissions to Strength	160·800		167·015		139·930		101·445		118·410		88·934		64·788		82·608		137·265		Corps, &c. included in this Return.		
Do. Deaths do.	1·324		1·802		2·389		1·864		2·510		2·459		4·225		10·144		2·111				
Do. do. to Admissions	0·830		1·079		1·707		1·838		2·120		2·764		6·521		12·280		1·538				

General Return of European Sick, exhibited according to Age, for 3 *years*—1848, 1849, *and* 1850-51.

Years of Age	20		25		30		35		40		45		50		Above 50		Total.		Per centage of Admissions to Strength.	Per centage of Deaths to Strength.	Per centage of Deaths to Admissions.
Strength	3,864		8,733		9,108		4,626		1,928		480		112		94		28,945				
	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.	Admitted.	Died.			
Fevers	963	8	2,478	14	2,875	14	1,145	6	480	5	71	1	14	0	5	0	8,031	48	27·745	0·165	0·597
Cholera	11	6	26	13	16	11	17	12	4	2	..	..	..	..	..	..	74	44	0·255	0·152	59·459
Dysentery, acute and chronic	225	11	737	43	834	64	320	27	124	12	24	4	1	0	2	0	2,267	161	7·832	0·556	7·101
Diarrhœa	341	2	730	11	716	13	276	2	130	6	16	0	7	0	7	3	2,223	37	7·680	0·127	1·664
Other Diseases of the Stomach and Bowels	386	2	818	6	820	8	393	6	186	2	41	0	7	0	7	0	2,658	24	9·182	0·082	0·902
Hepatitis, acute and chronic	154	5	453	23	672	23	320	10	139	8	29	0	0	0	2	0	1,769	69	6·111	0·238	3·900
Diseases of the Lungs	295	5	814	18	992	34	375	13	170	8	44	2	2	1	4	1	2,696	82	9·314	0·283	3·041
Ditto Brain	125	0	366	9	422	16	228	10	133	6	31	1	8	0	5	3	1,318	45	4·553	0·155	3·414
Rheumatic Affections	290	0	864	0	985	5	453	3	248	2	60	0	2	0	4	0	2,906	10	10·039	0·034	0·344
Venereal ditto	1,413	0	3,146	4	2,855	6	816	2	237	1	44	0	2	0	1	0	8,514	13	29·414	0·044	0·152
Dropsies	7	0	17	4	15	3	7	3	6	2	4	1	2	1	0	0	58	14	0·200	0·048	24·137
All other Diseases	1,669	5	3,663	17	4,069	22	1,582	12	713	10	217	1	28	1	26	1	11,967	69	41·343	0·238	0·576
Total	5,879	44	14,112	162	15,271	219	5,932	106	2,570	64	581	10	73	3	63	8	44,481	616	153·674	2·128	1·384
Per centage of Admissions to Strength	152·148		161·593		167·665		128·231		133·298		121·041		65·178		67·021		153·678		Corps, &c. included in this Return.		
Ditto Deaths to Strength	1·138		1·855		2·404		2·291		3·319		2·083		2·678		8·510		2·128				
Ditto Ditto to Admissions	0·748		1·147		1·434		1·786		2·490		1·721		4·109		12·698		1·384				

FOREIGN INTELLIGENCE.

DENMARK.—*The "Brandforsikkring for Huse og Gaarde" (Fire Insurance Company for Buildings)*, at Copenhagen, has published the results of the business for the year ending the 30th September, 1854.

The risks in force on that day amounted to Rthlr.61,281,150 (£6,575,684) on buildings in the town of Copenhagen, being Rthlr. 1,400,000 (£155,556) more than at the last year. The funds of the Society amount to Rthlr.1,560,675 (£173,418), Rthlr.13,459 (£1,495) more than last year. The amount of claims has been, in the year 1853-54, Rthlr.36,125. 94sh. (£4,014), viz.—

	Rthlr.	sh.	£.
25 claims each less than Rthlr.500 (£56)	1,864		207
5 " " " 3,000 (£333)	6,107	94	679
1 "	6,272		697
1 "	21,882		2,431
	Rthlr.36,125	94	£4,014

The income was—

	Rthlr.	£.
Premiums	20,199	2,244
Interest	67,600	7,511
	Rthlr.87,799	£9,755

Rthlr.23,000 (£2,556) have been paid to the Royal Committee, for fire engines, &c.

It is surprising that the system of mutual fire insurance, limited to the buildings of a single town, still prevails on the Continent, notwithstanding that many fires of great extent have proved that such a system may for many years seem advantageous, and yet at last prove to be ruinous. The great accumulation of risks is increased by the circumstance that no other insurance of a building is allowed.

The Life Insurance and Annuity Society at Copenhagen, established in 1842, publishes the following accounts from Jan. 1 to Dec. 31, 1854:—

Annuity branch—

	Rthlr.	sh.	£.
Income	359,675	86	39,964
Payments	106,941	27	11,882
Surplus	252,734	59	28,082

The funds on the 31st December, 1854, were Rthlr.2,353,468. 11sh. (£261,496).

Widows' annuity branch—

	Rthlr.	sh.	£.
Income	85,658	32	9,51
Payments	5,626	57	625
Surplus	80,031	71	8,89

The funds on the 31st December, 1854, were Rthlr.212,229. 83sh. (£23,581).

Life insurance branch—

	Rthlr.	sh.	£.
Income	141,028	10	15,680
Payments	95,564	84	10,618
Surplus	45,463	22	5,062

The funds on the 31st December, 1854, were Rthlr.345,470. 82½sh. (£38,386).

BREMEN.—*Duration of Voyages from Bremen to the undermentioned Ports, in* 1852–54.

From BREMEN to	Number of Vessels.		Average Duration of Voyage.			Shortest Duration of Voyage.			Longest Duration of Voyage.		
	1853.	1854.	1852.	1853.	1854.	1852.	1853.	1854.	1852.	1853.	1854.
			Days.	Days.	Days.	Days.	Days.	Days.	Days.	Days.	Days.
New York. *Steamers*	14	15	21	21	20	17	19	15	31	29	25
Ditto. *Sailing Vessels*	155	214	46	49	47	23	35	29	83	75	76
Baltimore	41	56	54	50	50	32	32	36	82	78	88
New Orleans	31	43	57	58	54	41	41	42	73	69	69
Philadelphia	11	12	51	55	48	38	39	37	80	80	67
Galveston	8	14	61	59	60½	46	46	52	79	84	78
Charleston (S. C.)	1	..	47	46							
Rio de Janeiro	1	..	71	68	..	62	..	..	81		
Ditto, *viâ* Newcastle	..	6	..	..	85	..	..	67	..	..	103
Ditto, „ Cardiff	..	2	..	..	82	..	..	73	..	..	91
Bahia	2	2	56	52½	51½	49	46	42	64	59	61
Rio Grande do Sul	1	3	120	81	63	91	..	52	148	..	70
Pernambuco	1	..	41	91							
Havanna	9	5	68	56	51	53	49	43	83	75	66
Trinidad de Cuba	2	3	69	65	58	38	61	50	101	69	72
S. Jago de Cuba	3	5	50	48	55	35	46	47	60	50	64
Cienfuegos	2	1	45	57½	51	..	43	..	..	72	
Manzanilla de Cuba	1	1	56	72	42	46	..	..	66		
Matanzas	1	..	81	55							
Cuidad Bolivar	6	3	69	74	57	62	52	48	83	113	68
Ditto, *viâ* Liverpool	..	5	108	..	76	84	..	72	126	..	87
La Guayra	..	1	71	..	50	42	..	..	96		
Aguadilla	1	7	51	43	40	40	..	33	69	..	55
San Juan	..	1	77	..	33	54	..	..	102		
Ponce	..	..	73								
Mayaguez	1	..	58	43							
St. Thomas	3	..	40	51	..	38	36	..	43	78	
Ditto, *viâ* Newport	..	3	87	..	84	56	..	68	132	..	97
Cape Hayti	2	2	42	45	49	..	38	38	..	61	60
Porto Plata	4	..	39	48½	..	..	38	..	..	52	
Port au Prince	1	..	..	64							
Gonaives	..	1	..	..	52						
Buenos Ayres	1	2	..	69	68½	..	..	62	..	..	75
Kingston (Jamaica)	1	..	66	41	..	51	..	..	82		
Quebec (Canada)	2	3	53	52½	48	35	52	43	96	53	56
Canary Islands	3	2	..	19	21½	..	18	19	..	20	24
Batavia	..	..	112	..	..	110	..	..	114		
Odessa	12	..	53	60½	..	41	39	..	61	107	
Galatz	4	..	75	75	..	60	57	..	96	100	
Constantinople	1	..	89	68							
Gronland, out & home	12	11	158	149	148	137	109	128	175	194	168

Duration of Voyages from the undermentioned Ports to Bremen, in 1853–54.

To Bremen from	Number of Vessels.		Average Duration of Voyage.		Shortest Duration of Voyage.		Longest Duration of Voyage.	
	1853.	1854.	1853.	1854.	1853.	1854.	1853.	1854.
			Days.	Days.	Days.	Days.	Days.	Days.
New York. *Steamers*	13	16	17	18	13	15	24	29
Ditto. *Sailing Vessels*	31	75	33	35	22	18	49	50
Baltimore	34	34	37½	39	24	28	49	67
New Orleans	27	39	55	54	44	37	87	79
Philadelphia	..	4	..	28	..	22	..	35
Galveston	5	5	70	55	57	39	94	66
Charleston (S. C.)	7	6	44	41	37	32	57	56
Richmond (V. A.)	13	17	38	41	27	27	49	67
Rio de Janeiro	11	6	78	67	54	51	116	94
Bahia	22	26	68	62	49	33	115	84
Rio Grande do Sul	3	1	86	87	77	..	98	
Pernambuco	4	4	71½	62	55	56	88	71
Santos	4	4	81	70	70	43	103	89
Havanna	9	10	54	55	41	40	68	75
Trinidad de Cuba	11	16	53	56½	44	43	61	71
S. Jago de Cuba	13	16	53	53	42	30	68	85
Cienfuegos	..	2	..	56	..	53	..	59
Manzanilla de Cuba	7	16	66	55	50	40	101	79
Matanzas	..	2	..	52	..	38	..	66
St. Cruz de Cuba	4	..	51	..	43	..	57	
Gibara	2	2	48½	68½	42	41	55	96
Nuevitas	2	..	59	..	57	..	61	
Cuidad Bolivar	2	10	60½	44	48	42	73	119
La Guayra	..	3	..	48	..	42	..	52
Aguadilla	6	7	44	47	32	40	51	58
San Juan	..	2	..	52	..	52	..	52
Ponce	2	6	54	72	53	32	55	115
Arecibo	7	6	44	51	32	39	51	70
Guayama	2	3	47½	44	44	38	51	49
Mayaguez	3	1	58	59	49	..	72	
Cape Hayti	2	5	71½	57	42	40	101	75
Porto Plata	13	14	43	57	33	41	55	95
Port au Prince	1	..	95					
Gonaives	1	1	42	58				
Buenos Ayres	1	3	78	83	..	77	..	92
Kingston (Jamaica)	7	..	58	..	42	..	81	
Canary Islands	3	..	39	..	32	..	44	
Fayal (Azores)	1	2	36	24½	..	24	..	25
Mexico	3	5	104	113	62	63	129	175
Sta. Martha	3	2	99	71	83	58	117	83
Akyab	4	10	137	143	122	132	151	157
Batavia	..	4	..	139	..	124	..	155
Odessa	16	26	89	73	55	42	140	136
Galatz	24	6	100	88	67	63	146	106
Constantinople	3	1	91	74	70	..	108	

CORRESPONDENCE.

ON THE APPLICATION OF THE DIFFERENTIAL AND INTEGRAL CALCULUS TO "INTEREST" QUESTIONS.

To the Editor of the Assurance Magazine.

SIR,—Having been kindly favoured by Professor De Morgan with some remarks on my Paper, lately read before the Institute, and to be inserted, I believe, in your present Number, your readers may be glad to have the following additional illustration of the application of the Calculus to Interest questions, as afforded by so eminent a mathematician. The illustration may be used to indicate, that if but one mean annual rate of interest be assumed for all periods alike, whether long or short, as is commonly done, that such rate can only be properly taken as typical of average accumulation, by supposing all investments broken up into yearly periods, with annual repayment of principal and annual chance of unproductiveness, or of reinvestment at such rates as may yearly happen to obtain.

Without enlarging on the manner in which the differentials of the second and higher orders disappear, the Professor says:

"Let $\phi x dx$ be the chance that the year's rate of interest per £1 lies between x and $x+dx$; let $x_1, x_2, \ldots x_n$ be the years' rates actually occurring in the n separate years; then the £1 will amount to $(1+x_1)(1+x_2)\ldots.(1+x_n)$, and the chance of such event is $\phi x_1 dx_1 . \phi x_2 dx_2 \ldots\ldots \phi x_n dx_n$: whence the equivalent certainty for such a case is $(1+x_1)(1+x_2)\ldots.(1+x_n).\phi x_1 \phi x_2 \ldots. \phi x_n . dx_1 dx_2 \ldots. dx_n$; and for all possible cases, α and β being the extreme possible rates,

$$\int_\alpha^\beta \int_\alpha^\beta \ldots \int_\alpha^\beta (1+x_1)(1+x_2)\ldots(1+x_n).\phi x_1 \phi x_2 \ldots \phi x_n . dx_1 dx_2 \ldots dx_n,$$

which, by reason of the independence of the limits, and the factorial separability of the functions, is

$$\int_\alpha^\beta (1+x_1)\phi x_1 dx_1 \times \int_\alpha^\beta (1+x_2)\phi x_2 dx_2 \ldots. \times \int_\alpha^\beta (1+x_n)\phi x_n dx_n,$$

reducible to

$$\left\{\int_\alpha^\beta (1+x)\phi x\, dx\right\}^n.$$

If all cases be equally likely, $\phi x dx = \dfrac{dx}{\beta-\alpha}$, and

$$\frac{1}{\beta-\alpha}\int_\alpha^\beta (1+x)dx = \frac{(1+\beta)^2-(1+\alpha)^2}{2(\beta-\alpha)} = 1+\frac{\beta+\alpha}{2},$$

so that the mean value for the end of the nth year is $\left(1+\dfrac{\beta+\alpha}{2}\right)^n$, or the common result when the mean rate of interest is assumed for all periods."

A more striking instance, perhaps, could not have been given, of how completely the Calculus adapts itself to the investigation of even the commonest assumptions in actuarial subjects; and thus enables us to ascertain the exact conditions with which such assumptions are really connected.

Thus, while the above illustration shows that the common assumption

of the mean rate of interest between the limits is a fair one when loans and repayments are dealt with year by year, the table in the Paper continues the series and indicates how such an assumption should be modified to represent the average accumulations of longer transactions. A real and not a fictitious tabular simplicity of results may thus, it is hoped, be gradually brought about in actuarial calculations; and with the greater and greater effect, as the supposed difficulties of variation, instead of being evaded, become more and more thoroughly studied.

To prevent undue inferences, it is right to state that the eminent mathematician alluded to, is not to be considered as in any way answerable for the contents of the paper in question, nor indeed as an implied authority either for or against the principles therein enunciated.

Your obedient Servant,
EDWIN JAS. FARREN.

Hanover Chambers, Buckingham Street, Strand,
London, February 2nd, 1855.

ON THE FACILITIES AFFORDED BY MR. THOMSON'S ACTUARIAL TABLES IN MAKING CERTAIN CALCULATIONS.

To the Editor of the Assurance Magazine.

Sir,—It would, I think, be useful, if you were to invite communications from your readers of such questions as they may meet with in practice, and which are not to be found in the text books. If you approve of this suggestion, and think the accompanying case worthy of a place in your *Magazine*, perhaps you will kindly insert it. The facility with which the formula is worked out affords another instance of the usefulness of Mr. Thomson's Actuarial Tables, and of the consequent benefit they confer on the profession.

I am, Sir, yours truly,
ROBERT TUCKER.

Lombard Street, 8th February, 1855.

What single and annual premium should be charged to secure £100 per annum to A, aged 32, after the death of B, aged 40, provided B die within 5 years (Carlisle 3 per cent.)?

The value of that portion of the annuity which may be enjoyed by A during the first five years is evidently ${}_{5|}A - {}_{5|}AB$; and it is equally clear, that the value of the remaining portion is ${}^{5}A \times {}_{5|}\mathfrak{B}$ ∴ the total value of the annuity is ${}_{5|}A - {}_{5|}AB + {}^{5}A \times {}_{5|}\mathfrak{B}$.

Thomson, Table 1, Single Lives.		Table 2, Single Deaths.
A = 19·13521	*AB = 14·30229	$\mathfrak{B}$ = ·47158
$A_{-\overline{5}|}$ = 14·69049	$-{}_{5|}AB$ = 9·91515	$-{}_{5|}\mathfrak{B}$ = ·40884
${}_{5|}A$ = 4·44472	${}_{5|}AB$ = 4·38714	${}_{5|}\mathfrak{B}$ = ·06274
4·38714		
${}_{5|}A - {}_{5|}AB$ = 0·05758		

* AB being taken equal to a single life of 50.

$$ {}^{5}A \times {}_{5|}\ddot{a}_{\overline{20}|} = 17{\cdot}9287 \times {\cdot}06274 = 1{\cdot}124846 $$
$$ {\cdot}05758 $$
$$ {}_{5|}A - {}_{5|}AB + {}^{5}A \times {}_{5|}\ddot{a}_{\overline{20}|} = 1{\cdot}182426 $$

Annual premium $= \dfrac{1{\cdot}182426}{1 + {}_{5|}AB} = \dfrac{1{\cdot}182426}{4{\cdot}588} = {\cdot}25772$ for £100 per annum. Premium in one sum, £118. 4*s.* 10*d.*, or £25. 15*s.* 5*d.* annually.

FORMULA FOR AN APPROXIMATE VALUE OF ANNUITIES AT SIMPLE INTEREST.

To the Editor of the Assurance Magazine.

SIR,—In looking over some old letters, I found one, dated some years back, from Professor De Morgan, in which he gives the following elegant approximation to the value of $\dfrac{1}{1+r} + \dfrac{1}{1+2r} + \dfrac{1}{1+3r} +$, &c. $\dfrac{1}{1+nr}$.

He says the best approximation is

$$ \frac{2{\cdot}3205851}{r} \cdot \log. \frac{1+nr}{1+r} + \frac{1}{2}\left(\frac{1}{1+r} + \frac{1}{1+nr}\right) + \frac{r}{12}\left(\frac{1}{\overline{1+r}|^{2}} - \frac{1}{\overline{1+nr}|^{2}}\right) $$
$$ - \frac{r^{3}}{120}\left(\frac{1}{\overline{1+r}|^{4}} - \frac{1}{\overline{1+nr}|^{4}}\right); $$

error only in the sixth decimal when $r = {\cdot}1$, or interest at 10 per cent.,

$$ \frac{1}{1{\cdot}1} + \frac{1}{1{\cdot}2} + \frac{1}{1{\cdot}3} + \ldots\ldots \frac{1}{2{\cdot}0}. $$

Approximation	6·687715
Truth	6·687714

I am, Sir,

Your obedient Servant,

London Assurance, March 10, 1855. PETER HARDY.

NOTICES OF NEW WORKS.

On the Loans raised by Mr. Pitt during the first French war, 1793—1801, *with some Statements in Defence of the Methods of Funding employed.* By WILLIAM NEWMARCH, one of the Honorary Secretaries of the Statistical Society. Effingham Wilson, Royal Exchange; Harrison, 59, Pall Mall; and Nissen and Parker, 43, Mark Lane.

WE had occasion some time back (*see* Vol. IV., page 78) to call the attention of our readers to an essay by this gentleman, "On the new supplies of gold," and to express an opinion of its value and importance to all en-

gaged in inquiries on such subjects; and it is with pleasure that we meet him again in a wholly different arena, but in one which, judging from the specimen before us, the writer is peculiarly qualified to excel. The almost universal condemnation of the financial policy of Mr. Pitt, so far as it was exhibited in the contraction of loans, seems to have stimulated Mr. Newmarch to investigate its real character; and it must be no doubt a pleasing reward for the laborious diligence devoted to the inquiry, that he is enabled not only to stem the tide of censure which has so long set in against that celebrated minister, but to demonstrate that the measures taken by him for raising supplies during the perilous times of his administration were really the best that could have been adopted, whether looked at in a theoretical or practical point of view. The arguments brought forward by Mr. Newmarch in support of his propositions are backed by a remarkable amount of statistical evidence, drawn from sources which probably few beside himself would have traced with equal success, and the origin of which he is always careful distinctly to indicate. Considering the circumstances in which the country is placed at the present time, and the many points of resemblance between its position now and at the period of which Mr. Newmarch treats, nothing could be more opportune than the appearance of such a publication, or be better calculated to assist the Government in its deliberations as to the best means of discharging the balance of the enormous expenditure already incurred, and of providing for the still more formidable addition to its burdens which there is every reason to anticipate the nation will be unhappily compelled to submit to.

A Treatise on the Enfranchisement and Improvement of Copyhold, Life-Leasehold, and Church Property; with Rules and Tables for the formation of Copyhold Enfranchisement and Freehold Land Societies, and a Mathematical Appendix. By Arthur Scratchley, M.A., F.R.A.S. Third Edition, enlarged. Charles Mitchell, 12, Red Lion Court, Fleet Street.

This publication, the author says in his preface, "has for its object the development of a comprehensive system for the general enfranchisement and improvement of property held by copyhold or customary tenure. The main elements are the establishment of Copyhold Enfranchisement Societies and Freehold Land Societies, with the application of the life assurance principle." Mr. Scratchley has contrived to make his subject, which is necessarily a somewhat dry one, of more interest than ordinary, by an introduction in which he has collected some curious particulars in reference to the origin of manorial rights, and the extraordinary customs connected with them, prevailing even at the present day; and as regards the practical part of his treatise, it appears to contain all that persons seeking information in such matters can desire to have.

The Act of 17 and 18 Victoria, cap. 116, in relation to Church property, contains clauses of much importance, and introduces several improvements connected with the regulation of it—one, in particular (suggested, it seems, by Mr. Scratchley himself), prohibiting the use of the Northampton Table in any calculations thereafter to be made in reference to such property.

Government Regulations for the Examination of Candidates for the Appointments to the Civil Service of the East India Company, &c. Stanford, Charing Cross.

ALTHOUGH the above heading is that of a mere sixpenny pamphlet, it may fairly be considered as referring to principles of especial interest to such of our readers as are members of the Institute of which our *Magazine* forms the Journal. Such members, on perusal of the above pamphlet at our recommendation, will, we think, immediately detect, that what may hitherto have appeared to some to be so narrow a question as almost to be confined within the walls of the Institute of Actuaries, has now become in high quarters a question of the greatest public, financial, and administrative importance—viz., the question of educational qualification *versus* official patronage, as the basis of fiscal and civil employment. The result arrived at is defined by the words of the Committee upon whose report the regulations are founded, when they say—

"The educated youth of the United Kingdom are henceforth to be invited to engage in a competition in which about 40 prizes will, on an average, be gained every year. Every one of these prizes is nothing less than an honourable social position, and a comfortable independence for life."

"Hitherto, the admissions have been given by favour. They are henceforward to be gained by superiority in an intellectual competition."

Without pretending that the parallel is a close one, the lines of relation are still not so far distant as not almost instantly to recall to the well informed actuary what process was forming the only one current in life assurance appointments, and to thus second the public expediency of the modern process of examination on financial subjects, as originated by the Institute itself. Considering, indeed, that such examinations have now been in public operation for some years among members of the Institute, it would not be unfair, though perhaps not exactly correct, to surmise that such a provision for the efficiency of the officers of Joint Stock Insurance Companies may also have had some effect in suggesting the propriety of instituting a change of procedure as to the appointment of officers in the financial or civil service of the joint stock East India Company. Be this as it may, the open recognition of detailed education as the basis of qualification for financial employments is all that the originators or supporters of the Institute have ever sought to obtain even by charter, and it is for this reason that we think it right on public grounds to thus bring a pamphlet on an allied subject under the notice of the general body. It would be to overlay so limited a publication, to formally review it at any length; but the following extracts may be acceptable to our own readers as illustrative of the parallel, however distant, to which we have alluded.

The Committee upon whose report (dated Nov., 1854), the Government regulations have been founded, comprised the following eminent names—T. B. Macaulay, Ashburton, Henry Melvill, Benjamin Jowett, and John George Shaw Lefevre.

"We think it most desirable that the examination should be of such a nature that no candidate who may fail shall, to whatever calling he may betake himself, have any reason to regret the time and labour which he spent in preparing himself to be examined."

"The whole examination ought, we think, to be carried on by means

of written papers. The candidates ought not to be allowed the help of any book; nor ought they, after once a subject for composition has been proposed to them, or a paper of questions placed before them, to leave the place of examination till they have finished their work.

"It is, of course, not to be expected that any man of 22 will have made considerable proficiency in all the subjects of examination. An excellent mathematician will often have little Greek, and an excellent Greek scholar will be entirely ignorant of French and Italian. Nothing can be further from our wish than to hold out premiums for knowledge of wide surface and of small depth. We are of opinion that a candidate ought to be allowed no credit at all for taking up a subject in which he is a mere smatterer. Profound and accurate acquaintance with a single language ought to tell more than bad translations and themes in six languages. A single paper which shows that the writer thoroughly understands the principles of the differential calculus ought to tell more than twenty superficial and incorrect answers to questions about chemistry, botany, mineralogy, metaphysics, logic, and English history.

"It will be necessary that a certain number of marks should be assigned to each subject, and that the place of a candidate should be determined by the sum total of the marks which he has gained."

"We have, with an anxious desire to deal fairly by all parts of the United Kingdom, and by all places of liberal education, framed the following scale, which we venture to submit for your consideration:—

English language and literature—	
Composition	500
History	500
General literature	500
	1,500
Greek	750
Latin	750
French	375
German	375
Italian	375
Mathematics, pure and mixed	1,000
Natural sciences	500
Moral sciences	500
Sanscrit	375
Arabic	375
	6,875

"It seems to us probable, that of the 6,875 marks, which are the maximum, no candidate will ever obtain half. A candidate who is at once a distinguished classical scholar and a distinguished mathematician will be, as he ought to be, certain of success. A classical scholar who is no mathematician, or a mathematician who is no classical scholar, will be certain of success if he is well read in the history and literature of his own country."

"When the result of the examination has been declared, the successful candidates will not yet be civil servants of the East India Company, but only civil servants elect. It appears from the 40th clause of the Act to be the intention of the legislature that, before they proceed to the East, there should be a period of probation and a second examination."

"This examination should, of course, be in the four branches of knowledge already mentioned as those to which the attention of the probationers

ought to be specially directed. Marks should be assigned to the different subjects, as at the first examination."

"The time of probation ought not, we think, to be less than one year, nor more than two years."

"When the marks have been cast up, the probationers who have been examined should be arranged in order of merit. All those who have been two years probationers, and who have, in the opinion of the examiners, used their time well and made a respectable proficiency, should be declared civil servants of the Company."

"Thus a salutary emulation will be kept up to the last moment."

"Early superiority in science and literature generally indicates the existence of some qualities which are securities against vice—industry, self-denial, a taste for pleasures not sensual, a laudable desire of honourable distinction, a still more laudable desire to obtain the approbation of friends and relations. We therefore believe that the intellectual test which is about to be established will be found in practice to be also the best moral test that can be devised."

With such evidence of the encouragement that is being given for the "advancement of learning" in other spheres, it should certainly not be unpleasing to the actuary to find that some efforts, however humble, have already been made for a similar purpose by the members of his own.

REPORTS OF ASSURANCE COMPANIES.

Amicable Society.—Account of the Receipts and Payments of the Corporation of the Amicable Society for a Perpetual Assurance Office, for the Year ending the 4th day of April, 1854.

Dr.	£.	s.	d.	£.	s.	d.
Balance of cash, April 4, 1853				12,152	0	10
Contributions received				69,809	6	5
Fines, penalties on readmission, &c.				183	13	6
Rents, viz.:—St. James's Street Estate	1,669	16	8			
„ for No. 13, Serjeants' Inn	291	5	0			
				1,961	1	8
Interest, viz.:—Dividends on Stock	15,059	1	9			
„ Long Annuities	13,106	5	0			
„ On loans on mortgages	4,797	10	0			
„ On loans on the Society's policies	1,465	5	9			
„ On Exchequer Bills paid off	232	12	0			
„ Discount on advances on claims	58	0	2			
				34,718	14	8
Repayment of loans on policies				4,177	0	0
New South Sea Annuities paid off				14,300	0	0
Sale of £22,000, Exchequer Bills				22,054	11	8
				£159,356	8	9

Or.	£.	s.	d.
Purchase of £10,000, Three and a Quarter per Cent. Annuities	10,375	0	0
Loans on mortgages	29,500	0	0
Loans on policies of the Society	8,154	6	8
Carried forward	£48,929	6	8

	£.	s.	d.	£.	s.	d.
Brought forward				48,029	6	8
Purchase of £5,000, Exchequer Bills				5,085	0	10
Claims of the year ending April 4, 1853				50,160	0	0
Ditto ditto, 1854				41,978	0	6
Contributions paid after death, and returned				26	6	0
Redemption of policies				2,154	13	7
Redemption of bonus additions to policies				74	14	2
Allowance to directors and auditors	650	0	0			
Salaries	1,250	0	0			
Medical adviser	100	16	0			
Commission to agents	170	19	9			
Rates, taxes, and insurance of Society's houses	228	15	0			
Paid on account of Society's Bill in Parliament	250	0	0			
Solicitor's bills	59	0	8			
Tradesmen's bills	180	8	4			
Miscellaneous expenses, viz.:—stamps, advertising, postage, messenger, &c.	338	3	11			
				3,228	3	8
Balance of cash, April 4, 1854				8,620	3	4
				£159,356	8	9

General Statement of the Affairs of the Society, as they stood on the 4th day of April, 1854.

	£.	s.	d.	£.	s.	d.
Present value of annual contributions for assurances on single lives for the whole of life, with participation in profits	624,743	13	2			
Present value of contributions for limited periods, in lieu of annual contributions for assurances on single lives for the whole of life, with participation in profits	1,984	0	5			
Present value of annual contributions for assurances on single lives for the whole of life, without participation in profits	11,645	13	2			
Present value of annual contributions for assurances on joint lives and contingencies	3,186	14	2			
Total value of contributions payable				641,560	0	11
Value of investments in Government securities	498,471	14	5			
Value of estates	55,358	9	0			
Lent on mortgage of estates	155,600	0	0			
Lent on mortgage of the Society's policies	36,738	6	8			
				746,168	10	1
Arrears, viz.:—Half a year's dividends on stock	10,312	13	7			
" Contributions due	4,161	3	4			
" Interest on loans	419	11	9			
" Rents of St. James's Street Estate, and						
" 13, Serjeants' Inn	563	1	8			
" Old South Sea Annuities redeemed	4,000	0	0			
				19,456	10	4
Balance of cash on April 4, 1854				8,620	3	4
Total assets				£1,415,805	4	8

	£.	s.	d.	£.	s.	d.
Present value of assurances on single lives for the whole of life, with participation in profits	1,262,642	18	7			
Present value of assurances on single lives for the whole of life, without participation in profits	12,462	10	0			
Present value of assurances on joint lives and contingencies	8,757	19	0			
Total value of sums assured (carried forward)				£1,283,863	7	7

	£.	s.	d.
Brought forward	1,283,863	7	7
Present value of additions to policies on the bonus plan . . .	1,529	8	7
Amount of claims allowed, but unpaid, April 4, 1854 . . .	20,972	5	0
Total liabilities	1,306,365	1	2
Balance in favour of the Society	109,440	3	6
	£1,415,805	4	8

Dividend per share, payable on claims of the year, commencing April 5, 1854, as directed by the charters and Act of Parliament of the Corporation, £225. 8*s*.

Anchor Assurance Company, August, 1853.—The total receipts for the last financial year amounted to £44,444. 17*s*. 11*d*., and the expenditure and investments to £38,932. 8*s*. 5*d*., leaving at bankers a balance of £5,512. 9*s*. 6*d*. The Company's assets and liabilities, balanced by the sum in bank, amounted to £116,191. 14*s*. 6*d*. The number of policies issued in the present financial year exceeds, by one third, the number of those issued in the previous year. The mortality has been under two thirds of that for which provision has been made by the Company's tables. As regards the departments of fire insurance, this may be termed the third year on which the directors have to report their experience. During this period, the new policies issued covered insurances to an amount exceeding £2,000,000, yielding upwards of £5,000 of new premiums; and which, when added to the premiums on policies granted in former years, gives a total of fire premium exceeding £12,000. The losses during the year in question amount to £6,978. 12*s*. 8*d*.; and as an index of the increase of the Company's business in the fire department, the Government duty for the three quarters ending Michaelmas, 1852, amounted to £1,959. 6*s*. 8*d*., while the duty for the succeeding Christmas quarter amounted to £1,511. 2*s*. 7*d*.

Clergy Mutual Assurance Society.—Report of the Directors for the Year ending May 31, 1854.—There have been 302 proposals for assurances received in the last year, of which 241 were upon life. Of the above 241 proposals upon life, 213 were accepted, to the amount of £184,919; and 28, amounting to £23,700, were declined. The number of life policies at present existing is 2,499, and the amount of annual premiums payable upon such policies is £58,735. 11*s*. 3*d*.; the total amount assured upon life by such policies is £2,272,409.

Twenty-four members holding life policies have died during the year, and the aggregate amount of the claims upon their assurances is £24,250. Assurances upon life, amounting to £17,650, have been purchased of members desirous of discontinuing them. Twenty-one members, having assurances for "provision during sickness," have received from the funds of the Society, according to their respective claims, the sum of £1,051. 10*s*.; and one policy has been discontinued.

Claims under endowment policies amounting to £6,806. 11*s*. 11*d*., under education annuities to £2,190, and under deferred and temporary annuities to £941. 19*s*., have been paid during the year.

The total income of the Society on the 1st of June, after reductions in annual premiums made from time to time in way of bonus, was £89,577. 4*s*. 11*d*.

After payment of all claims and expenses during the year, an increase

of £40,634. 10*s.* 4*d.* has been made to the property of the Society, making an aggregate capital of £619,121. 13*s.* 8*d.*

The number of applications made for life assurance policies during the past year is only five less than in the year previous to it; and the sum assured would have been about the same in 1853-4 as in 1852-3, but for the greater number of proposals which the directors were under the necessity of declining.

Clerical, Medical, and General Life Assurance Society.—Thirtieth Anniversary Meeting.—Report for the Year ending June 30th, 1854.—The number of policies issued was 558, by which the sum of £252,698 has been assured. The new premiums receivable therefrom amounted to £8,293 per annum. During the period embraced in this report, the sum of £50,448 has been carried to the consolidated fund, which is now increased to £972,766; and it is important to observe that this large amount, which has been accumulated for the benefit of the assured, is exclusive of the proprietors' fund. The annual income of the Society is now £148,698.

For the information of those who were not present at the last annual meeting, it may be interesting to state that the following important alterations were proposed and adopted:—1st. Granting whole world policies, which give the life assured permission to go at any time to all parts of the globe, on payment of a fixed but moderate rate of premium. 2nd. Affording perfect security to parties interested in policies on the lives of others, in the event of the life assured going abroad without their knowledge. 3rd. Rendering assurances made by persons on their own lives, and of twelve months' standing, valid, should death occur by duelling, suicide, &c.; whilst policies granted in favour of another, and assigned policies, are valid from the date thereof.

Friends' Provident Institution.—Twenty-first Report, 2nd of 8th Month, 1854.—220 new policies have been issued during the past year, the greater number of which, as heretofore, has been in the class of life assurance.

The following statement exhibits a summary of the transactions of the Institution from its commencement to the 20th of 11th Month, 1853, a period of 21 years:—

Receipts.	£.	*s.*	*d.*
Net amount of single premiums	118,340	15	11
Do. of periodical premiums	439,991	13	6
Interest on investments	156,666	1	9
Entrance money, &c.	1,438	11	5
	£716,437	2	7

Disbursements.	£.	*s.*	*d.*
Annuities	75,321	19	7
Endowment assurances	19,815	3	5
Deferred sums	4,974	5	7
Life assurances	150,757	15	0
Purchase of policies	30,076	19	5
Returns on policies lapsed by death of parties before taking effect	2,001	1	0
Property tax	3,229	13	6
Carried forward	£286,176	17	6

	£.	s.	d.
Brought forward	286,176	17	6
Banker's commissions	231	2	0
Printing and stationery	1,081	3	5
Other expenses (average, £861. 17*s.* per annum) . .	18,098	16	8
Balance, being amount of property on 20th of 11th Month, 1853	410,849	3	0
	£716,437	2	7

In their last report, the directors had the pleasure to announce that the sum of £57,485. 6*s.* 3*d.* had been apportioned among the members of Class IX., in respect of the profits accrued in the five years ending 20th of 11th Month, 1852. The calculations for the other classes have since been completed. In Class I. (deferred annuities) there was a deficiency of £328. 6*s.* 11*d.*, which, in accordance with the rules of the Institution, has been made good out of the surplus in the other classes. In Classes II., III., IV., V., and VII., as intimated in the last report, the surplus was not sufficiently large to make it expedient to declare a bonus.

The following statement shows the surplus realized and divided in the several classes:—

	Realised. £.	s.	d.	Divided. £.	s.	d.
Class II.	515	17	2			
„ III.	1,639	15	7			
„ IV., V.	74	15	11			
„ VI.	2,023	10	4	1,759	18	8
„ VII.	46	14	6			
„ VIII.	351	1	8	304	18	8
„ IX.	63,120	7	6	57,485	6	3
„ X.	1,353	13	5	290	14	7

Only 29 policies have been issued in Class X.; and, the average in so small a number being liable to great disturbance, it was not considered prudent to divide more than the sum mentioned.

The increased value of money, whilst tending on the one hand rather to diminish the amount of new business, has, on the other, afforded greater facilities for investment; and the directors have availed themselves of the opportunities which have been presented, of placing out on safe and profitable securities a large balance lately in the hands of the National Debt Commissioners.

The total amount advanced on loans to members, on the security of their policies, is upwards of £20,000.

Within the last few years it has been the policy of the legislature from time to time to limit the scope and operation of the Acts relating to Friendly Societies, under which this Institution was originally enrolled; and various privileges conferred by those Acts have been successively withdrawn. The frequent occurrence of these alterations in the law has been a source of much trouble and perplexity, involving the necessity, session after session, of watching Bills introduced into Parliament, which, more or less, affected the interests of the Institution. It is therefore a great satisfaction to the directors, that an Act has been obtained in the present session of Parliament (17 and 18 Vict., c. 56), whereby this Institution, in common with a few others similarly circumstanced, has ceased to be a "Friendly Society;" and whilst its constitution as a Mutual Assurance Association is effectually secured, it is now placed on an equal footing, as to the extent and character of its operations, with other Assurance Companies.

The number of deaths reported since the last general meeting is 30, viz: 1 deferred annuitant, 9 immediate annuitants, 1 child for whom an endowment had been provided, 18 persons on whose lives assurances had been effected in Class IX., and 1 survivorship annuitant, Class X.—making the total number of deaths since the commencement of the Institution, 356; of these, 203 have been in the department of life assurance, in which class the total amount paid or accrued to the representatives of deceased parties has been upwards of £157,100.

The total amount assured on policies in Class IX. existing at the date of this report is £1,397,364. 3*s*. 6*d*., exclusive of bonuses.

The total number of policies which have been granted, from the opening of the Institution in the 11th Month, 1832, to the 7th Month, 1854, both inclusive, is as follows:—

		Policies.
Class I.	Deferred annuities	137
„ II.	Deferred annuities, with a condition annexed, making the premiums returnable without interest, on the death of the annuitant before the assurance takes effect	138
„ III.	Immediate annuities (averaging £23. 15*s*. 2*d*. each)	386
„ IV.	Endowments for children, payable at 14	4
„ V.	Endowments payable at 21 or 25	80
„ VI.	Endowments payable at 14, 21, or 25, the premiums returnable as in Class II.	333
„ VII.	Deferred sums	69
„ VIII.	Deferred sums, the premiums returnable as in Class II.	113
„ IX.	Life assurances (averaging about £660 each)	2,713
„ X.	Survivorship annuities	29
	Total number of policies	4,002

A Statement of Receipts and Payments from the 20th of 11th Month, 1852, to the 20th of 11th Month, 1853.

Receipts.	£.	*s*.	*d*.
Balance on the 20th of 11th Month, 1852	388,712	6	9
Single premiums received on assurances (agents' commission deducted)	7,431	17	8
Periodical premiums received on assurances (agents' commission deducted)	32,218	18	4
Interest on moneys invested with the National Debt Commissioners and on other securities	16,750	8	4
	£445,113	11	1

Payments.	£.	*s*.	*d*.
Annuities, Class I.	580	0	0
„ Class II.	740	0	0
„ Class III.	6,079	18	4
„ Class X. (survivorship)	65	0	0
Endowment assurances, with bonuses, Class V.	200	0	0
Ditto ditto, Class VI.	4,069	11	8
Deferred sums, assured in Class VII.	100	0	0
Ditto ditto Class VIII.	396	1	3
Life assurances, Class IX., on 15 lives	16,665	17	4
Premiums returned on policies in Classes II. and VI., lapsed by the death of parties before taking effect	166	3	4
Policies and bonuses purchased by the Institution	2,033	11	9
Printing and stationery	44	9	0
Bankers' commission	5	19	2
Income tax charged upon interest on investments	411	7	9
Carried forward	£31,557	19	7

	£.	s.	d.
Brought forward	31,557	19	7
Directors' fees, being the amount voted by the general meeting of 1853, for two years ending 20th of 11th Month, 1853	1,000	0	0
Expenses of management, including rent, salaries, postages, &c.	1,254	0	10
Actuary's fees, for valuation and apportionment of profits	420	0	0
Receipt stamps	32	7	8
Balance in favour of the Institution on the 20th of 11th Month, 1853	410,849	3	0
	£445,113	11	1

A Statement of Funds and Effects belonging to the Institution on the 20th of 11th Month, 1853.

Property belonging to the Institution.	£.	s.	d.
Principal invested with the Commissioners for the Reduction of the National Debt	23,855	7	11
Principal invested on mortgage securities	357,705	0	0
Interest accrued thereon	4,699	7	6
Loans on policies	17,192	5	7
Interest accrued thereon	226	12	9
Deposit with Overend, Gurney, & Co.	3,200	0	0
Interest accrued thereon	81	8	3
Policy and receipt stamps on hand	232	5	5
Balance at bankers in London	4,086	1	7
Balance in the hands of secretary	96	14	7
Balance due from agents and assurers	43	1	5
	£411,418	5	0

Debts owing by the Institution.	£.	s.	d.
Assurances fallen in, but not yet payable	507	8	0
Balance due to agents and assurers	61	14	0
Balance in favour of the Institution on the 20th of 11th Month, 1853	410,849	3	0
	£411,418	5	0

Law Life Assurance Society.—Summary of the Accounts of the Society for the Year 1850.

	£.	s.	d.	£.	s.	d.
Balance of the guarantee fund on 31st Dec., 1849	457,229	6	6			
Ditto assurance fund on same date	2,945,497	8	9			
Together				3,402,726	15	3
Received during the year 1850:—						
New premiums	14,723	3	9			
Renewal premiums	297,606	19	2			
Profit and loss	97,879	8	2			
				410,209	11	1
				3,812,936	6	4
Paid during the year 1850:—						
Claims on death—Sums assured	166,066	0	0			
„ Bonuses	48,007	0	0			
For surrenders of policies	13,045	2	8			
Charges for management	6,200	8	11			
				233,318	11	7
Total balance (the whole of which is invested in Government or real securities), being the amount of assets on 31 Dec., 1850				£3,579,617	14	9

The Society has, since its establishment in 1823, paid the sum of £2,427,000 upon the deaths of parties whose lives were assured. Three divisions of profits have been made, and reversionary bonuses amounting to £1,929,085 have been added to the several policies.

The Mutual Life Assurance Society.—Report of the Directors for the Year ending 31st December, 1850.—In the course of the year 200 proposals have been made to the Society, for assuring the aggregate sum of £120,213. Of this number 155 have been accepted, assuring the sum of £86,015, the first premiums on which amount to £2,934. 12*s.* 5*d.*; at the end of the year, 16 proposals, for assuring £8,150, were standing over for completion or further consideration; and 29 proposals, for assuring £26,048, have been declined or otherwise not proceeded with. The total number of policies granted since the commencement of the Society to the 31st December last is 1,786, and the sums assured thereby £1,256,628.

Comparing the new business of the last with that of the preceding year, it will be noticed that, whilst the policies effected are five less in number, the amount assured is a trifle more, though at a somewhat lower premium, owing to the parties assured being on an average of a younger age.

The policies cancelled in the year, either by claims on death or other causes, are 90 in number, assuring the sum of £65,300; and the amount of annual premiums written off, and of commuted premiums which have expired, is £2,846. 3*s.* 10*d.* The number of policies which have become claims by death is 21, assuring the total sum of £15,853. The total amount of additions on these claims is £3,305. There remained in force at the close of the year 1,312 policies, assuring altogether the sum of £848,805, and producing in annual premiums an income of £28,251. 13*s.* 9*d.* The expenditure has been less than in the preceding year by £484. 3*s.* 6*d.*, and than in the year 1848 by £701. 17*s.* 1*d.* After providing for all liabilities, the directors have to state that a surplus remained, on the 31st December last, of £81,290.

General Cash Account for the Year ending December 31st, 1850.

Dr.	£.	*s.*	*d.*
Balance in hand, January 1st., 1850, viz.—Cash, £6,369. 16*s.* 7*d.*; policy stamps, £112. 15*s.* 0*d.*	6,482	11	7
Premiums received on 155 new policies, assuring £86,015	2,934	12	5
Ditto on policies renewed	25,602	4	6
Policy stamps on new assurances	231	0	0
Dividends on stock in the funds	9,862	5	10
Interest on mortgages	1,860	6	11
Ditto on loans, bankers' balance, and half yearly premiums	238	6	6
Proceeds of £10,000, New 3¼ per Cent. Annuities, sold	9,900	0	0
Repayment of mortgages	1,600	0	0
Repayments of loans to members	135	0	0
Commissions on reassurances, &c.	5	8	5
	£58,851	16	2

Cr.	£.	*s.*	*d.*
The amount paid for 21 claims by death	17,953	0	0
Additions to these claims	3,500	0	0
Amount paid for 33 policies surrendered	2,204	6	10
Cash invested, viz.—			
On mortgage	9,000	0	0
In purchase of £10,000, New 3¼ per Cent. Annuities	9,725	0	0
On loan to members, on the security of their policies	784	0	0
On the Society's house and premises	18	6	9
Policy stamps £260 10 0			
Receipt stamps 10 10 6			
	271	0	6
Carried forward	£43,455	14	1

	£.	s.	d.	£.	s.	d.
Brought forward				43,455	14	1
Commission				270	4	4
Reassurances				29	9	4
Local boards, Gloucester (final payment for all local boards)				180	0	0
General disbursements, viz.—						
Law charges	107	8	3			
Advertisements	278	19	10			
Printing and stationery	120	3	9			
Rent and insurance	169	8	10			
Directors, trustees, and auditors	796	4	0			
Salaries to office establishment, viz.—Actuary and clerks	1,118	15	0			
Medical officers	210	0	0			
House expenses	60	0	0			
Town agent	93	15	10			
Sundry disbursements, viz.—Wages, rates, taxes, postage, country medical fees, repairs, and other incidental expenditure for the year	509	13	4			
				3,464	8	10
Stamps in hand				142	5	0
Balance in the bank				11,309	14	7
				£58,851	16	2

Balance-sheet, 31*st December*, 1850.

Dr.	£.	s.	d.
The present ascertained value of £848,805, sums assured under 1,312 policies, being all the existing policies	481,122	4	11
The amount of claims by death allowed, but unpaid, on the 31st December, 1850	2,587	0	0
Surplus	81,290	0	0
	£564,999	4	11

Cr.	£.	s.	d.	£.	s.	d.
The present ascertained value of £28,251. 13*s.* 9*d.* per annum, life premiums payable under the existing policies				412,503	0	0
The present value of the Society's assets, viz.—Stock in the funds, consisting of £5,760 per annum, Long Annuities; £4,110, January and July Annuities; £5,000, 3½ per Cent. Annuities	81,229	18	0			
Cash advanced on mortgages and bonds	51,466	13	4			
Cash advanced on loan to members, on their policies	3,353	0	0			
Value of the Society's house and furniture	2,862	8	10			
				138,912	0	2
Interest on loans and mortgages, and premiums due but unpaid, 31st December, 1850				2,132	5	2
Stamps in hand				142	5	0
Balance at bankers, 31st December, 1850				11,309	14	7
				£564,999	4	11

The Mutual Life Assurance Society.—Eighteenth Annual Report of the Directors for the Year ending 31*st December*, 1851.—The total number of proposals laid before the board during the year has been 167, for assuring the sum of £83,771. Of this number 126, for £60,946, have been completed; 9 proposals, for £4,600, were standing over for completion or further consideration at the end of the year; and 32 proposals, for assuring £18,225, have been declined or not proceeded with.

The claims by death during the year have been remarkably small; being only £10,050, on a total sum assured varying from £848,805 at the beginning to £874,347 remaining in force at the end of the year. This is considerably less than the mortality on which the premiums are computed,

and is only about four fifths of the average amount of claims for the preceding four years.

The policies remaining in force on the 1st January, 1852, were 1,367, assuring £874,347, and producing an annual income from premiums of £29,058. 8*s*. 9*d*.

The following short table has been constructed to show the progress of the Society, at equal intervals of six years—

Year.	Assurances remaining in force.		Income from Premiums receivable.	Value of invested Capital.
	No. of Policies.	Amount Assured.		
		£.	£. s. d.	£.
31st December, 1839	505	312,521	10,790 19 8	27,045
" " 1845	882	610,175	20,804 13 5	86,816
" " 1851	1,367	874,347	29,058 8 9	174,170

The total assets of the Society on the 31st December last amounted to £604,505. 18*s*. 5*d*. (including the value of future premiums receivable), and the total liabilities to £514,933. 18*s*. 5*d*. (including the present value of all the sums assured), leaving a clear surplus for division of £89,572.

It may be proper to mention, that the whole of the invested capital of the Society is either in the Government funds, or secured on the mortgage of freehold or other available property—none being lent out on personal security; and that the amount of it has increased during the past year from £152,496. 11*s*. to £174,170. 10*s*. 6*d*.

General Cash Account for the Year ending 31st December, 1851.

Dr.	£.	s.	d.
Balance at the bank, January 1st, 1851	11,309	14	7
Premiums received on 126 new policies, assuring £60,946	2,811	3	10
" " policies renewed	26,775	2	4
Policy stamps on new assurances	183	12	6
Dividends on stock in the funds	9,011	15	4
Interest on mortgages and bonds	2,961	11	1
" on loans to members	162	2	0
" on half yearly premiums and bankers' balances	157	0	3
Fines	8	10	9
Proceeds of the sale of £1,500 per annum, Long Annuities, and £1,400, Annuities for Terms (January and July)	20,916	4	8
Repayment of mortgages	1,100	0	0
Repayments of loans to members	431	0	0
Loan from London Joint Stock Bank	2,000	0	0
Commissions on reassurances, &c.	21	16	4
	£77,849	13	8

Cr.	£.	s.	d.	£.	s.	d.
Amount paid for 14 claims by death				5,200	0	0
Additions to these claims				1,724	1	4
Amount paid for 25 policies surrendered				1,148	12	0
Cash invested, viz.—						
On mortgages and bonds	60,850	0	0			
On loans to members, on the security of their policies	528	0	0			
				61,378	0	0
Repayment of loan to London Joint Stock Bank	2,000	0	0			
Interest on ditto	3	5	9			
				2,003	5	9
Carried forward				£71,453	19	1

	£.	s.	d.	£.	s.	d.
Brought forward				71,453	19	1
Policy stamps				158	11	0
Commission				268	6	5
Reassurances				29	9	4
General disbursements, viz.—						
Directors, trustees, and auditors	806	14	0			
Advertisements	305	5	9			
Printing	130	0	0			
Stationery	51	15	11			
Town agent	82	2	0			
Rent and insurance	169	8	10			
Medical officers	210	0	0			
Ditto, on altering dates of payments to 31st December	160	19	0			
Country medical fees	55	19	0			
Salaries to actuary and clerks	862	18	8			
House expenses	60	0	0			
Minor disbursements, viz.—Wages, rates, taxes, postage, repairs, and sundry expenses	401	15	0			
				3,276	18	2
Balance in the bank, 31st December, 1851				2,662	9	8
				£77,849	13	8

Balance-sheet, 31st December, 1851.

Dr.	£.	s.	d.
Present ascertained value of £874,347, sums assured under 1,367 policies, being all the existing policies	507,029	18	5
Amount of claims by death allowed, but unpaid, on 31st Dec. 1851	7,904	0	0
Surplus	89,572	0	0
	£604,505	18	5

Cr.	£.	s.	d.	£.	s.	d.
Present ascertained value of £29,058. 8s. 9d. per annum, premiums payable under the existing policies				430,335	8	0
Present value of the Society's assets, viz.—Stock in the funds, consisting of £4,260 per annum, Long Annuities; £2,710, Annuities for Terms (Jan. and July); £5,000, Three and a Quarter per Cent. Annuities	52,037	11	0			
Cash advanced on mortgages and bonds	111,216	13	4			
Cash advanced on loan to members on their policies	3,450	0	0			
Value of the Society's house and furniture	2,692	8	10			
				169,396	13	2
Interest on loans and mortgages, and premiums due but unpaid 31st December, 1851				1,994	4	1
Value of stamps in hand				117	3	6
Balance at bankers, 31st December, 1851				2,662	9	8
				£604,505	18	5

The Mutual Life Assurance Society.—Report for the Year ending 31st Dec., 1852.—The total number of proposals laid before the board during the year was 228, for assuring the sum of £126,329. Of this amount, 174 proposals, for assuring £93,681, were completed; 43 proposals, for assuring £27,248, were either declined or not proceeded with; and 11 proposals, for assuring £5,400, were standing over for further consideration at the close of the year.

The following statement will show the relative proportion of assurances in force at the end of 1852, as compared with the end of 1851:—

	No.	Sums Assured.			Premiums.		
		£.	s.	d.	£.	s.	d.
Assurances in force, 31st December, 1851	1,367	874,347	0	0	29,058	8	9
Written off in 1852	49	37,449	0	0	1,260	16	5
	1,318	836,898	0	0	27,797	12	4
New assurances in 1852	174	93,681	0	0	3,374	7	11
Additional sums assured by policies in force for increasing sums	..	449	10	0			
Assurances remaining in force, 31st December, 1852	1,492	931,028	10	0	31,172	0	3

Of the policies written off, only 11 were for claims by death, assuring altogether £8,350, and which, if compared with the amount assured on the 30th June, £909,583, will be found to represent a rate of mortality of little more than 9 per 1,000; 12 policies, assuring £8,500, were purchased by the Society; and the remaining 26, for £20,599, were either forfeited or expired.

The directors have taken advantage of the high price of the public funds during the past year to realize a considerable portion of the Society's stock, and to invest it on mortgages of land and other available securities. It is satisfactory to state that, after the payment of all claims and expenses, nearly £25,000 have been added to the invested capital of the Society during the past year, making the total very close upon £200,000; and if the securities were valued at the present market price, it would be considerably above that amount. The income of the Society has increased during the same period more than £3,000 per annum, and now amounts to upwards of £89,300 per annum. After a careful valuation of all the assets and liabilities of the Society, the clear divisible surplus on the 31st December last is ascertained to be £99,211. 2*s.*, as will appear by the balance-sheet.

The General Cash Account for the Year ending 31st December, 1852.

Dr.	£.	s.	d.
Balance at the bank, January 1st, 1852	2,662	9	8
Premiums received on 174 new policies, assuring £93,681	3,276	2	4
Ditto on policies renewed	28,355	7	2
Policy stamps on new assurances	263	0	0
Dividends on stock in the funds	5,306	16	5
Interest on mortgages and bonds	5,468	19	4
Ditto on loans to members	204	6	0
Ditto on half yearly premiums and bankers' balances	33	17	2
Fines, £92. 13*s.* 11*d.*; commission on reassurances, &c., £18. 12*s.* 2*d.*	111	6	1
Proceeds of sale of £3,260 per annum, Long Annuities; £910 per annum, Annuities for Terms; and £5,000 stock, New 3½ per Cents.	33,546	14	6
Repayments of mortgages	12,333	6	8
Repayments of loans to members	207	0	0
Costs received on behalf of the Society's solicitor, on the repayment of a mortgage	199	10	9
	£91,968	16	1

Cr.	£.	s.	d.	£.	s.	d.
Amount paid for 11 claims by death	9,150	0	0			
Additions to these claims	1,376	0	0			
				10,526	0	0
Amount paid for 12 policies surrendered				805	10	0
Carried forward				£11,331	10	0

	£.	s.	d.	£.	s.	d.
Brought forward				11,331	10	0
Cash invested, viz.—						
On mortgages and bonds	71,756	6	3			
On loans to members, on the security of their policies	1,458	0	0			
				73,214	6	3
Policy stamps, £257. 10*s.*; commission, £403. 9*s.* 8*d.*				660	19	8
Reassurances				29	9	4
Premiums returned, viz.—Original extra premiums	20	0	0	123	19	3
" Renewal premiums	98	19	3			
General disbursements, viz.—	£.	s.	d.			
Directors, trustees, and auditors	809	17	0			
Advertisements, printing, and stationery	602	6	2			
Rent and insurance	169	8	10			
Medical officers, and country medical fees	296	12	6			
Salaries to actuary and clerks	905	0	0			
House expenses	60	0	0			
Minor disbursements, viz.—Receipt stamps, £54. 14*s.* 6*d.*; agencies, circulars, postage of prospectuses, &c., £350. 0*s.* 8*d.*; returned fines, &c., £39. 13*s.* 8*d.*; other expenses, rates, taxes, &c., £285. 6*s.* 10*d*	729	15	8			
				3,573	0	2
Costs paid to the Society's solicitor (as per contra)				199	10	9
Balance in the bank, 31st December, 1852				2,836	0	8
				£91,968	16	1

Balance-sheet, 31st December, 1852.

Dr.	£.	s.	d.
Present ascertained value of £931,028. 10*s.*, sums assured under 1,492 policies existing	549,093	0	11
Amount of claims by death allowed, but unpaid, 31st December, 1852	7,919	0	0
	557,012	0	11
Surplus	99,211	2	0
	£656,223	2	11

Cr.	£.	s.	d.
Present ascertained value of £31,172. 0*s.* 3*d.* per annum, premiums payable under the existing policies	457,067	8	0
Present value of the Society's assets, viz.—			
£1,000 per annum, Long Annuities; and £1,800 per annum, Annuities for Terms of Years (Jan. and July)	£17,570 10 0		
Cash advanced on mortgages and bonds	169,883 6 8		
Cash advanced on loan to members on their policies	4,701 0 0		
Value of the Society's house and furniture	2,522 8 10		
	194,677	5	6
Interest on loans and mortgages, and premiums due but unpaid, 31st December, 1852	1,530	15	3
Value of stamps in hand	111	13	6
Balance at bankers, 31st December, 1852	2,836	0	8
	£656,223	2	11

The Mutual Life Assurance Society.—Report for the Year ending 31st December, 1853.—The number of proposals for assurance laid before the board during the past year has been 253, for assuring the total sum of £137,892. 2*s.*; of this number, 205, for assuring the sum of £112,144. 2*s.*, have been completed; 15, for assuring £9,999, have been declined; and the remaining 33, for assuring £15,749, have been either not proceeded with, or were standing over for completion or further consideration at the close of the year.

The increase of business during the year will be seen in the following statement:—

	No.	Sums assured.			Premiums.		
		£.	s.	d.	£.	s.	d.
Assurances in force 31st December, 1852	1492	931,028	10	0	31,172	0	3
Written off in 1853	46	25,848	4	0	1,275	3	4
	1446	905,180	6	0	29,896	16	11
New assurances in 1853	205	112,144	2	0	4,027	19	9
Increase on policies for assuring increasing sums	..	47	10	0			
Assurances in force 31st December, 1853	1651	1,017,371	18	0	33,924	16	8

In the two years ending on the 31st December last, the assurances in force have increased, in round numbers, from £874,000 to £1,017,000, the total income from £36,200 to £42,400 per annum, and the invested capital from £174,000 to close upon £213,000. The claims by death paid during the same period, independent of bonuses, have only averaged £13,250 per annum.

After a very careful investigation, and the usual valuation of the assets and liabilities of the Society, the surplus profits on the 31st December last are ascertained to be £108,082, which will allow to every member an increase on his bonus of the preceding year.

The directors have recently taken measures for extending the usefulness of the Society by the adoption of a set of tables prepared by the actuary, for the transaction of business on the non-profit system. They have further, after a careful investigation by the actuary into the rates of mortality amongst assured lives, materially lowered the premiums for assurances on joint lives, survivorships, and for terms of years, &c. Another important subject has recently engaged the attention of the directors. Under the conviction that many money transactions, involving the necessity of a life assurance, fail on account of an apprehension on the part of the lender that the borrower may go beyond the prescribed limits for foreign residence, and so cause a forfeiture of the policy, they have taken the subject of foreign risks into consideration, and have come to the conclusion that, in cases in which they can be satisfied that the party whose life is to be assured has no present intention of going abroad, beyond the prescribed limits, the Society may safely, for a very trifling extra premium, grant him a license to go to and reside in any part of the world, and so relieve the lender from all anxiety on the subject.

The General Cash Account for the Year ending December 31st, 1853.

Dr.				£.	s.	d.
Balance in the bank, January 1st, 1853				2,836	0	8
Premiums received on 205 new policies, assuring £112,144. 2s.				3,956	2	6
" " policies renewed				30,207	2	10
Policy stamps on new assurances				115	10	0
Dividends on stock in the funds				873	15	0
Interest on mortgages and bonds	£7,317	1	5			
" on loans to members	190	9	4			
" on half yearly premiums, bankers' balances, &c.	157	13	3			
				7,665	4	0
Fines				36	12	6
Commission on reassurances, loans, &c.				3	12	1
Carried forward				£45,693	19	7

	£.	s.	d.	£.	s.	d.
Brought forward				£45,693	19	7
Proceeds of the sale of—						
£1,000 per annum, Long Annuities	6,429	9	0			
£1,800 per annum, Annuities for Terms (January and July)	11,098	12	4			
				17,528	1	4
Repayments of mortgages and bonds				51,083	6	8
„ of loans to members				270	0	0
				£114,575	7	7

Cr.	£.	s.	d.	£.	s.	d.
Amount paid for 24 claims by death	18,049	0	0			
Additions to these claims	6,510	0	0			
				24,559	0	0
Amount paid for 6 policies surrendered				265	0	0
Cash invested, viz.:—						
On mortgages, bonds, and loans	71,075	3	3			
On loans to members, on the security of their policies	773	0	0			
				71,848	3	3
Policy stamps				134	0	0
Reassurances				29	9	4
Commission				426	15	1
Premiums returned, &c.—Renewals, £18. 19*s.*; interest, £4. 10*s.* 9*d.*				23	9	9
General disbursements, viz.:—						
Directors, trustees, and auditors	807	15	0			
Advertisements, printing, and stationery	427	3	11			
Rent and insurance	169	8	10			
Medical officers, £105; country medical fees, £105	210	0	0			
Salaries to actuary and clerks	955	0	0			
House expenses, alterations in offices, &c.	128	7	7			
Minor disbursements, viz.—Agency expenses, rates, taxes, postage of prospectuses and circulars, &c.	974	5	4			
				3,672	0	8
Balance in the bank, 31st December, 1853	13,122	18	1			
Cash in house	494	11	5			
				13,617	9	6
				£114,575	7	7

Balance-sheet, 31*st December*, 1853.

Dr.	£.	s.	d.
Present ascertained value of £1,017,371. 18*s.*, sums assured under 1,651 policies existing	594,404	5	3
Amount of claims by death allowed, but unpaid, 31st Dec., 1853	7,888	0	0
	602,292	5	3
Surplus	108,082	0	0
	£710,374	5	3

Cr.	£.	s.	d.	£.	s.	d.
Present ascertained value of £33,924. 16*s.* 8*d.* per annum, premiums payable under the existing policies				497,485	16	0
Present value of the Society's assets, viz.:—						
Cash advanced on mortgages and bonds	£189,931	1	3			
„ „ on loan to members, on their policies	5,204	0	0			
Value of the Society's house and furniture	2,352	8	10			
Interest on loans and mortgages, and premiums due but unpaid, 31st Dec., 1853	1,783	9	8			
Balance at bank, 31st Dec., 1853	13,617	9	6			
				212,888	9	3
				£710,374	5	3

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

First Ordinary Meeting, Session 1854-55.—*Monday, 27th November*, 1854.

E. J. Farren, Esq., Vice President, in the Chair.

The minutes of the last annual general meeting were read and confirmed.

The Secretary announced several donations to the library.

A catalogue of the library, prepared by Mr. Wheatley, was laid on the table, and the Chairman explained to the meeting the principles upon which it was compiled. It was arranged in one general alphabet, as being the most useful, and the readiest form for reference; the library itself being arranged in classes on the shelves. To render it, as nearly as possible, a correct mirror of the contents of the library, each work had but one principal descriptive entry. The shelf-mark was confined to this main entry: duplicate references leading to frequent and unavoidable errors. This entry was under the *author's* name, when given on the title-page or otherwise known; it being the only arrangement which allowed one general rule to be followed throughout the catalogue. Anonymous works, whose authors' names were unknown, were placed under the subject to which they related; or if on no definite subject, under their usual, popular, or arbitrary title. Any articles placed under a writer's name, of which he was not the author, but which were anonymous answers to or criticisms on his work—anonymous memoirs, under their subjects—collections, under their editors—and any entries whatever in which the heading name was not that of the author—were distinguished by a certain mark following the name. The cross-references, or secondary entries, were made—1, From the subjects of biographies, to the authors; 2, from the principal anonymous and pseudonymous works, to the writer's real name; 3, from works included in, or noticed in the title-pages of, other publications, to those publications. To obviate the imperfections necessarily attendant on an alphabetical arrangement of authors only, and for the greater facility of reference, short classifications were introduced, of the subjects on which the books in the library treated, referring to the names of the authors in the same general alphabet; thereby uniting the advantages of the alphabetical and classified systems, and acting in some measure as a key to the prevailing character of the library, by showing its riches or defects in particular classes. The names to be referred to, or cross-references, were singly underlined; but when the reference was only collateral to another heading, and not direct to an author, it was distinguished by being doubly underlined. Where no place of publication was given, London was to be understood.

The following candidates, duly nominated at the last ordinary meeting, were elected Associates of the Institute, viz.:—

R. P. Hardy,	G. J. Jago,
Arthur Jago,	William White.

Mr. Jellicoe, Vice President, read a paper "On the relation which should obtain between the amount assured upon lives, and the sum reserved at the expiration of given terms to meet it."

Second Ordinary Meeting, Session 1854-55.—*Monday, 8th January*, 1855.

Charles Jellicoe, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The Chairman announced the result of the annual examinations in London and Edinburgh, by which it appeared that five candidates offered themselves for matriculation examination in London, and two passed, viz.:—Almaric Rumsey, with great credit; and Henry Bishop, very creditably. Four presented themselves for the second year's examination, and all passed; their names appearing in the following order of merit:—

J. B. Allan,	Robert Hatton, and
Samuel Younger,	D. R. C. Robinson.

The two first, Messrs. Allan and Younger, passed with great credit; the remaining two, Messrs. Hatton and Robinson, very creditably.

In Edinburgh, four candidates came forward for the matriculation examination, and two passed, viz.—A. H. Turnbull, very creditably, and H. J. Johnston, creditably; and one for the second year's examination, viz.—W. F. Birkmire, who passed very creditably.

The third year's examination, entitling those who pass to a certificate of competency from the Institute, is to take place in London, for the first time, in December, 1855.

The examiners remark that some of the candidates, both here and in Edinburgh, have put down answers to questions without giving the methods of solution. Although it is evident that this frequently arises from mere misapprehension, the examiners think it should, nevertheless, be ruled on future occasions, that no marks will be given for answers unaccompanied by the methods of solution.

The following candidates, duly nominated at the last ordinary meeting, were elected Associates of the Institute:—

J. A. Avarne,	G. M. Ogilvie,
Adam Crawford,	Charles Price,
D. L. Crawford,	C. F. Racine,
Stewart Helder,	Almaric Rumsey,
H. J. Johnston,	A. H. Turnbull,
William Lance,	William Wallace,
Charles Mallandain,	T. Y. Wardrop,

J. V. Yatman.

Mr. Farren, Vice President, read a paper "On the improvement of life contingency calculation."

Third Ordinary Meeting, Session 1854-55.—*Monday, 29th January,* 1855.

E. J. FARREN, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The following candidates, duly nominated at the last ordinary meeting, were elected Members of the Institute, viz.:—

Fellows.

J. S. Crocker, | Edward Sang.

Associates.

M. A. Black,	David Ritchie,
John Finlaison, Jun.	J. S. Smith,
C. P. Mann,	Cornelius Walford, Jun.

Mr. Samuel Brown, one of the Honorary Secretaries, read a paper "On the rate of sickness and mortality amongst the members of Friendly Societies in France."

Fourth Ordinary Meeting, Session 1854-55.—*Monday, 26th February,* 1855.

CHARLES JELLICOE, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

Benjamin Gompertz, Esq., F.R.S., was elected an Honorary Member.

The Chairman announced that Mr. Archibald Day, an examined Associate, had been admitted a Fellow in conformity with Rule V. of the Constitution and Laws.

G. W. Berridge, duly nominated at the last ordinary meeting, was elected an Associate of the Institute.

A discussion then took place "On the methods pursued at the present day for estimating the value of contingent reversionary interests," which was opened by Mr. Robert Tucker, Vice President.

THE

ASSURANCE MAGAZINE,

AND

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OF THE

INSTITUTE OF ACTUARIES.

*On the Comparison of various Tables of Annuities.** *By* J. W. Lubbock, Esq. (*now* Sir J. W. Lubbock, Bart.), B.A., F.R. & L.S., *of Trinity College, Cambridge.*

[Extracted, by permission of the Author, from the *Transactions of the Cambridge Philosophical Society.*]

1. A SHORT time back I transmitted to the Philosophical Society of Cambridge some remarks upon the construction of tables of annuities: my object in that paper was to show how the probabilities upon which annuities depend should be deduced from tables of mortality, and I gave in illustration some tables of annuities calculated from observations of the mortality at Chester, by Dr. Haygarth, which appear to have been made with very great care. I have since compared these tables with a great many others, and I now present the result of this comparison.

2. Very few registers of mortality give the deaths at every year throughout life; they generally give the deaths between birth and 5, 5 and 10, 10 and 20, 20 and 30, and so on for every decade. When the deaths are given between birth and 5, the *living* at 5, at 20, 30, &c., are known, and in order to form a complete table of mortality it is necessary to interpolate the number of living at each intermediate age.

If the probability of an individual aged m years living n years be called $p_{m,n}$, if r is the rate of interest, and if the same hypo-

* This is the paper referred to in the note at page 205, vol. iv., of this *Journal.*

thesis of probability be adopted as in my former paper, which amounts to increasing by 1 the deaths at every age,

$$p_{m,n} = \frac{\text{living at } n+101-n}{\text{living at } m+101-m},$$

the value of a payment of unity after n years is $\frac{p_{m,n}}{(1+r)^n}$, and the value of an annuity is $\Sigma\left\{\frac{p_{m,n}}{(1+r)^n}\right\}$, m being constant in this expression, and n variable.

Instead however of interpolating values of $p_{m,n}$ between those values which are known, it is better to interpolate at once between the values of $p_{m,n} \times (1+r)^{-n}$ which are given; but even this labour is unnecessary, because $\Sigma \frac{p_{m,n}}{(1+r)^n}$, or the value of the annuity, is a function of those terms only of the series which are given.

Let $y_0, y_i, y_{2i}, \ldots y_{ni}, y_{(n+1)i}$, &c., be successive values of any variable y,

$$y_0 = y_0$$

$$y_i = y_0 + i\Delta y_0 + \frac{i.i-1}{1.2}\Delta^2 y_0 +, \text{ &c.},$$

$$y_{2i} = y_0 + 2i\Delta y_0 + \frac{2i.2i-1}{1.2}\Delta^2 y_0 +, \text{ &c.},$$

$$y_{ni} = y_{ni}$$

$$y_{(n+1)i} = y_{ni} + i\Delta y_{ni} + \frac{i.i-1}{1.2}\Delta^2 y_{ni} +, \text{ &c.},$$

$$y_0 + y_i + y_{2i} + y_{3i} \ldots + y_m + y_{(n+1)i} +, \text{ &c.} + y_{(mni-1)}$$
$$= n(y_0 + y_{ni} + y_{2ni} \ldots\ldots\ldots + y)$$
$$+ i\{1+2+3\ldots\ldots n-1.\}\{\Delta y_0 + \Delta y_{ni} +, \text{ &c.} + \Delta y_{(n-1)ni}\}$$
$$+ \frac{i}{1.2}\{1.i-1+2.i-2+, \text{ &c.} + n-1.1\}\{\Delta^2 y_0 + \Delta^2 y_{ni} +, \text{ &c.}\},$$

$$\Delta y_0 + \Delta y_{ni} +, \text{ &c.} + \Delta y_{(n-1)ni} = y_{mni} - y_0,$$

$$\Delta^2 y_0 + \Delta^2 y_{ni} \ldots\ldots + \Delta^2 y_{(n-1)ni} = \Delta y_{mni} - \Delta y_0,$$

$$\Delta^3 y_0 + \Delta^3 y_{ni} \ldots\ldots + \Delta^3 y_{(n-1)ni} = \Delta^2 y_{mni} - \Delta^2 y_0.$$

When $ni = 1$, $i = \frac{1}{n}$, the sum of the series is equal to

$$n(y_0 + y_1 + y_2, \text{ &c.} \ldots\ldots + y_{n-1}) + \frac{1}{n}\{1+2+3\ldots\ldots+n-1\}\{y_n - y_0\}$$
$$- \frac{1}{1.2n^2}\{1.n-1+2.n-2+3.n-3\ldots\ldots+n-1.1\}\{\Delta y_n - \Delta y_0\}$$
$$+ \frac{1}{1.2.3n^3}\{1.n-1.2n-1+2.n-2.2n-2\ldots\ldots$$
$$+ n-1.1.n+1\}\{\Delta^2 y_n - \Delta y_0\} +, \text{ &c.}$$

The coefficient of $\Delta^q y_m - \Delta^q y_0$ is equal to the coefficient of x^{q-1}

in the development of $\frac{(1+x)^{ni}-(1+x)}{1-(1+x)^i}$: or, in other words, if this coefficient be called z_p $\frac{x\{(1+x)^{ni}-(1+x)\}}{1-(1+x)^i}$ is the generating function of z_q; and since $ni=1$,

$$\frac{(1+x)^{ni}-(1+x)}{1-(1+x)^i}=\frac{x}{(1+x)^i-1}$$

$$=\frac{1}{i}-\frac{i-1}{2i}x+\frac{i-1.i+1}{12i}x^2-\frac{i-1.i+1}{24i}x^3+, \&c.$$

$$=n+\frac{n-1}{2n}x-\frac{n-1.n+1}{12n}x^2+\frac{n-1.n+1}{24n}x^3+, \&c.$$

The sum of the series is

$$n(y_0+y_1+y_2, \&c.+y_{n-1})+\frac{n-1}{2}\{y_n-y_0\}-\frac{n-1.n+1}{1^2.n}\{\Delta y_n-\Delta y_0\}$$

$$+\frac{n-1.n+1}{24.n}\{\Delta^2 y_n-\Delta^2 y_0\}+, \&c.$$

Laplace has given (in the fourth volume of the *Mécanique Céleste*, p. 206) the particular value of this series which obtains when the interval i which separates the values of y is indefinitely diminished.

In this case the coefficient of $\Delta^q y_m-\Delta^q y_0$ is found by integrating

$$\frac{i.(i-1)(i-2).....(i-q)di}{1.2.3.....q+1}$$

from $i=0$ to $i=n$, if $n=1$, the sum of the values of y or the area of the curve between y_0 and y_m,

$$=\tfrac{1}{2}y_0+y_1+y_2.....+\tfrac{1}{2}y_n-\frac{1}{12}\{\Delta y_n-\Delta y_0\}+\frac{1}{2^4}\{\Delta^2 y_n-\Delta^2 y_0\}.$$

In applications of the former series to the calculation of annuities, reversionary payments, &c., y_m, Δy_m, $\Delta^2 y_m$, &c., $=0$.

The first term in the series of the values of y or y_0 is the value of a present payment $=1$, if we neglect the term

$$\frac{n-1.n+1}{12.n}\{\Delta y_n-\Delta y_0\}$$

and the following, and suppose the values of the annual payments to be in arithmetical progression, the value of an annuity on the life of a person aged 20 to commence at the end of the first year.

If $n=10$, $y_0=1$, $y_1=\frac{p_{20,\,10}}{(1+r)^{10}}$,

$$=10\left\{1+\frac{p_{20,10}}{(1+r)^{10}}+\frac{p_{20,20}}{(1+r)^{20}}+,\ \&c.\right\}-\frac{9}{2}-1,$$

$$=10\left\{\frac{p_{20,10}}{(1+r)^{10}}+\frac{p_{20,20}}{(1+r)^{20}}+,\ \&c.\right\}+\frac{9}{2},$$

the values of annuities at 0, 5, &c. may be obtained in a similar manner. This value of the annuity will be a very close approximation: the error, whatever it be, will be nearly constant for different tables of mortality; and as the first correction, which is in this case $\frac{9}{2}$, is constant, the whole correction may be considered as constant. It may therefore be determined easily by calculating the annuity first accurately, and afterwards by the approximate method from any table of mortality in which the deaths are given for every age: the difference between the two values so obtained will be the correction required. By means of the Chester Table for Males, I determined the correction as follows, supposing the table of mortality to give the living at 0, 5, 10, 20, 30, 40, 50, &c., and that the annuity commences at the end of the first year.

Age.	Value.	Age.	Value.
At Birth	2·481	30	4·109
5	3·692	40	4·024
10	4·167	50	3·920
20	4·242	60	3·792

Thus the value of an annuity at 20 is

$$10\times\left\{\frac{p_{20,10}}{(1+r)^{10}}+\frac{p_{20,20}}{(1+r)^{20}}\ .....\&c.\right\}+4{\cdot}242.$$

How close an approximation this method gives may be seen in Table II.,* where I have placed underneath the results which I have obtained those which have been obtained by other writers. The same series shows that the value of an annuity of £1 paid half yearly is the value of the same annuity paid yearly $+\frac{1}{4}$, and the value of an annuity of £1 paid weekly is the value of the same annuity paid yearly $+\frac{\pounds 1}{10^{\frac{1}{2}}}$; the annuity being supposed to commence at the end of a year, and the first weekly payment to commence at the end of a week.

When the table of mortality which is made use of gives the deaths at every age, the preceding method can only be considered as an approximation; but in all cases I believe the error due to this method will be less than the error due to the errors of the observations.

* *See* page 292, and the note there.

The same series furnishes a method, which I think is the simplest which can be proposed, of calculating approximately the values of annuities or insurances on two or three lives.

The value of an insurance on one life is easily deduced from the value of the annuity; in fact, if A is the value of the annuity, the value of the insurance in a single payment is

$$\{1+\mathrm{A}\}\frac{1}{1+r}-\mathrm{A},$$

and the value of the premium is

$$\frac{1}{\mathrm{A}(1+r)}+\frac{1}{1+r}-1.$$

When the persons observed, upon whom the table of mortality is founded, are few in number, and the deaths are given for every year, they will present considerable irregularities, owing partly to the effect of accidental causes, and partly to the unavoidable errors of the observations; but these causes may be considered in theory as identical. If e be the probability that an individual died in the year in which he is recorded to have died, e_1 the year after, e_n the n^{th} year after, &c., and if the table of mortality be founded upon a population observed from birth throughout life, upon the same hypothesis of probability, *à priori* as before, the formula which I gave in my former paper on this subject* shows that, if d_n be the number of deaths recorded to have taken place at the n^{th} age, the probability at the birth of a child that he will die at the n^{th} age is

$$\frac{\Sigma\{d_{n+m}\times e_m\}+1}{\Sigma d_n+p};$$

Σd_n being the total number of persons observed, and p the number of cases possible, or ages at which deaths are supposed to take place. The values of e are to a certain extent arbitrary. If e be supposed to be constant and $=\frac{1}{m+1}$, and that the values of e are $e_{-\frac{m}{2}}\ldots e_{-1}$, e, $e_1\ldots e_{\frac{m}{2}}$, this amounts to taking the mean of the deaths which are recorded to have taken place within $\frac{m}{2}$ years of the age n. Generally, however e be supposed to vary, $\Sigma e_m=1$. This theory shows how a table of mortality should be corrected, for the irregularities which present themselves, when the observations are not numerous.

The number p may also be considered as arbitrary; and by

* *See* page 197.

altering this, which amounts to increasing the deaths at every age by an arbitrary quantity, the table may also be corrected: but the former method is simpler.

With the assistance of Mr. Deacon, I have calculated the tables of annuities at the end of this paper * by the approximate method given above, and the data or table of observations from which they are taken is prefixed to each.

8. Table I. contains different registers of mortality, giving first the actual number of living deduced from the recorded deaths, and then the same reduced to the radix 1000.

The table for Paris is taken from the *Annuaire du Bureau des Longitudes.*

The Breslau Table is taken from Dr. Halley's paper in the *Transactions of the Royal Society;* it was formed from observations communicated to the Royal Society by Mr. Justell. Dr. Halley has not given the observations themselves.

Kerseboom's Table was formed by him from registers of life annuitants in Holland and West Friesland—Desparcieux's tables, from lists of the nominees in the French Tontines. These two must be considered as formed upon very select life.

The tables for Brussels and Amsterdam are taken from the *Recherches sur la Population dans le Royaume des Pays Bas,* by M. Quetelet.

The table for Sweden was formed "from observations of the proportion of the living to the numbers who died at all ages for 21 years, from 1755 to 1776, in the kingdom of Sweden" (*see* Dr. Price, vol. ii. p. 140). The table for Montpelier is from a Memoir by Mr. Morgue, in the first volume of the *Mémoires de l'Institut.* The Northampton Table is taken from the deaths in All Saints' Parish, Northampton, from 1735 to 1780 (*see* Dr. Price, vol. ii. p. 95). The Carlisle Table of Mortality, as given by Mr. Milne, was formed by him from the observations of the mortality which are given in the next column, combined with two enumerations of the population. The numbers upon which this table is formed are very small. The expectation of life is given at the foot, calculated from each by a method similar to that I have explained for calculating annuities.

Table II. contains annuities deduced from the preceding.

Table III. contains tables of mortality in which the sexes are distinguished, and Table IV. contains annuities deduced from them. It will be observed, that all these tables agree in giving to

* *See* page 292, and the note there.

females a greater longevity than to males; a fact which is further confirmed by the circumstance that in all countries—with the exception, I believe, of Russia—notwithstanding the male births exceed the female, the number of females in the population exceeds that of the males.

Mr. Griffith Davies has published tables of annuities taken from statements of Mr. Morgan in his addresses to the general courts of the Equitable Society, and in notes added by him to the latter editions of Dr. Price's *Observations on Reversionary Payments*. In Mr. Morgan's address to the general court held on the 24th of April, 1800, he stated that the decrements of life among the members of the Equitable, for the preceding 30 years, had been, to those of the Northampton—

From	10	to	20,	as	1	: 2
„	20	„	30	„	1	: 2
„	30	„	40	„	3	: 5
„	40	„	50	„	3	: 5
„	50	„	60	„	5	: 7
„	60	„	80	„	4	: 5

which statement is confirmed in his subsequent addresses.

In a recent publication, Mr. Morgan admits that he was not then aware of the great number of instances in which there are several policies on one and the same life, and he says that this circumstance very materially affects Mr. Davies's calculations.

Such statements as these appear to me too vague to be made the basis of calculations, although the experience of the Equitable Society would be most valuable, if we were acquainted with all the details concerning it.

Mr. Finlaison has very recently published extensive tables of mortality formed from the Government Tontines and Annuitants, which are rendered equally valuable by the accuracy of the materials from which they have been deduced, and the very great care and attention which has been bestowed on them by the author. Mr. Finlaison has done me the favour to prepare for me a summary of these tables, which is to be found in Table V. in a form in which it may be easily compared with the other tables which I have given.

Mr. Finlaison, in his report to the Lords of the Treasury, explains at length the manner in which he made use of the records of the Tontines. Mr. Finlaison observes, that "the facts shown in these observations bear conclusive testimony that the rate of mortality in England has, during the last century, diminished in a

very important degree, on each sex equally, but not by equal gradations, nor equally at all periods of life; and that while, in regard to the males, it seems in early and middling life to have remained for a long time as it stood about fifty years ago, in respect of the females it has during the same time visibly and progressively diminished, to this day, by slight but still sensible gradations." This fact is at variance with the opinion that the improvement which has taken place in life is to be attributed to the introduction of vaccination. Epidemics, however, are of much less frequent occurrence in England than they were formerly, which circumstance must tend materially to diminish the rate of mortality.

The great plague years in London were 1592, 1593, 1603, 1625, 1636 and 1665, in which the burials were as follows:

	1592.	1593.	1603.	1625.	1636.	1665.
Total Deaths	25,886	17,844	37,294	51,758	23,359	97,306
Deaths of the Plague	11,503	10,662	30,561	35,417	10,460	68,596

Now the average number of deaths in London is about 20,000, and the actual number varies very little.

Observations such as those presented by Mr. Finlaison, where the deaths are given at every age, are particularly well calculated to determine delicate points, such as any small increase of the rate of mortality at different ages. A small increase of mortality, according to Mr. Finlaison's tables, takes place about 23; thus, in observation 19 (p. 56 of Mr. Finlaison's report), it appears that there is a minimum of mortality at 13, a maximum at 23, and a minimum again at 33. This does not obtain in Mr. Finlaison's observations on females. It is very remarkable, that the same circumstance is to be observed in the Chester Tables, though here it is found equally in the tables for males and females: this appears to me a great proof of their accuracy, and of the fidelity with which Dr. Haygarth recorded the facts which were presented to him. Dr. Price says—"the bills (for Northampton) give the numbers dying annually between 20 and 30 greater than between 30 and 40; but this being a circumstance which does not exist in any other register of mortality, and undoubtedly owing to some accident and local causes, *the decrements were made equal between* 20 *and* 40," &c. (vol. ii. p. 97.)

However accurate the observations be upon which Mr. Finlaison's results are founded, it must be recollected that the lives were selected from a selected class; and it remains to be shown that the mortality in the lower classes of society is the same as in the higher, and that selection produces no effect on the results.

4. Tables of mortality which are founded upon registers of deaths only are subject to an error arising from the supposition that the population is stationary, as was long ago noticed by Dr. Price (vol. ii. p. 251).

The probability of an individual dying in a given n^{th} year of his life, if the effect of migration be neglected, is the number of deaths of that age divided by the number of births in one year, n years previously—which, if the population were stationary, would be the same as the total of deaths in any year.

If therefore the births, n years previously, are $\begin{matrix}>\\<\end{matrix}$ than the total of deaths at all ages in the year of the observation, the probability of an individual dying at the n^{th} age is $\begin{matrix}<\\>\end{matrix}$ than the quotient of the deaths at that age divided by the total of the deaths at all ages. In America this effect is, I think, clearly perceptible, and has led some persons to conclude that the population in that continent is more unhealthy than in Europe.

The following table has been formed from the bills of mortality for Boston, New York, Philadelphia, and Baltimore, in 1820—

Age.	Living.	Age.	Living.
0	1000	40	254
5	587	50	160
10	549	60	96
20	495	70	53
30	371	80	24
Expectation of life at birth, 24·959.			

which table is much lower than any of the others; but the annual rate of increase of the population in the United States, between 1810 and 1820, was about 1·034. In England, at the same time, it was only 1·016. In order to show directly the effect which an increase in the population produces in the table of mortality, I have calculated three tables from the Chester tables of mortality, supposing the deaths at the time of the observation to be equal to

the deaths 40 years previously (which was nearly the case in this country in the last century), and the births to increase annually in a geometrical progression of which the common ratio is given.

The column	A	supposes	the	ratio	of increase to be 1·005
„	B	„	„	„	1·010
„	C	„	„	„	1·015

The column D is calculated in the same way for females, and supposes the ratio of increase to be 1·005. The ratio 1·005 is very nearly what obtained in England during the last century, according to the Parliamentary Reports. The births in all England, in the year 1700, were 138,979, and in 1780, 201,310, making the mean annual rate of increase 1·0046. In the county of Chester, taken by itself, in 1700 they were 2,690, and in 1780, 4,592, making the mean annual rate of increase 1·0061; therefore the columns A and D, which I have given at length in Table VII., must approach very nearly to exactitude: and, considering attentively the limits of the errors of which observations of this kind are susceptible, I think that it is improbable that the longevity in this country generally, when the Chester Table was formed, was quite so great as that indicated by Mr. Finlaison's tables and the experience of the Equitable Society. It may have improved since.

When the law of mortality in any country, and the number of births in each year during the century previous to any given epoch, are known, it is easy to assign the total number of persons living at every age; for if $p_{0,n}$ be the probability of a child at birth surviving n years, b_n the births n years previously, the number of living in the population at the n^{th} age is $p_{0,n} \times b_n$, and the ratio of the living at that age to the whole population is

$$\frac{p_{0,n} \times b_n}{\Sigma(p_{0,n} \times b_n)}.$$

I have calculated Tables VIII. and IX. in order to show the effect which is produced by a given increase of the births. Table VIII. shows the proportion of the living at each age, and of the deaths to the whole population, when the law of mortality obtains which is given by Table VII. The male births are supposed to be to the females as 104 to 100. Table IX. is calculated upon the supposition that the law of mortality obtains which is given by the Carlisle Table in Mr. Milne's work (vol. ii. p. 564). The following are the results which are given by these tables:—

Ratio of Increase of the Births yearly	1		1·005		1·010		1·015		1·020	
	Chester.	Carlisle.	Chester.	Carlisle.	Chester.	Carlisle.	Chester.	Carlisle.	Chester.	Carlisle.
Ratio of the Births to the Population	$\frac{1}{37\cdot381}$	$\frac{1}{39\cdot219}$	$\frac{1}{32\cdot092}$	$\frac{1}{33\cdot783}$	$\frac{1}{27\cdot964}$	$\frac{1}{29\cdot274}$	$\frac{1}{24\cdot660}$	$\frac{1}{25\cdot654}$	$\frac{1}{21\cdot959}$	$\frac{1}{22\cdot722}$
Ratio of the Deaths to the Population	$\frac{1}{37\cdot381}$	$\frac{1}{39\cdot219}$	$\frac{1}{38\cdot417}$	$\frac{1}{39\cdot440}$	$\frac{1}{38\cdot405}$	$\frac{1}{40\cdot054}$	$\frac{1}{38\cdot544}$	$\frac{1}{40\cdot065}$	$\frac{1}{38\cdot187}$	$\frac{1}{39\cdot781}$
Ratio of Increase of the Population yearly	..	..	1·005	1·005	1·010	1·010	1·015	1·015	1·020	1·020
Population doubles in	..	..	138 yrs.	138 yrs.	69 yrs.	69 yrs.	46 yrs.	46 yrs.	35 yrs.	35 yrs.
Deaths are equal to the Births after ..	..	..	36 yrs.	31 yrs.	33 yrs.	31 yrs.	30 yrs.	30 yrs.	27 yrs.	28 yrs.

The ratio of the deaths to the population is nearly constant, according to both these tables, whatever be the rate of increase of the births; when the ratio of the births to the population is constant, the rate of increase of the population is necessarily the same as that of the births. The rate of increase of the births has been supposed to be constant—a small inequality in this rate, unless it be of long period, will not produce any sensible difference in the results; but, although the total number of deaths which take place in a given population is not much influenced by the rate of increase, the apparent table of mortality is much altered. In order to show the extent of the error which is likely to arise from this circumstance, I have given the apparent tables of mortality corresponding to each rate of increase of the births.

According to Mr. Rickman, in the *Population Abstract*, 1821, the ratio of the deaths to the population in England, at that time, was 1 to 57. This ratio is considerably less than would be given by any table of mortality; and it is probable, therefore, that the number of unentered burials is much greater than Mr. Rickman has supposed. Since the ratio of the deaths to the population is nearly constant when the law of mortality is given, this rate would be an excellent criterion of the longevity of different countries, if it could be accurately ascertained; to this, however, many difficulties are opposed.

In the Tables VIII. and IX. the rate of increase of the births is arbitrary. In order to see how far the mortality in this country coincides with that given by Table VII., I have formed Table X., taking the values of p from that table, and supposing the births in the century previous to 1821 to have been the same as the christenings that are given in the *Population Abstract* before referred

to; and since the ratio $\frac{p_{0,n} \times b_n}{\Sigma(p_{0,n} \times b_n)}$ involves only the ratios of the births, which must be nearly the same as the ratios of the christenings, the error introduced by this hypothesis is altogether insensible.

I have placed, for the sake of comparison, the results given by the census of 1821 with the results deduced from theory; and they agree, I think, within the limits of the errors of which the census is susceptible, and much nearer than the results of different counties agree with each other. The number of deaths in a population of 1,000 males and females, according to the law of mortality of Table VII., is 271, making the ratio of the deaths to the population about $\frac{1}{37}$. Calculating the deaths between 0 and 5, to which period Mr. Finlaison's observations do not extend, from the same table, and those at the succeeding ages from Mr. Finlaison's observations 11 and 19, the total number of deaths which results in a population of 1,000 males and females is 244, nearly, and the ratio of the deaths to the population about 1 to 41: which is far greater than the ratio given by Mr. Rickman.

The following are some of the elements of the population of England and France. Those for England are deduced from the returns in the *Population Abstract* of 1821, before referred to; and those for France, from the *Annuaire du Bureau des Longitudes* for 1829.

	England.	France.
Ratio of males to females	·95764 : 1	
„ male births to female	1·0435 : 1	1·0656 : 1
„ „ deaths to female	1·0024 : 1	1·0180 : 1
„ „ legitimate births to female	.. ..	1·06795 : 1
„ „ illegitimate births to female	.. ..	1·04844 : 1
„ population to marriages in one year	122·50 : 1	132·619 : 1
„ „ births in one year	32·274 : 1	31·535 : 1
„ „ deaths in one year	54·296 : 1	39·423 : 1
„ births to marriages	3·5902 : 1	4·205 : 1
„ legitimate births to marriages	.. ..	3·912 : 1
„ increase of the population annually	1·0167	1·00634

The population of England, according to the census of 1811, was 9,538,827, and according to that of 1821, 11,261,437, making the mean annual rate of increase of the population 1·0167. The baptisms in 1810 were 298,853, and in 1820, 343,660, making the mean annual rate of increase 1·0140.

Mr. Rickman considers the census of 1821 more accurate than

that of 1811: if therefore we suppose the ratio of the births to the population to have been constant during this short interval between these enumerations, so that the real rate of increase of the population was only 1·0140, we have 9,792,600 for the population in 1811, instead of 9,538,827, and 1,468,837 for the increase of the population between 1811 and 1821. A comparison of the registered baptisms and burials during the same time gives an apparent increase of only 1,245,000. (*See* Mr. Rickman's observations prefixed to the *Population Report*, 1821.)

Hence, if the increase was really 1,468,837, the average yearly excess of unentered baptisms over unentered burials is 22,383; and if, with Mr. Rickman, we admit the average number of unentered burials yearly to be 8,770, the average number of unentered baptisms will be 31,153. The baptisms in England, in 1820, were 328,230.

$$\frac{328{,}230+22{,}383}{11{,}261{,}437}=\frac{350{,}613}{11{,}261{,}437}=\frac{1}{32{,}044},$$

which ratio does not materially differ from that given above, in deducing which the average yearly number of unentered baptisms was supposed to be 20,696. The ratio of the population to the deaths was found by adding 8,770 to 198,634, the total of the burials in 1820; and to the marriages, by adding 191 to 91,729 (the marriages in the same year), and dividing by 11,261,437. (*See* p. 145 of the Report above alluded to.)

M. Benoiston de Châteauneuf, in the *Annales des Sciences Naturelles*, 1826, gives the following numbers as the ratio of the births to the marriages:—In Portugal, 5·14; Bohemia, 5·27; Lombardy, 5·45; Muscovy, 5·25; and in several of the southern departments of France, above 5.

In the territory of the two Sicilies, the ratio in 1828, according to the report of the Secretary of State, was 5·716 : 1.

This ratio is increased by two causes—either by the prolificness of the sex, or by the prevalence of concubinage. In the report above alluded to, the ratio of the marriages to the population is 1 : 154—in England it is 1 : 122, which difference is sufficient to account for the difference in the ratio of the births to the marriages, without supposing the former of the two causes indicated above to exist.

If the ages at which deaths take place, and the number of births, were accurately registered in a great empire, the probabilities of life would be known with the greatest accuracy, the multitude of the observations destroying any small sources of

inaccuracy; and the number of the population ($=\Sigma p_{0,n} \times b_n$) would be known far more accurately than by the laborious process of actual enumeration, for in a large district the effect of migration would be wholly insensible. It seems indeed worthy of consideration, whether it might not be possible to publish annually the bills of mortality for every parish in the empire, as is now done in London and in some great towns. If this were done, many interesting questions in science would be determined, the comparative healthiness of different districts and of different periods of the year would be ascertained, and great light might be thrown upon the efficacy of the manner in which different diseases are treated. So many questions in which property is involved are connected with the accuracy of the parish books, that it seems extraordinary that greater attention is not paid to their exactness.

No data have yet been published by which the additional premium can be determined which should be paid when the subject of the policy has any chronic disease. The only case of which I have endeavoured to determine the risk is childbirth. The deaths in childbirth during the ten years from 1818 to 1827, by the London bills, were 2,117, the number of christenings 241,352, and the number of stillborn 7,575 : which would give $\frac{2,117}{241,352 + 7,575}$, or $\frac{1}{117}$, for the probability that a woman does not survive giving birth to a child—making the extra premium of insurance about 17*s*. At Strasburg the deaths in childbirth are 1 in 109. At the City of London Lying-in Hospital, in 1826, the deaths were 1 in 70; in the Dublin Hospital, in 1822, there were 12 deaths among 2,675 women delivered, or 1 in 223; in the Edinburgh Hospital the mortality is 1 in 100; in the whole kingdom of Prussia, in 1817, the deaths were 1 in 112. (*See* Dr. Hawkins's *Medical Statistics*.) Most extensive returns of sickness have been furnished to the Society for the Diffusion of Useful Knowledge, by Friendly Societies, and these will no doubt furnish much valuable information upon the subject of the duration of sickness. If returns could be obtained from hospitals, of the ages at which individuals come in afflicted with different complaints, with the time they continue under treatment, and the number who die, these would also furnish the means of determining the probability of a sick person continuing sick for any given time, and the probability of an individual sick dying. From this, and the probability of an individual dying at the given age which is given by the tables, the probability of an

individual falling sick at a given age, with his *expectation* of sickness at that age, might be determined. The bills of mortality in London give the diseases by which deaths are occasioned, but unfortunately the sexes are not distinguished.

Table IX. shows the ratios of the diseases to which the deaths have been attributed at different periods in the London bills: measles seem to have increased. So little dependence, however, is to be placed on these documents, that I forbear making any further comments upon them. The column headed America is taken from the bills of mortality for Boston, New York, Philadelphia, and Baltimore; and that for Carlisle, from Mr. Milne's work on annuities.

I have also endeavoured to determine from the bills of mortality, as given in the *Annual Register* for the ten years from 1810 to 1820, the mortality and the births in London at different seasons (*see* Table XII.) The burials amounted during this period to 197,695, and the christenings to 245,287. The returns, however, are made so very irregularly, that these results, notwithstanding the very large numbers from which they are formed, are by no means accurate; for the parish clerks, as I find by examining the weekly bills, generally return the deaths and christenings of several weeks together. I have annexed observations of a similar kind given by M. Quetelet and Mr. Milne; and a table for Glasgow, which I have deduced from the bills of mortality for that city for the years 1821 to 1827. The total number of burials during that time was 31,245.

In London the mean monthly price of wheat varies very little, if at all; the same is the case with the barometer: the variation, therefore, which takes place in the number of deaths and christenings, must be principally owing to the variations in the temperature. The mean number of christenings in any month, in a given place, will also be affected by the mean time which christening is delayed after birth in that place. All the results given in Table XII. have been reduced to the radix 1200, and are corrected for the unequal lengths of the months.

I have thus endeavoured, as briefly as possible, to present the data which we now possess for determining questions connected with the duration of human life. The accordance of the results which have been deduced proves that no considerable error can obtain; for the slight difference which exists between Table VII., which I have formed from the observations at Chester, and the Table formed by Mr. Milne from those at Carlisle, is of the order

of the inevitable errors of these observations, and of the hypothesis I made with respect to the rate of increase of the population during the century previous to the observation: and in order to get rid entirely of this slight discrepancy, it would be only necessary to make the rate of increase about 1·007 instead of 1·005, as I supposed it to be. The Northampton Table, treated in the same way, would give results nearly similar.

No doubt our information on this subject will soon be much improved; for when we consider the accuracy which has been introduced into every other branch of philosophical inquiry, it appears surprising that this should have remained so far behind.

Annuities at 3 per Cent.

Age.	From Desparcieux.		From Northampton Table.	
	By approximate method.	By usual method.	By approximate method.	By usual method.
0	..	..	12·389	12·270
5	22·575	22·597	20·524	20·474
10	22·750	22·756	20·617	20·663
20	21·242	21·168	18·590	18·639
30	19·446	19·492	16·824	16·922
40	17·051	17·183	14·613	14·848
50	13·717	13·899	12·026	12·436
60	10·315	10·522	9·065	9·774
70	6·963	..	5·592	6·734

NOTE.—As the reader will no doubt infer, a considerable quantity of tabular matter accompanied this paper in the publication from which it is extracted. Had our space been less limited, we should have been glad to reprint a larger portion of it, particularly the tables illustrating the learned author's observations on the rate of increase in the population. The nature of these tables, however, will be easily gathered from the paper itself; whilst the more accurate information obtained of late years in reference to such matters deprives them, for the most part, of their otherwise intrinsic value. The registers of mortality referred to at page 282, as comprised in Table I., will be found more or less in the second volume of Mr. Francis Baily's well known work on assurances. The short table above given forms part of that referred to in the paper as Table II.—ED. *A. M.*

An Examination of the Objections urged against the plan of Decimal Coinage proposed by the Royal Commissioners and by the Select Committee of the House of Commons. *By* CHARLES JELLICOE, *Esq., one of the Vice Presidents of the Institute of Actuaries.*

[Read before the Institute, 26th March, 1855, and ordered by the Council to be printed.]

IT is now somewhat more than twelve months ago that the decimal coinage question was brought under the notice of this Institute, and that a resolution was thereupon unanimously come to approving of the plan recommended by the Committee of the House of Commons. Since that decision, public attention has been repeatedly called to the subject; and although various plans have been proposed and discussed, founded on a different basis from the one thus recommended, it must, I think, be conceded, that none of them have made any serious impression on the public mind, but that, on the contrary, the one proposed by the Committee has made some if not considerable progress in the course of the sifting which the question generally has undergone.

Aided by the powerful efforts of the Decimal Association, it is probable that this progress would have been more decided, but for the distraction created by the arguments in favour of other plans, the strenuous exertions made by the supporters of them, and the oft repeated objections to the proposition of the Committee, to be met with in the speeches and publications of these adverse parties.

That there is a certain force in some of these objections I do not mean to deny, and am quite ready to allow them their proper weight; but the majority of them may, I think, be shown to have no foundation whatever, or, to say the least, in many cases to be built upon a very slender one. As it is extremely desirable that the question be brought to issue as soon as practicable, and as the exposure of any fallacies connected with it must have a tendency to promote that object, I propose now to examine briefly such statements put forward by recent writers as appear to me to be erroneous, and calculated to impress the public mind with false ideas upon this important subject.

Mr. Theodore W. Rathbone has been amongst the most active and energetic of the opponents of the Committee's plan, and most earnest in the recommendation of one of his own; and it may be well, therefore, to begin with some of the arguments of that gentleman. At page 9 of his *Comparative Statement* he says—

"Now a very serious objection arises in the first instance to this millesimal division of the pound, from the *form of account* which is proposed, and which it necessarily involves; and it may be questioned whether this one requirement would not of itself render the loss to the country, to say nothing of the inconvenience, greater than any possible gain from a decimal, or rather millesimal, system such as this. In the words of the little work lately published by 'One of the Million,' 'the commonest, and certainly the most laborious, work of an office, is *addition;* and the great defect of the present system is the necessity of dividing the pence by twelve, instead of reckoning them by tens:' but to this principal and most laborious work of ordinary accounts, the scheme under consideration, by bringing in a fourth or mil column of figures, adds unavoidably upwards of five-and-twenty per cent. Regardless of the evidence on this point of even their own witnesses—(Mr. Miller, a cashier of the Bank of England; Mr. Bevan, a private banker, &c.), who distinctly told the Committee '*that it would be exceedingly inconvenient in large transactions to force farthings or mils into the calculations that we should have to deal with; that there would be an objection, and a strong feeling, against the introduction of farthings into mercantile accounts*'—the plan recommended in the Report renders this wholly inevitable in *all* accounts, large and small. The necessary form of account on this plan rendering the next denomination to the *mil*, the *cent*, consisting of twopence and two fifths of a penny (almost 2½*d.*), it would be altogether impossible to deal with *mils* as at present with farthings and halfpence, and to make the proposed *cent*, like the penny, the lowest denomination in our ordinary accounts; and hence we must evidently have, as proposed in the Report and by all the advocates without an exception of schemes on this basis, an additional *fourth* money of account and column of figures to add up and deal with in *all* our accounts—consisting either of farthings, or the still smaller amount, the *mil*—also no aliquot part of pence, or halfpence, or farthings. Great as would undoubtedly be the advantage in so many points of view of that universal labour-saving machine, decimal coinage and numeration, it is surely questionable whether they would not be more than all thrown away by this one unfortunate and egregious inconsistency with a real and perfect decimal system of accounts."

The objections here made are greatly exaggerated. There can be no doubt that in all transactions on a large scale mils would be disused: it is probable that they would not be taken any account of in Banks or Insurance Companies, several of which latter institutions make their charges, as it is, in not less fractions than threepences. But were it otherwise, the labour would not be increased one fourth,* as Mr. Rathbone asserts, since the mil would only occur here and there. We may imagine, for instance, that an ordinary column of sums for addition would appear as follows:—

40	7	„	„
20	3	4	5
17	8	6	„
54	7	9	3
19	5	7	„

* Were the labour augmented as Mr. Rathbone imagines, it would be increased one third, not one fourth only.

the sums set down for the most part not including mils at all. Mr. Rathbone will surely admit that the additional trouble here is balanced by having tens only to carry, instead of 12 and 20. As regards the actual coinage, the labour of counting would be greatly reduced, since halfcrowns, shillings, sixpences, fourpenny and threepenny pieces, and pence, might ultimately be withdrawn, leaving only four varieties necessarily to be dealt with. On this head the plan proposed by Mr. Rathbone would break down altogether; for if the coinage consisted of pence and tenpenny pieces only, a banker receiving such a sum for instance as £50. 4*s.* 6*d.* would have to count upwards of 1,220 coins, instead of 59; and this could be avoided only by retaining the pound in his scheme, in which case the accountant would have to divide his additions of one column by 24, a more inconvenient divisor than either of those existing at present.

Throughout his pamphlet Mr. Rathbone insists on the insuperable difficulties arising out of the circumstance of the new coins being incommensurable with the old, and states that "the former must be suddenly introduced, as the two will not work together." It is to be presumed that this means, that the new must be made at once to supersede the old; or, in other words, that the latter must be suddenly withdrawn. This is quite a misapprehension. It is proposed at first merely to issue cent and mil pieces, calling in those which it is intended shall thus be superseded as the new coinage comes into use. In a short time there will be cents' worth and mils' worth of all kinds of articles, as there are now fourpenny and pennyworths; and as the new least coin will be of less value than any now in use, the poorer classes will thus get more accurately adjusted values for their money than they did before. As for such fixed charges as postage rates and tolls, the old coinage will remain applicable to them till the requisite changes in them be effected; and it must be borne in mind, that even here no great inconvenience will arise, except only where single pieces are concerned—thus, a dozen penny stamps will be purchasable for 5 cents or 50 mils, and a sixpenny cab fare can be paid with two cents and five mils. It is not denied, however, but that during this transition state some embarrassment will arise, nor can any change whatever be made without; it is the natural penalty to be paid before a better order of things can be attained, and for the purpose in question the payment of such a penalty is well worth the while. The principal inconvenience, however, will arise in accounting. Thus, bankers and others will no doubt

receive coins of the old and new denominations at the same time, and will have to pay cheques and bills drawn in both currencies. But a little management will suffice to remedy in a great measure any complication which may arise from these sources. It will be necessary only to add in account books as now constructed one column for the new system of coinage, and make the entries accordingly. Such an arrangement as the following, for instance, would meet any temporary difficulty of this kind:—

	£.	s.	d.	£. f. c. m.
A. B.	25	7	6	
C. D.	..	..		100·730
E. F.	16	8	6	
G. H.	..	..		50·682
I. K.	..	..		75·350
	152	16	0	226·762

The totals in each case being ascertained, the shillings and pence can be easily converted into corresponding values in the new system, and the sum of the whole then taken in that character.

It is not very easy to understand the object of the passages quoted from Mr. Laurie's work by Mr. Rathbone, and which he appears to look upon as of vast importance, but which, probably, few other persons with a knowledge of the subject will regard as of any. In one portion of them, complaint seems to be made that some of the present coins can be expressed in decimals of a pound only by a great many figures; and in another, great evils are said to arise from our not having coins of smaller value than those now existing. These objections apply to the present system, but they are precisely those which will be remedied by the proposed one. Considering that Mr. Laurie, according to Mr. Rathbone's account, "was one of the most wonderful men that ever lived, as regarded his knowledge and application of figures," that gentleman seems to have derived remarkably little support from him.

Dr. Gray and Mr. Minasi, as well as Mr. Laurie, have both endeavoured to conjure up some frightful consequences attendant upon the sacrifice of what they term the "poor man's penny." From these gentlemen's statements it might be supposed that all sorts of articles in general use would still be made up in pennyworths, and multiples of pennyworths, whilst the poor man's pocket contained nothing but cents and mils; or that the shopkeeper would have coins only of one system, whilst his customer had those only of the other. Nothing can be really more fallacious than this.

The poor man will make his purchases, probably, having shillings, pence, cents, farthings, and mils for the purpose, and the shopkeeper will have a quantity corresponding with either of them. Let us suppose that a portion of some article of commerce has been cut off from a larger quantity, and that, being weighed, its price is proportionately not quite a farthing more than was required: will it be denied that the mil is other than a convenience in such a case, and that it will tend rather to save the poor customer from loss? If the necessaries of life were still to be sold at prices indicated by our present coinage, whilst the purchaser had only the new in his possession, there might be some weight in the objections thus raised; but that not being the case, they fall wholly to the ground. The effect of the change will be much the same as that which a foreigner experiences on his first arrival. His embarrassment in making payments ceases directly he has obtained our coins in lieu of those of his own country; and so it would equally cease if we changed our charges to suit his coins. The result of the whole matter is, that during the transition there will be a greater range of prices and a corresponding variety of pieces with which to meet them—in other words, that there will be increased facilities of exchange.

But a real difficulty which will arise has been in a great measure overlooked by these gentlemen. They have omitted to observe that where an old charge—such, for instance, as one for rent or taxes—is made in the new currency, doubts will arise in the minds of unskilled persons as to whether the one is really an equivalent of the other; they will not readily see, for example, that £25·375, or twenty-five pounds three florins seven cents and five mils, is really the equivalent of £25. 7*s*. 6*d*., and it will, no doubt, take some time to make this apparent. On the other hand, it must be remembered that these cases will not occur to the same individual every day—that when they do he will have ample time to consider them—that he can pay the old sum in the old coin, at least till he has become more familiar with the new; and that meanwhile he will have abundant means of resolving his doubts by such contrivances as tables of equivalents and "ready reckoners," of all sorts and sizes, which the exigences of the time will, we may be sure, abundantly call forth.*

From the foregoing considerations we may observe, that no

* Two publications of this kind have already appeared—one by Mr. Robert Mears, and another by Mr. Walford. They are both very ably got up, and are calculated to be of much practical utility, should the proposed change take place.

embarrassment will arise except where a price in terms of the existing coinage has to be translated into terms of the proposed one; and as this necessity need never arise in ordinary buying and selling, where quantities will adapt themselves to either system, no inconvenience, as I have already said, need be anticipated in such transactions. But where the terms must be translated, another alleged difficulty presents itself, which has been much insisted upon by Mr. Minasi. Those who are familiar with such matters are well aware that some of our present pieces cannot be represented as decimals of a pound in less than four or more figures; and since the new system will have coins corresponding with three places of decimals only, it becomes impossible, with two exceptions, to represent the one exactly in terms of the other. Thus, decimally—

1	penny	=	·004166	+	of a pound.
2	pence	=	·008333	+	,,
3	,,	=	·0125		
4	,,	=	·016666	+	,,
5	,,	=	·020833	+	,,
6	,,	=	·025		
7	,,	=	·029166	+	,,
8	,,	=	·033333	+	,,
9	,,	=	·0375		
10	,,	=	·041666	+	,,
11	,,	=	·045833	+	,,
12	,,	=	·05		,,

Here we see that the sixpence and shilling alone can be truly represented in the new system—the former being exactly equal to 25 mils, or two cents and a half, and the latter to 5 cents, or 50 mils.

Now, it must be remembered that exchanges will have to be made between the two currencies—or, what is the same thing, payments registered in the old will have to be made in the new, either of a single coin for its corresponding value, or of a sum made up of several coins for the like corresponding value; but the single-coin payments, consisting of tolls, postages, and one or two similar charges, have been already disposed of, and we come to larger payments—such, for instance, as a workman's wages, which for our present purpose we may suppose to be 14*s.* 8*d.* a week. In such a case it will be seen that the employer must pay 733 or 734 mils ($\frac{7}{10}$ths of a mil too much, or $\frac{3}{10}$ths of a mil too little); that is to say, that about one fourteen-hundredth part of such a week's wages is thus placed in jeopardy. Had the wages been half the amount we

have supposed, then about one seven-hundredth part of them might be lost or gained in this way; and so on, the proportion increasing as the sum to be paid diminishes. Now, it is well known that the poorer class of persons have very few payments to make at all of a recurring kind; and so we arrive at the conclusion that in their case the inconvenience arising on this head will scarcely ever occur, and that, when it does, it will be wholly insignificant. But Mr. Minasi contrives to magnify the difficulty on this head in a very ingenious manner. He first assumes a price at an inconvertible rate; then translates the rate, of necessity inaccurately, into the new coinage; and, multiplying this last 9,000 times, discovers that the product is so much in excess or deficiency of what it would be if the original rate had been multiplied instead. Thus he supposes 9,000 articles on sale at 10*s.* 2½*d.*, and, on Professor De Morgan's authority, which he quotes for the purpose, represents that rate by 511 mils: but 9,000 times 511 mils amount to £4,599, whereas 9,000 times 10*s.* 2½*d.* would amount to £4,593. 15*s.* only; "and thus," says Mr. Minasi, "an error of no less than five guineas would arise in such a case." Mr. Minasi may rest assured that very few people would go to work in that way. So long as any article is quoted at a price in the old currency, that rate will be dealt with. Ten pounds of sugar at 10*d.* a pound will be ten times 10 pence, not ten times 41 mils. But the shopkeeper, for his own interest' sake, will soon have quantities adapted to the new currency; and then, and not till then, will calculations of price be made in it. Before we are charged 250 mils for ten pounds of coffee, it will be settled that the coffee is at 25 mils a pound.

Mr. Minasi adduces several other arguments against the Committee's plan, but it will be seen that they fall to the ground on examination. Thus he shows that certain multiplications, such as £58. 17*s.* 1½*d.* by 35, can be performed in the ordinary way by means of a less number of figures than by making the computation decimally; but he omits altogether to take into account the time and labour which the one process occupies as compared with the other. The example is as follows—

		£.	s.	d.
58·856		58	17	1½
35				5
294280		294	5	7½
176568				7
2059·960		2059	19	4½

where the former calculation requires 26 figures and the latter only 22 (counting a fraction as one); but if time and labour be taken into account, there can be very little doubt which is the preferable process.

Again: quoting from a writer in the *Eclectic Review*,* who, it seems, speaks of the compound rules of addition, subtraction, multiplication, division, reduction, practice, rule of three, and vulgar fractions, disappearing from our school books, Mr. Minasi protests against any such banishment of these useful modes of calculation, and urges that, even should a decimal coinage be established, they afford the means of arriving at results more quickly "*than by decimals.*" By reference to Mr. Minasi's paper it will be seen that what he means is, that such rules as that called the rule of three will always be useful whether a decimal system be introduced or not.

The objections made by Mr. Thomson to the retention of the pound may, I think, be soon disposed of. That gentleman says—

"How would it be possible, for instance, in writing down 8·124 (8 pounds 1 florin 2 cents and 4 mils) to make it intelligible to a working man, or even one higher in the social scale, that 1 (the first decimal figure) means two shillings—for we shall still for some time count in shillings; that 2 (the second decimal figure) means two tenths of two shillings; and that 4 (the third decimal figure) means four hundredths of two shillings, or four thousandths of one pound?"

The answer to this is, that 8·124 need not, at first at all events, be written in that way, but in the manner in which pounds, shillings, and pence are now—that is, as 8·1·2·4: in which way no one could have any difficulty in understanding them, for it will not be necessary to translate them into the values of the old coinage, when we have what they represent in the new; and

* I believe, Mr. John Middleton Hare, Jun., B.A., of St. John's College, Cambridge—the author also of a very able letter on the subject, addressed to the Chancellor of the Exchequer. The following paragraph, which occurs in his letter, very happily exposes the kind of arguments occasionally brought forward by the opponents of the Committee's plan:—

"The next comparison is the most ludicrous of any, and one that it could scarcely be conceived possible for anyone to have had the rashness to put forward. I give it word for word as it occurs.

"'The pound, florin, cent, and mil scheme requires an abstruse decimal system of calculation, difficult to be comprehended by the less informed, and involving great risk of error in its use. The pound, tenpenny, and penny scheme does not even require a decimal notation, except when a less sum than a penny is required, and except where wanted in calculation that may be marked or not by a vulgar fraction.'

This would seem to be hardly a fair comparison, following as it does one which describes the Committee's plan as unworthy of the name of decimal . . . It is most amusing to find the tenpenny scheme recommended because it does not require a decimal notation, whereas it is this very decimality that is so much desired, and that will produce so many benefits. But is it not absurd that any proposed scheme calling itself a decimal one should adduce as a circumstance in its favour that vulgar fractions may be still used under it?"

Mr. Thomson appears to forget that each coin will eventually have its value marked upon it, as a mil, a cent, a florin, &c., and that the correspondence between the coins themselves and a statement of them on paper will thus be complete. His objection that the cent will be inconveniently small has more weight. The threepenny piece is, as he says, apt to slip through our fingers; but this is, perhaps, merely owing to our not being long accustomed to small silver coins, and a remedy for the inconvenience might be found in making the cent a little thinner and larger.

Mr. Ryley says—

"1. Our objection to the denominations of pounds, cents, and mils, is, that it is using up for purposes of account names which have a larger signification. This is Professor De Morgan's objection to cents and mils. We wonder that so acute an observer did not perceive, and that so candid a speaker did not say, that the same objection applies to the pound, which continually necessitates the use of the qualification 'sterling.' The pound is, besides, a very heavy and high denomination for the unit—very much, out of all measure, higher than is in use in other countries. Moreover, as no one would propose to carry its decimals further than the third place of figures, it does not offer so many facilities for approximating the payments of very small sums in the existing coinage to their nearest equivalents in the new as would be afforded by the selection of a lower unit, divided similarly into thousandths.

"2. It involves the use of the florin, which, with the retention of the shilling, under the designation of the half-florin, or 50-mil piece, will very greatly impede the thorough introduction of a really decimal system into the thoughts and language of the people."

I think it will be admitted by Mr. Ryley himself that these are not very formidable objections. It has been shown over and over again, and with particular force by Mr. Brown, the chairman of the Decimal Association (to whose exertions in this movement, success, if it be attained, will be mainly attributable), that the attempt to introduce and preserve a similarity between the coins of this and other countries is quite hopeless, and that it would be useless could we achieve it. The pound, being so valuable, is an equivalent for a great many other coins, and hence a vast deal of labour is saved to those engaged in the actual manipulation of money. As regards the value of the coin which is to occupy the third place in the decimal rank, it is less than that now having the place of least value, and is therefore, so far, an improvement in the present system. The change in the thoughts and language of the people will no doubt be slow; but that is unavoidable, whatever system be adopted. Nevertheless it must, I think, be admitted, that the florin is already beginning to familiarize itself,

and that it is not at all uncommon, even now, to hear it quoted in naming the price of things. Moreover, the decimal system would obtain quite independently of the conditions which Mr. Ryley seems to consider necessary. It is likely that hereafter five-mil pieces and five-cent pieces would be combinations in more request than any others; but the accounting decimally would still go on, and there would still be always coins in use corresponding with those of account. It is not essential that these last should be the most popular.

Amongst the anonymous objectors to the plan of the Committee, none have been more active than some gentlemen of the press in Liverpool, who also greatly favour Mr. Rathbone's views. One of these writers, in an article which appeared about the time of the meeting of the British Association in that town, remarks that

> "Amongst the changes necessary for the introduction of the proposed system would be the revision and alteration of the whole of the taxation based on the penny—namely, the income tax, the customs rates, the rates of tolls granted by Act of Parliament to railway companies, bridges, highways, &c. &c."

Now nothing can exhibit more forcibly the reckless manner in which objections are brought forward than this, or the facility with which the public mind may be misled on the subject. The objection at first appears to be a serious one, but a moment's consideration will suffice to show that it is entirely without foundation. Thus the income tax, it is true, is assessed at so many pence in the pound; but this is merely a mode of expressing that it is to be levied at a certain rate per cent.; so that, under the present arrangement of sixteen pence in the pound, we arrive at the charge by taking one twentieth of the sum taxed and adding a third of the quotient to itself—an operation more easily performed with a decimal system than with the present one. Thus, if we suppose £100. 12*s.* 6*d.* to be the income, one twentieth of this is equal decimally to 5·031; and adding the third, we get for the result 6·708 for the amount of the tax, which is simply one fifteenth part of the income, the rating of which is altogether independent of any such changes as those under discussion. The same may be said as to other kinds of taxation "based on the penny"; for the most part they are merely per centages—or, in other words, a certain definite portion of the amount taxed—and need not, in this point of view, be affected by any alteration in our present monetary system.

I believe I have now noticed most of the principal objections to

the plan proposed by the Committee—with exception, perhaps, of one very constantly dwelt upon by all who oppose it, and that is, the necessity for calling in at once all the copper coinage, and such silver pieces as would be no longer part of the decimal system. So far from there being a necessity to do this, it is, as it seems to me, of great importance that it should not be done for a considerable time. In fact, the present coinage, or much of it, might remain, without any prejudice to the new arrangements, till it became necessary to withdraw it on the score of its being worn out; or, at all events, the process might be so gradual as to be almost imperceptible; and when thus spread over a good many years, such a process could not be regarded as an expensive one. As I have already said, the change must in its nature occupy a very considerable time, and will be for that reason the less felt.

From what has been said, it will be seen that the number of opposing writers (for, with the exception of Mr. Milward, Mr. Davidson, and some anonymous gentlemen, I believe I have noticed nearly all) is not very great, and that the difficulties as represented by them are not very formidable. Into the arguments directly in favour of the change I do not propose to go: suffice it to say, that it is recommended by a host of persons of great intelligence, amongst whom may be numbered some of the profoundest thinkers, and men having the widest range of knowledge and experience of this or any age. Considering, indeed, what a phalanx of ability they represent, the length of time during which they have digested the proposed innovation, and the unanimity with which they join in the recommendation of it—it is somewhat surprising that a Government which has with the easiest *nonchalance* issued several descriptions of coins nobody ever asked for, should look with such coyness on the small request now so powerfully supported—viz., that they would issue simply two more comparatively insignificant pieces to complete a system which they themselves have initiated, and which, without such addition, is altogether imperfect and inoperative. Our threepenny and fourpenny coins were issued, it is to be presumed, for the sake of public convenience; and the same plea will surely justify the introduction of the cent and mil, even were the more cogent reasons for such introduction altogether wanting.

Answers to Fallacies on the Decimal Question: Balance of Trade, Exchanges, and Common Coins. By WILLIAM BROWN, Esq., M. P., *Chairman of the Decimal Association.*

[Read before the Institute, 26th March, 1855, and ordered by the Council to be printed.]

LET us suppose France and the United States to enter into a convention with us to issue pieces of gold of the same weight and fineness as our sovereign—that is, containing 113 grains of fine gold; the three countries would then have a common standard of gold money. Let us examine whether, in the face of the operations of commerce, the international quality of value of these coins could at all times be maintained. The temporary balance of trade is always fluctuating in favour of one or the other country; a permanent balance against any country is impossible.

Our transactions with the United States of America will be sufficient to illustrate the disturbance of common value to which the coins in the three countries must be continually subject.

If the American imports from our country, and the claims that England may have on them for indirect transactions, amount in any one year to £15,000,000, and if America by her direct shipments to us and by indirect operations only pays £10,000,000 of this debt, it leaves, of course, £5,000,000 to be paid and sent here by some other means. The parties having those payments to make will endeavour to get bills to remit; but so many being wanted to liquidate this debt, they will rise in value.

We will suppose the premium to rise from 8 per cent. to 10 per cent. Parties having to pay £100 in England must then give £110 in America, if they make the remittance by bills. The moment the bills have exceeded this premium it will become cheaper to send the payment to England in gold. There will then be a pressure in America for gold. The convention coin of 113 grains of fine gold will be more in demand, as it would clearly be the interest of parties having remittances to make to send them in gold, if they could get it at a less premium, after freight, insurance, &c., are considered, than the bills. The convention coins would therefore immediately rise in value in America for export, and there would be an end, for the time, of their international equality with the same coin in England. Nor is this an imaginary case; the ordinary trade quotations prove that both gold and silver are frequently at a premium in the United States.

The rate of exchange is, it is true, kept down, because many houses in foreign countries have sufficient credit with their English correspondents to allow them for a time to overdraw their accounts—either on the strength of collateral securities placed here, or without them—until the course of trade adjusts itself. It does so by our lessening our exports when they will not leave a profit of sufficient margin above the premium on the bills.

The action is precisely the reverse in the United States: the shippers of produce being able to sell their bills at a high price, they can afford to give more for the produce they buy for shipment, or else to sell it cheaper here, which stimulates and increases their exports. We decrease, they increase, their shipments: it equalizes our transactions, and brings the exchanges* to par.

Those who are not conversant with the nature of the fluctuating rate of exchange may fancy that it would have no existence if all countries used the same coins.

Let us again suppose that England and France agreed to use the very same money; say, our sovereigns. In order to prevent any suspicion of difference in weight and fineness, let them employ the United States to coin for both; so that, when 10,000 sovereigns are struck off at New York, it just depends on the first letters received by the foreign post whether that lot of sovereigns shall go to England to be English money, or to France to be French money.

From the very day of the arrangement the English and French money will be made unequal by the operations of trade, and become, for the time, of more value in one country than the other, and the English sovereign will be quoted as the French sovereign, more or less as the case may be. The reason, independently of other trade questions, is, that 100 sovereigns in London, and which must go to Paris, are not worth so much to the English holder as 100 sovereigns actually in Paris; and this because he must pay the insurance, loss of interest, freight, and expenses that are necessary to get them there. Mercantile calculations in English money would not then, any more than now, represent the results in French money.

It is true that the traveller would no longer be perplexed by exchanges of the money which he carried in his pocket. But he would have paid for the passage of his money, and stood the risk of travelling with it; that is, as to all he carried in his pocket. When he came to cash his letter of credit, he would feel the effect of the exchange, and, if he wanted to check his banker's account,

* English exchanges in the United States are always calculated 4*s.* 6*d.* to the dollar.

would have the same sort of calculations to make under the convention money as at present.

Now let us see how the matter stands in the United States. There accounts are kept in dollars and cents, and both gold and silver were legal tenders to any amount until last year. A drain of silver taking place, they reduced the value of their silver coins from 17 dwts. 4½ grains the dollar to 16 dwts. the dollar, in order to keep them at home. Of these reduced coins they have issued, since the Act was passed (March 3rd, 1853), 8,654,161 dollars. They are mere tokens, and made a legal tender for but four dollars, and a still further reduction is spoken of;* whilst, in order to attract gold, they some years since raised the value at which our sovereign is taken from 4·44 dollars to 4·84 dollars. From such fluctuations it is clear that no convention could have protected the coins of America. They do not refuse our sovereign, they do not dispute either its coined weight or established fineness; they admit both for customs duties. The case is precisely the same as if the 113 grains of gold were coined under an international convention in America. The United States declared the coin worth more than its value in England, and so they would have done had they coined it themselves; for, in consequence of the requirements of commerce, 113 grains of gold became more scarce and were more in demand, and therefore worth more, on that side the Atlantic, than on this.

An exchange of 10 per cent. and a fraction brings gold from the United States here, whether in the shape of bars, dust, eagles, or sovereigns; and an exchange under 6 per cent. sends it back. It therefore follows, that the necessity of carrying gold from the United States here, or sending it back, solely depends on where it is of most value; and this commerce regulates. Its intrinsic value may remain the same in weight and fineness, but its purchasing value varies with the circumstances of trade and the cost of transmission.

It has been the misfortune of England, from bad harvests, war, and other temporary causes, to have had such payments to make to

* The director of the United States Mint, reporting upon the alteration, states that "The depreciation of the standard weight of the half dollar and lower denominations of silver coins, authorized by the Act of 3rd March, 1853, has been attended with good results . . . The new issue has reached the sum of 8,654,161 dollars, which is a larger amount than we have struck during the five years preceding . . . The appreciation of silver made the alteration necessary . . . In England, since 1816, a silver currency has been maintained by similar means . . . There will constantly be a varying per centage of fluctuation (in the relative values of gold and silver), and this will some time be so great as to compel a legal modification of standards . . . Although our own standard of silver has so lately been changed to suit the market, there is reason to fear that the reduction of weight was not sufficient, and that another recoinage, at no distant date, may become necessary."—*Read by Mr. Franklin at Society of Arts, February 14th.*

other countries as have embarrassed our banks and merchants, and caused a severe drain on our gold, then at a premium for export.

If it were possible to induce France, the United States, England, and other countries, to adopt a gold coin of equal weight and fineness, it would save the trouble of weighing it whilst new, but nothing more. To keep any coin at a fixed purchasing power in any country is impossible, so long as that country has mercantile transactions with other nations.

After very mature consideration by our Government, it was deemed important to have but one standard of value—gold. We could not now adopt practically the American dollar, for it has become a mere token. If we were to take the Spanish dollar, or the French franc, as our principal coins, we should require a double standard—gold and silver; for if we kept our accounts with the rest of the world in silver, they would refuse to receive the payments in the tokens which now circulate. We must either make our silver coins of the full value in silver, or pay in gold.

About forty years ago the Mint commenced coining 66 shillings out of the pound of silver, in place of 62 shillings. If we take the same weight of silver bullion—its present market price being five shillings and a penny per ounce—it is evident that, when this quantity can be got for £3. 1*s.*, we shall not send away our silver tokens, which pass for £3. 6*s.*, and other countries will not take them for £3. 6*s.* when their value, even when they are new and full weight, is but £3. 1*s.* Gold and silver, as coins, must represent a higher value than the metal they contain, to prevent their leaving us, or being melted for purposes of manufacture. So far as the value of the metal goes, they are only merchandise, which, from day to day, fluctuates in value in the same way as other articles; and it is therefore impossible to prevent the value of the coins from fluctuating in foreign countries. Some think that coined gold has not, as any other commodity, a market price—we contend that it has. They lose sight of the fact that the value of bank notes is quite conventional. Let us suppose for a moment that in the markets of the world a sovereign should take only the purchasing power of half a sovereign—no one can doubt that the ounce of gold would partake of that fall in value, and still be represented by £3. 17*s.* 10½*d.* in bank notes. We may also say that bank notes are deposit receipts for the delivery of a certain number of sovereigns of a certain weight and fineness, without the least reference to the fluctuating value of gold in this or other markets of the world. If ever the United States or France

should give up their present silver currency, and adopt altogether a gold standard, you will never get them to abandon their present mode of keeping their accounts in dollars and francs. Therefore we think all we can do is to decimalize our own currency, weights, and measures: for other nations will not change what is already decimalized, and which is found so convenient—the change would only give them trouble without any adequate benefit.

But suppose it were possible to induce France, Germany, and other countries, to coin money of the same weight and fineness, what security have we that they would continue this practice? All history shows that nations, to meet the exigencies of the moment, have deteriorated and debased their coin. This may have answered for a short time in the countries themselves, but other nations were too sagacious to overlook the depreciation, and the expected advantage was counteracted by the exchanges; for no one would give so much for a bill of exchange to be paid in the debased coinage as for a bill payable in coin of a higher standard.

In *Rees's Cyclopædia* it is shown that in the time of William II., 1087, the pound troy, containing 11 oz. 18 dwts. and 18 grs. of fine gold, and 1 dwt. 6 grs. alloy, was coined into £9; and after undergoing many mutations and alterations, we now coin 11 oz. troy of pure gold, and 1 oz. of alloy, into £46. 14*s*. 6*d*. The exact weight of fine gold in our sovereign is now $113\frac{1}{6\frac{1}{2}3}$ grs., and with alloy it weighs $123\frac{1}{6\frac{1}{2}3}$ grs. The silver has also undergone many changes. At the time of William the Conqueror, 11 oz. 2 dwts. of fine silver, and 18 dwts. of alloy, were then coined into 21*s*. 4*d*., and now into 66*s*.

Since we have so altered the value of coins of the same name in various reigns, how can we expect that other nations will refrain from doing the same? Where there is a double standard of both gold and silver, they will be compelled against their will to appreciate or depreciate, as one or other coinage is found leaving them, as the United States have done and are doing with our sovereigns and their own silver. To have an international money, therefore, appears altogether hopeless. The United States may or not give up their dollar, or France the franc, as their moneys of account; but either being retained as the chief coin of circulation, or retained at all at their present value in silver, seems more than doubtful. We have nothing to gain upon the facilities we already possess for the adoption of a decimal system by waiting upon the chances of further changes in France, America, or other countries. Nothing can be more simple than the decimalization of our coinage from the pound, as it stands as the chief unit; and it is scarcely

possible to conceive anything more chimerical than the adoption of any other unit than the pound. If we took a penny or a tenpenny unit, a double standard would hardly be escaped: this would entail infinitely greater changes than acting upon the recommendation of the Parliamentary Committee and the Royal Commissions. If the tenpenny unit did escape a double standard, it would entirely change our existing accounts—the amount of every entry must be calculated in the *tenpennies* or *pennies*—the columns in existing account-books would be rendered useless—there must be multiplications and reductions, either by 24 or 240, for every payment and entry—our moneys of account and our moneys of circulation would be entirely different—we should be engaged in the child's play of endeavouring to seem very rich, by changing all our pounds into 24 *tenpennies* or 240 *pence*. We should be carving out work for ourselves in every counting-house, and bank, and shop, by undertaking to write, instead of £9, 216, if we took the tenpenny device; 2160, if we took the penny; and with all we should have to pay the sum, however it might happen to be written, in the 9 sovereigns. It would only meet the wish of a few, who have worked themselves into a belief that their schemes are perfect, and not that of practical men of business, nor of the country.

The Bank of England formerly made ingots of attested weight and fineness, but abandoned the plan, having got rid of only two. The United States Mint is authorized to coin discs of 1, 2, 3, 4, and 5 ozs., as well as to attest ingots or bars weighing from 10 ozs. upwards with an official stamp of weight, fineness, and local value. Yet neither our Mint nor the Bank of England accepts the United States Mint attestations, but melts and assays anew—indeed, it is found practically that the assays do not exactly agree.

About five years ago, at Frankfort-on-the-Maine, there was an attempt to adopt a currency which would circulate through all Germany; but nothing was done to accomplish this end. Another currency congress, for the same object, recently met at Vienna, but has broken up without doing anything. The existence of such difficulties even in the German Confederation, so closely connected in nationality and trade, is a proof of the impracticability of making a general international currency between nations of different blood. It could be attained only upon the manifestly impossible condition, that a convention of nations should establish a perfect equality at all times in the trade, and payments, and credit, *direct* and indirect, of the contracting parties; and this is the only way in which the sending to and fro of bullion could be rendered unnecessary.

On the Valuation of Government Securities. By EDWIN JAMES FARREN, *Esq., Fellow of the Statistical Society, and one of the Vice Presidents of the Institute of Actuaries.*

[Read before the Institute the 30th April, 1855, and ordered by the Council to be printed.]

AMONG the indirect advantages to be derived by the public from the establishment of Life Assurance Companies, may be fairly cited that of fostering a system of elaborate calculation so closely allied to the fiscal requirements of the times, that, allowing for hyperbole, the nation's finance minister might in the present day be in some degree characterized as the nation's actuary. Whether the incidental possession of analogous qualifications to those expected in insurance managers will ever be indirectly exacted by the general community from its finance ministers, would perhaps be presumptuous in us as actuaries to further discuss; but certain it is that, apart from insurance and its ramifications, no other definite school has hitherto been presented in which financial questions, and especially those relating to contingent finance, have ever been so elaborately and usefully developed. The interest and annuity tables alone, of insurance writers, form indeed an addition to financial literature of considerable importance.

Such being the allied nature of actuarial studies, it was perhaps to be expected that some demonstrative and yet ready mode of valuing Government securities would be current among the profession, considering the leading part such securities commonly play in our balance-sheets. From the distinctions, however, which have gradually grown up between the conditions of private and public loans, and the heterogeneous elements thereby brought into play to influence prices, the valuation of Government securities is at present such debateable ground among actuaries that I believe no less than four very distinct methods are in common use. I propose to briefly examine each of these methods, after having glanced at the distinctions, already referred to, which exist between the conditions of private and public loans.

In a private loan, the money lent is generally represented by some counter-security of professedly greater value—in order to guard, not only against the loss of principal, but of interest. In the earlier public loans we find a similar specification of security was also presented, by the assignment generally of some particular tax. Gradually, however, no such special security was afforded,

but the loans were obtained upon the general resources of the country. This want of assigned security in public, as compared with the best form of private loans, is the first distinction to which I wish to draw attention.

In a private loan, moreover, the money is either borrowed for a specific time, or is repayable under notice from either party; while in a public loan, in place of definite repayment, a power of transfer is given, and the right of notice from the lender withheld. There are other distinctions between private and public loans; but the two I have traced out—viz., the want of specific security, and the withholdance of the lender's right to give notice, but with an allowed power of transfer—are sufficient for the purpose I have in view. There is, indeed, an important species of private, and indeed of certain foreign public loans, in which the interest demanded is not merely a recompense for the hire of the money, but is also in some degree a premium of insurance for chance of loss of the principal itself, as in the various contracts on personal security. At some future time it may be useful to investigate the conditions under which such loans are most properly undertaken by Insurance Companies, and also the proportions in which loans on public, private, and personal securities should enter in a well constituted balance-sheet; but at present I must limit myself to contrasting merely such public and private loans as are considered to be effected on the best class of security only. The conditions, then, of a private loan on good security, are obviously such as scarcely admit, as a matter of argument, of profit and loss, for in all such cases the money lent is professedly the same in amount as the money to be repaid, while the right of notice on each side hypothetically prevents the interest being remarkably excessive or deficient. In a public loan, however, the right of notice being withheld from the lender, and a power of transfer substituted, the door is immediately opened to the consideration, not merely of the money lent, but of the supposed value of the contract. I say "supposed," because it is well known that prices are influenced by a multitude of so-called time bargains and other transactions that could never occur to disturb the terms of a private loan on good security. The elements of profit or loss upon the principal being thus introduced, any estimate to be made of the future value to be obtained upon transfer in place of notice necessarily involves the consideration already alluded to, in the want of specific security, of a quasi premium of insurance against any partial loss of principal, balanced as this may be wholly or in part by the chance of a relative gain. In terminable annuities,

however, the conditions of private and Government loans are more nearly assimilated; and had the fair operation of the income tax been adjusted, such annuities would have supplied nearly all the requirements of a good private and therefore of a good Government loan: for if a real loan or grant be sought, and not merely a speculation, the true conditions are that principal be repaid, by instalments or in gross, with proper interest; and it is obvious that terminable Government annuities fulfil such conditions. The reimposition, however, of the income tax, in an exaggerated form, with chance of inordinate repetition, reopens the question of part loss of principal in terminable as in perpetual stock, and reduces us, in the absence of particular Insurance Companies for guaranteeing fixed prices to stocks (for which purpose, however, in some form or other, Offices may eventually be established), to the adoption of one or other of those methods of valuation to which I have already alluded, and which I now proceed to explain.

The first method is what may be called "the purchase-price method," by which Consols, for instance, would be taken at the same price from time to time as the original price paid when purchasing them. The principal merit of this method appears to be, that it represents the whole as a mere matter-of-fact money transaction, and claims credit for the same money as matter of account that was disbursed as a matter of cash. It moreover gets rid both of the necessity of assumptive estimate, and, *pro tanto*, of a profit and loss account. Its demerits I conceive to be, its neutralizing the distinction between profitable and unprofitable stock transactions. Thus a Company, really unfortunate in such investments, would be still taking credit for not having misemployed their money, although it might be abundantly apparent that the prices originally paid were really excessive, and had become indicative of money lost rather than of capital sustained. On the other hand, a transaction skilfully conducted and bearing an obvious premium is neutralized by ranking no higher than a mere deposit of so much original money, and accordingly reckons but as such; while, on sale of stock, the system ignores the integrity of the purchase-price method, and appeals to a profit and loss account.

The second method is what may be called the "selling-price method," by which stock is valued at the selling price of the particular day upon which the valuation account itself is made up. The merit of this method is its appealing to practical minds under the universality of the proverb that "the worth of a thing is what it will bring." It also gets rid of any imputation as to the price

being an assumed price to suit any particular view or end. The price taken is a public price, and is dealt with as such. The demerit I should tax this method with is, that the price of the day is a price that, though it fit the circumstances of a day, has but little if any claim to remain persistent for more than a day in an account extending in its after-effects over a series of years. Indeed, it is easy to imagine that a day's prices might be so erratic that a Society shown by such means to be abundantly solvent in the morning might have to be considered the reverse in the afternoon. I would consequently venture to remind the lovers of financial apothegms that it is only true that "the worth of a thing, is what it will bring," "provided it hold, the thing must be sold"; for though a Company who had bought Consols at 80 might be content to be compelled to sell when prices were at 90, yet a Company who held at par would not be sellers, but buyers.

The third method, I believe, in use, is what may be called the "interest-price" method, by which all Government securities are valued as if yielding a particular rate of interest. This method appears to be a favourite one in old insurance charters and deeds, and thereby evidently suggests its rise as from a simple-minded wish to assimilate the fluctuating price of stocks with the fixed rates of interest assumed in the old methods of calculating annuity and reversion tables. Could this notion of assimilating practice with theory be effectually borne out by so obvious an expedient, such a method would doubtless at once become the favourite among modern as originally it seems to have been among the elder school of actuaries; but the truth appears to be, that so simple an assumption at starting involves, as such simple assumptions at starting are apt to do, such complex consequences in its train, that its pristine simplicity is completely buried beneath the multiplicity of details to which it gives rise. Thus, if Consols be taken at 75 as commensurate to a fixed rate of interest of four per cent., not only the price itself becomes fictitious, but the dividend becomes anomalous, for it is only at a set price that a set dividend or interest as to rate is to be uniformly obtained; numerous subsidiary calculations must therefore be entered into to show the real effects flowing from such a supposition. Moreover, a change in the rate of interest assumed in the tables, as to whether advisable or not for the future, ought to be argued upon grounds wholly independent of the temptation to adopt such a rate as will bring out prices of stock favourable to the existing balance-sheet.

The fourth method may be denominated the "average-price

method," and it is to a form of this I confess that my own opinion strongly inclines. The ordinary mode of determining the average is, I believe, to take the highest and lowest prices for a series of years, and to constitute the average thereby determined by the past as the representative of that in years to come; and it is in this generalization that the merit of this system exists. The defect of such a principle I think is, that the range over which the average is to be taken is by no means obvious, and may lead, under different hands, to a variety of average prices, which it is the very object of the average principle to avoid. The modification I would suggest is what I may term the "quarter-average method," or making up the average by taking a quarter of each of four terms, representing the price given, the price of the day, and the highest and lowest market opening prices that have intervened between the day of original purchase, or set price, and day of making up the valuation account. It is true that this is but a compromise of difficulties; but we nevertheless thereby obtain a result which partakes both of past and present, both of money laid out and money realizable. It is thus a fair arbitrative result, composed of obvious elements, and obviously sufficiently specific, according to such elements, to neutralize any undue means of arriving at desired results in order to influence either in one direction or another the bearing of the balance-sheet. I find, moreover, upon a variety of trials I have been induced to make, that the "quarter average" appears to practically represent the real gist of what a more extended consideration of limits and fluctuations would but lead us to: it was, indeed, this consideration of the problem in its more general bearings that suggested the particular form thus presented, as one sufficiently combining the chief elements of the case for practical purposes, without pretending to that perfection of demonstration which the comprehensive analysis of the exact sciences alone can afford.

With respect to terminable annuities, the price of the day appears at present to afford the only expedient for assessing them, even for a general account; for what general principle can cope with such anomalies as one minister palliating an income tax in full upon terminable annuities, because intended to be but a three-years' levy, while another not only justifies the continuance of the full assessment or its double, but even adds extra taxation upon them immediately after a loan in them has been entered into? The price of the day thus appears, from the abnormal rating of the income tax upon these investments, to be the sole method at present

justifiable for realizing terminable annuities: so wayward, from the incidents of taxation, have their relative values become.

I have thus endeavoured to rapidly bring under review some of the leading features of the methods at present adopted for the valuation of Government securities. It is a subject, I think, that will be at once admitted as a proper one for discussion among ourselves, and the more so because it very forcibly illustrates the imperfection of that view of actuarial education which relies on fixed tables of value alone, and fails to inculcate the study of those great public and fiscal questions upon which the practical bearing of all such tables so mainly depends.

On the Analogy existing between the aggregate Effects of the Operations of the Human Will and the Results commonly attributed to Chance. *By* WILLIAM A. GUY, M.B. Cantab.; *Professor of Forensic Medicine, King's College; Physician to King's College Hospital; and one of the Honorary Secretaries of the Statistical Society.*

[Read before the Institute of Actuaries, 28th May, 1855, and ordered by the Council to be printed.]

I HAVE not found it easy to select a proper title for this paper; and I am very conscious of the difficulty of explaining my reasons for instituting the somewhat laborious experiments of which I am now to state the results. Perhaps, indeed, I ought to apologize for offering to the Institute of Actuaries a communication which some of its members, who are conversant with the leading treatises on the doctrine of probabilities, may know to have been anticipated and rendered unnecessary by the labours of men much better qualified than I can pretend to be, to do justice to so profound a subject. I have some reason, however, to believe that the experiments I am about to describe are new; inasmuch as, though I have consulted one or two works which are likely to have contained some reference to such experiments, had they been already made, and have questioned more than one eminent member of your Society upon the subject, I have not been able to learn that any such experiments are upon record.

In perusing the works of M. Quetelet, it is impossible not to be struck with the remarkable analogies which he proves to exist between events brought about by the operation of the human will

and events due to physical forces over which man can exercise no sort of control; nor can we easily avoid sympathizing with him in the feeling which dictated the following well known sentiment:— "It must be confessed that, distressing as the truth at first appears, if we submit to a well-followed-out series of observations the physical world and the social system, it would be difficult to decide in respect to which of the two the acting causes produce their effects with most regularity." * On reflection, however, this feeling of distress or disappointment, no less than the very natural misgiving which springs up in the mind lest this close resemblance of the effects of moral and physical causes might be turned to the injury of morality and religion, is gradually weakened, and at length disappears. We come at last to perceive and admit that the reproduction year by year of nearly the same figures as the ultimate expressions of the workings of the will in a multitude of persons, need not excite more surprise than the annual recurrence of nearly the same proportionate number of births or deaths in the population of the same country, or even than the maintenance of nearly the same rate of profit year by year in some gambling speculation. The numbers which express the aggregate or ultimate results of the conjoint operation of a number of causes of variable and inappreciable intensity may be expected to present many analogies and coincidences, though the causes be ever so different in their nature, and though the events or actions in which they issue be termed physical in one case and moral in another.

Upon some such considerations as these—upon the assumption that there is a strict analogy between numerical results which are usually attributed to chance, and others which are very commonly treated as beyond her domain—writers on the application of the doctrines and rules of the theory of probabilities to actual practice have acted without hesitation. Thus, Gavarret, an able French author upon medical statistics, criticises with some severity the conclusions of M. Louis respecting pulmonary consumption and fever, on the score of the insufficient number of his facts, and insists on applying to those conclusions corrections avowedly drawn from treatises on the doctrine of probabilities. Now, unless I am greatly mistaken, no attempt of any kind has yet been made to show that rules and calculations derived from abstract reasonings upon probabilities, backed by a few experiments on occurrences brought about by what is commonly designated "chance," are

* Sur l'Homme, et le Développement de ses Facultés. Par M. A. Quetelet. Conclusions, book iii. chapter 3.

applicable to events of a totally different order, brought about by the operation of the human will or by the multitudinous external influences which, acting on the human frame, preserve it in health or give rise to the diseases which impair its vigour and ultimately destroy it.

By some such reflections as these I was led to plan and execute the experiments of which I am now to describe the nature and detail the results. It occurred to me that as, when we make use of the word "chance," we really mean, if we mean anything, the result of the combined operation of a number of physical causes of which we are unable to measure the intensity, so we might find a counterpart of such result in the combined operation of mental or moral causes determining the actions of mankind in the aggregate. In order to put the soundness of this opinion to the test, I made the following experimental comparison:—I extracted from the out-patient books of King's College Hospital, in forty successive groups of 25 facts each, the number of men and the number of women in the order in which they were entered on the books. On summing up the result for men and women respectively of the thousand facts thus abstracted, I found that for 369 men attending as out-patients there were 631 women. Assuming this proportion of 369 men to 631 women, as derived from 1,000 facts, to be the true proportion, and looking upon the relative attendances of men and women taken by the 25 to be the ultimate result of the combined operation of a vast variety of physical and moral causes, of inappreciable intensity, acting together—physical causes determining the proportion of illness prevailing among persons of the two sexes; and moral causes such as a sense of convenience in respect of distance, leisure, and opportunity; or preference for the particular Hospital or the particular physician, arising out of former experience, or general repute, or the recommendation of friends—I was curious to know whether, if I substituted for such moral causes as these the equally inappreciable and variable movements of the hands which determine results in games of chance, or the more common experiments in illustration of the doctrine of probabilities, I should obtain similar results. The plan that I adopted with a view to the solution of this question was the following:—I furnished myself with 1,000 peas, of which 369 were white, and represented the number of male patients; and 631 black, and corresponded to the number of female patients. These peas I put into a large bag, which I shook and stirred till its contents were thoroughly mixed, and then withdrew them blindfold by small handfuls at a time, dropping them one by one upon

a surface of paper up to the number of 25, and restoring the surplus or making up the number by fresh drawings as the case might be. After every withdrawal the contents of the bag were stirred and shaken. I made two experiments in this way; the experiments differing in this, that in the first experiment the bag was gradually emptied, its contents being transferred by twenty-fives to another bag, while in the second experiment the drawings were always returned into the bag. The results of these two experiments will be found in the subjoined table, arranged side by side with the corresponding figures as abstracted from the Hospital books. The table is so arranged as to exhibit at a glance all the correspondences between the two experiments and the abstracts.

TABLE I.

Attendances of Men, in 25 Attendances.	White Balls. First Experiment.	White Balls. Second Experiment.	Attendances of Men, in 25 Attendances.	White Balls. First Experiment.	White Balls. Second Experiment.
0	—	—	—	9	9
—	—	3	—	9	9
—	—	3	—	9	9
4	4	4	—	—	9
4	—	4	—	—	9
4	—	—	—	—	9
5	5	5	—	—	9
5	5	5	—	—	9
5	5	5	10	10	10
5	—	—	10	10	10
6	6	6	10	10	10
6	6	6	10	10	—
—	—	6	10	10	—
7	7	7	—	10	—
7	7	—	—	10	—
7	7	—	11	11	11
7	7	—	11	11	11
7	7	—	11	—	11
—	7	—	11	—	—
—	7	—	12	12	12
—	7	—	12	—	12
8	8	8	12	—	12
8	8	8	13	—	13
8	8	8	—	—	13
8	8	8	14	—	14
8	8	—	14	—	14
—	8	—	—	—	—
—	8	—	16	—	16
—	8	—	—	—	17
—	8	—	—	—	17
—	8	—	18	—	—
—	8	—	18	—	—
—	8	—	18	—	—
9	9	9			

There is room for some difference of opinion as to the degree of similarity which this table establishes between the results of the operation of the two classes of causes, moral and physical, upon the same number and the same proportion of men and women on the one side, and of white and black balls on the other; but certainly the coincidences are sufficiently numerous to justify the experiments, and to prove the idea in which they originated to be not altogether unreasonable. For if we reckon up the coincidences in the table, we find that the three groups of 40 figures each yield no less than 18 triple coincidences, while the coincidences between the abstract column and the first and second columns respectively are so considerable as 25 in number;* and this last number of coincidences will appear the more remarkable when I add, that the coincidences between the figures in the second and third columns, which show the results of two modes of drawing, are only 21 in number. Those who are familiar with experiments of this class, and with the variable manner in which figures obtained by precisely the same process from variable numerical elements group themselves, will be ready to admit that the coincidences between two columns of figures obtained by identically the same process might not exceed in number those which Table I. presents.

Having observed, even before I began to arrange the foregoing figures in tables, an amount of coincidence which seemed to justify a further prosecution of this inquiry, I determined to extend it in a direction which might afford some interesting results irrespective of the light thrown on the question under examination. I accordingly selected the number of cases of pulmonary consumption presenting themselves among the out-patients of the Hospital relatively to the number of all other diseases, looking on the consumptive patient and all other patients as a compound class of persons forming part of a community attracted to the Hospital by a variety of motives, just as the men and women of the first experiment were attracted thither. I abstracted the facts from the books in groups of 25, as before; and as the number of cases of consumption, in comparison with cases of all other diseases, proved to be small, I continued my abstracts till I had collected 5,000 cases. Of these 5,000 cases, 172 proved to be cases of pulmonary consumption, and 4,828 cases of other diseases, the proportion being 1 to 28. These cases of consumption I represented by white peas, and the other diseases by black ones; and, after mix-

* These numbers are exclusive of three instances in which the same numbers (1, 2, and 15) do not occur in either of the columns.

ing them well together, proceeded to draw them out blindfold one by one, arranging them in a row on a groove of paper, that I might note the coincidences of two or more white balls coming together, as I had already noticed the like coincidences in abstracting the cases from the Hospital books. Having some reason to believe, from the large proportion of white balls in some of the last drawings, that the peas had not been sufficiently mixed together, I repeated this experiment; so that in the following table, as in Table I., I am able to compare one abstract with two experiments.

TABLE II.

Cases of Consumption in 25 Cases.			White Balls in 25 Drawings.		
Number of Cases.		Number of Groups.	No. of White Balls.	No. of Drawings.	
				Experiment 1	Experiment 2
0	in	101	0	92	83
1	„	50	1	73	70
2	„	32	2	18	35
3	„	12	3	10	10
4		4	4	4	2
5		0	5	2	0
6		1	6	0	0
7		0	7	1	0
		200		200	200

In this table, again, there is such an amount of coincidence as would appear to justify the theory which led to the performance of the experiments. The greatest number of white balls in any drawing of the two experiments amounts to 7 in the one experiment and 4 in the other, while the greatest number of consumptive cases in any group of attendances is 6; and though there is not any very striking numerical coincidence, there is at least this degree of resemblance, that large numbers correspond to large numbers and small figures to small figures. Perhaps, too, when it is borne in mind that the cases of consumption are less than 1 in every 25 cases of all diseases, a greater amount of coincidence is not reasonably to be expected; and I regret that I did not substitute groups and drawings of 50 for groups and drawings of 25. This omission may, however, be still repaired, by bracketing together by twos the groups and drawings which follow each other in the records of the abstracts and experiments. The results of thus substituting groups and drawings of 50 for groups and drawings of 25 will be seen in the following table:—

TABLE III.

Cases of Consumption in each 50 Cases.		White Balls in each Drawing of 50.		
Number of Cases.	No. of Groups.	No. of White Balls.	No. of Drawings.	
			Experiment 1	Experiment 2
0	19	0	26	17
1	31	1	31	29
2	24	2	21	26
3	17	3	9	18
4	5	4	6	8
5	2	5	3	2
6	2	6	2	0
—	—	10	2	0
	100		100	100

When the two orders of facts — the cases from the Hospital books, and the drawings of balls from the bag—are treated in this manner, it is impossible to overlook the numerous coincidences which exist between them. If we take the second experiment, as being, for reasons already stated, the most trustworthy, the coincidences are extremely striking; for though the relative number of cases of consumption is not exactly the same as the relative number of white balls, the differences are very slight. For 19 we have 17; for 31, 29; for 24, 26; for 17, 18; and for two groups of 5, two drawings of the same number. Even the first and least complete experiment exhibits two complete coincidences (31 against 31, and 2 against 2), and one or two close approximations.

I have already stated that, in performing these experiments on the relative number of white and black balls, representing the relative number of cases of consumption and of all other diseases, I took care to arrange the balls in such a manner that I might note the coincidences and compare them with those which were found to take place in the abstracts from the Hospital books. The results obtained in this manner are shown in the following table:—

TABLE IV.

	Cases of Consumption.	White Balls.	
		Experiment 1.	Experiment 2.
Sequence of two cases or two balls	13	4	3
Sequence of three cases or three balls	2	1	0

I must confess that I had expected to find a greater similarity in the figures of the three columns of this table. I was prepared

to find as many, or nearly as many, drawings of 2 and 3 white balls in succession, as I had found of cases of consumption entered consecutively in the books. The difference in this case may perhaps have arisen from a circumstance which may to a certain extent have affected both my abstracts from the Hospital books. Men suffering from the symptoms of consumption may have been brought together in the Hospital waiting room, and have followed each other into the physician's room, just as men and women are constantly, for convenience sake, sent into the physician in small groups of either sex. Be this as it may, the results are wider apart than I had expected to find them.

There is still one other comparison which it occurs to me to make, as having some bearing upon the question which I am now examining. The comparison in question is between the number of consecutive groups of 25 facts in which 0, 1, or 2 consumptive cases, or 0, 1, or 2 white balls, presented themselves, in the one abstract and the two experiments. This comparison is made in Table V.

TABLE V.

Cases of Consumption, or White Balls.	Consecutive Groups of 25.	Number of Times.		
		Hospital Attendance.	First Experiment.	Second Experiment.
0	in 2	15	12	15
0	„ 3	6	8	2
0	„ 4	2	4	1
0	„ 5	1	1	0
0	6	1	0	1
0	7	0	1	1
	Total..	25	26	20
1	in 2	7	14	10
1	„ 3	0	3	5
1	„ 4	0	0	0
1	„ 5	1	0	0
1	„ 6	0	1	0
1	7	0	1	0
	Total..	8	19	15
2	in 2	5	0	2
2	„ 3	0	0	0
2	4	0	1	0
	Total..	5	1	2

Here, too, the resemblance between the abstracts and the two experiments is not very considerable, unless it be in the figures

that represent the number of consecutive groups of 25 cases or 25 balls in which there was no case of consumption or no white ball. If we limit our attention to the abstracts of cases and the second and more complete experiment, we find that there are 15 instances in either column of results in which there was no case of consumption and no white ball twice consecutively, and also one instance in which there was no case of consumption and no white ball six times consecutively. In the other four horizontal columns of figures there is more or less divergence. In the other divisions of the table, which show the numbers of instances in which 1 or 2 cases of consumption, or 1 or 2 white balls, presented themselves in consecutive groups of 25, the divergence of numbers is considerable. Still, when we take into account the considerable differences which would most certainly have shown themselves between three sets of abstracts of attendance at hospital, and that do show themselves between two experiments differing only in the degree to which the blending of white and black balls was effected, there is certainly nothing in this table to militate against the theory which led to the making of these experiments, but, on the contrary, a fair confirmation of the soundness of the views which dictated them.

On a careful consideration, then, of the experiments which I have brought forward in this essay, and bearing in mind the great differences which are found to prevail between the aggregate results of successive observations and experiments on precisely the same order of facts, and made in exactly the same manner, I think that I am justified in asserting that there is a very marked analogy existing between the aggregate effects of the operations of the human will and the results ordinarily attributed to chance. In other words, there is reason to believe that a very marked analogy exists between the results brought about by the volition of a considerable body of individuals set in motion by moral motives, and the results brought about by those physical movements of one, two, or more individuals which bring about the results of games of chance, or of experiments made in illustration of the doctrine of probabilities.

On the Results of the Operations of the Gotha Life Assurance Bank for the first Twenty-five Years of its existence, particularly with respect to the Mortality amongst the Lives Assured. By Herr RATH G. HOPF, *Manager of the Gotha Life Assurance Bank, and Corresponding Member of the Institute of Actuaries in London, and of the Academy for Useful Sciences at Erfurt.*

THE Gotha Life Assurance Bank was founded at a time when there existed scarcely any knowledge of life assurance in Germany. Some former attempts to found such a Company had failed for this reason, and the few life assurances which were sought at that time by Germans, especially in the Hanseatic towns, could only be effected with English Companies. An essential change has taken place in this respect since the year 1827, when, by the establishment of the Life Assurance Bank for Germany at Gotha, the desire for life assurance was awakened, and its signification and meaning were explained and advanced by numerous publications. Since that time life assurance has risen in Germany as in no other country, with the exception of England. Excepting England, no country so abounds as Germany in self-denying heads of families, who find an impulse upon their minds to provide for their families beyond their own dissolution, and who endeavour to satisfy this impulse even with slender means. Such a susceptible soil could not but prove fertile to such as understood how to cultivate it. The greater part of the Life Assurance Companies which have been founded since have therefore gone on well, although the Life Assurance Bank of Gotha has hitherto reaped most of the fruits in this field.* As it was the first German Life Assurance Office, so it has risen to be the greatest of its kind in Germany, and, as concerns the number of the lives assured, to be the greatest in Europe. Though its establishment had been completed as early as 1801, and the statute to effect assurances had been made public, yet it required, as a mutual Society, a greater number of members, before it could make binding contracts of assurance. Such a number was gathered after a short time; however, to be quite sure, the policies were not delivered till the 1st January, 1829, on which day they were issued for not less than 794 members, and for a total sum assured of £200,000. Since then the Company has increased in quick progression, and has already made not only very considerable payments to the representatives of deceased members, but also

* Pp. 58, 159.

proportionably considerable returns from the surplus to the living members. The first 25 years of its existence closed with the year 1853—an epoch very proper to lead to reflections upon the manner in which the Company has gradually developed itself, and upon the experience it has gained as concerns the mortality of the lives assured. On the former point, the following extract from the table added to the report of the Office for 1853 gives information (*see* p. 327):—

This table shows that, of the 36,583 proposals, with the total sum assured of £7,667,712, presented as eligible by the agents to the Office, about 84 per cent. or £6,438,400 have been thought fit for acceptance. The greater part of the remaining 16 per cent. was refused, because those persons who wished to be assured did not enjoy sufficiently good health. Of those who are approved the Office does not indeed demand perfect or ideal health, which could scarcely be found, but a normal state—that is, the exemption from a predisposition to sickness which may endanger and shorten life. In order that the premiums may agree with the risk, he who wishes to be received must possess that measure of health which, according to science and experience, entitles him to the expectation that he will probably reach the mean period of life which forms the basis of the calculations of the Office. By this, and by an accurate graduation of the premiums according to the ages of the assured, the greatest possible equalization of risk amongst the members of the Association which the principle of mutuality requires is attained. When reasonable grounds for believing in such a degree of health are afforded by the certificates and the examination, the assurance is accepted: in all other cases, the proposal is refused; nor is it accepted at a higher premium than the normal one, because a sufficiently sure basis for computing the proportionate increase of the premium is wanting. The expectation of life, on which the admission of new members depends, is regulated by the table of mortality on which the premiums are founded. For this purpose, the well known table computed by Mr. Babbage according to the experience of the Equitable Society has been adopted by the Gotha Life Office—with the modification, however, that the mortality of the higher ages, which that table seemed to represent as too small, and which in general appeared to be greater on the Continent than in England, was augmented. It will be seen hereafter how useful and necessary such a change was. The mean age at death, according to the table thus modified, is

Age.			Age.	Age.			Ag
60	when	admitted at	15 to 17	69	when	admitted at	45 to 47
61	„	„	18 „ 21	70	„	„	48 „ 50
62	,	„	22 „ 25	71	„	„	51 „ 53
63	„	„	26 „ 29	72	„	„	54 „ 56
64	„	„	30 „ 32	73	„	„	57 „ 59
65	„	„	33 „ 35	74	„	„	60 „ 61
66	„	„	36 „ 38	75	„	„	62 „ 63
67	„	„	39 „ 41	76	„	„	64 „ 65
68	„	„	42 „ 44	77	„	„	66 „ 67

It results from Table I. (p. 327), that, of the 27,210 members who were assured, 4,264 went off in lifetime, and 4,519 have died: the number of those who died was therefore greater than that of those who went off in lifetime—which is remarkable, considering the short existence of the Gotha Company, and differs essentially from the experience of the English Life Assurance Offices. With these the case is entirely the reverse. For example: in the Equitable Society, at the end of 1829, after the Society had existed 67 years, the number of those who had died was 5,144, and of those who had left 9,324, the latter being therefore almost the double of the former. In 15 other Life Assurance Companies, on the experience of which a well known table of mortality is founded, and of which the oldest had existed not more than 29 years, the number of deaths was 3,928, and the number of discontinued policies 11,226. The German heads of families seem therefore to execute with greater perseverance the resolution they have once taken of being assured. The cause of it may partly be, that the revenue of that part of the public which takes a particular interest in Life Assurance Companies, in Germany, and to which more especially persons engaged in civil employments belong, seems to be exposed to fewer changes as to income, and that the assured are therefore seldom obliged to drop their policies on account of an insufficiency of the means of payment. The 4,519 deaths among the assured were not all paid in accordance with the conditions of assurance; there were among them not less than 100 cases of suicide; in addition to which, the payment of the assured sum for 31 deaths was refused on account of intemperance on the part of the assured, of fraudulent declaration and statements; 30 doubtful cases of them, on which lawsuits were instituted or intended, were adjusted by agreement and by partial payments of the assured sum: so that, upon the whole, independent of the full value or market value of the policies paid according to the statutes in cases of suicide and drunkenness, only 101 cases are deducted as not payable at all.

TABLE I.—*Assurances in the Gotha Life Assurance Bank during the Years* 1829–1853.

Year.	Assurances proposed.		New Assurances effected.		Assured in the course of the Year.		Decreased. By discontinued, expired, and lapsed Policies.		Decreased. By Death.		Decreased. Total Decrease.		Net Increase.		Existing at the end of the Year.		Average Sum on one Life.	Average Age of the Assured.	
	Persons.	£.	Persons.	£.	Persons.	£.	Persons.	£.	Persons.	£.	Persons.	£.	Persons.	£.	Persons.	£.	£.	Years.	Months.
1829	1,581	392,371	1,285	339,886	1,285	339,886	..	..	12	2,157	12	2,157	1,273	337,729	1,273	337,729	265	42	7
1830	607	160,243	504	149,243	1,777	486,971	14	3,743	16	4,900	30	8,643	474	140,600	1,747	478,329	274	42	9
1831	1,491	357,343	1,244	335,486	2,991	813,814	46	18,000	27	7,329	73	25,329	1,171	310,157	2,918	788,486	270	41	10
1832	1,574	359,071	1,165	309,729	4,083	1,098,214	212	70,014	55	14,671	267	84,686	898	225,043	3,816	1,013,529	266	42	4½
1833	1,351	282,186	1,041	248,300	4,857	1,261,829	120	51,243	46	10,414	166	61,657	875	186,643	4,691	1,200,171	256	42	9
1834	1,218	249,971	902	203,500	5,598	1,403,671	154	47,929	67	15,686	221	63,614	681	139,886	5,372	1,340,057	249	43	1
1835	1,318	268,271	989	225,371	6,361	1,565,429	155	42,614	86	24,200	241	66,814	748	158,557	6,120	1,496,614	245	43	3½
1836	1,635	341,486	1,213	276,071	7,333	1,774,686	170	54,829	110	25,986	280	80,814	933	195,257	7,053	1,693,871	240	43	5
1837	1,581	347,943	1,151	269,457	8,204	1,963,329	170	41,457	127	32,400	297	73,857	854	195,600	7,907	1,889,471	239	43	8
1838	1,555	317,843	1,154	269,700	9,061	2,159,171	156	48,314	126	30,643	282	78,957	872	190,743	8,779	2,080,214	237	44	
1839	1,419	306,143	1,024	249,029	9,808	2,329,243	195	54,129	127	30,100	322	84,229	702	164,800	9,481	2,245,014	237	44	4½
1840	1,485	277,543	1,089	224,057	10,570	2,469,071	193	50,657	143	39,714	336	90,371	753	133,686	10,234	2,378,700	232	44	8¾
1841	1,414	286,014	983	229,257	11,217	2,607,957	178	41,771	164	41,986	342	83,757	641	145,500	10,875	2,524,200	232	45	1
1842	1,352	277,114	1,013	227,814	11,888	2,752,014	167	49,800	198	44,957	365	94,757	648	133,057	11,523	2,657,257	231	45	5
1843	1,503	297,514	1,049	236,914	12,572	2,894,171	193	56,943	203	46,614	396	103,557	653	133,357	12,176	2,790,614	229	45	10
1844	1,493	300,243	1,073	249,643	13,249	3,040,257	183	41,686	200	50,829	383	92,514	690	157,129	12,866	2,947,743	229	46	
1845	1,412	292,286	1,018	235,257	13,884	3,183,000	178	46,729	238	54,814	416	101,543	602	133,714	13,468	3,081,457	229	46	4
1846	1,484	280,186	1,096	235,500	14,564	3,316,957	208	55,543	230	52,243	438	107,786	658	127,714	14,126	3,209,171	227	46	7
1847	1,706	346,457	1,235	278,314	15,361	3,487,486	240	56,643	293	68,100	533	124,743	702	153,571	14,828	3,362,743	227	46	10
1848	1,151	255,743	824	218,786	15,652	3,581,529	277	77,600	339	73,757	616	151,357	208	67,429	15,036	3,430,171	228	47	4
1849	1,377	278,071	1,011	232,829	16,047	3,663,000	239	64,214	337	81,771	576	145,986	435	86,843	15,471	3,517,014	227	47	7
1850	1,537	305,428	1,138	265,786	16,609	3,782,800	196	57,471	331	81,871	529	139,486	609	126,300	16,080	3,643,314	226	47	10
1851	1,667	338,457	1,262	288,400	17,344	3,931,857	190	51,286	299	65,957	487	117,100	775	171,300	16,855	3,814,614	226	48	
1852	1,868	381,657	1,423	332,371	18,278	4,146,986	217	60,171	346	82,757	563	142,929	860	189,443	17,715	4,004,057	226	48	2
1853	1,804	368,128	1,324	307,700	19,039	4,311,757	213	59,829	399	92,614	612	152,443	712	155,257	18,427	4,159,314	226	48	3
Total..	36,583	7,667,712	27,210	6,438,400	267,622	62,365,085	4,264	1,202,615	4,519	1,076,470	8,783	2,279,086	18,427	4,159,315	..	..	5,973		
Average for one year ..	1,463	306,708	1,088	257,536	..	..	171	48,105	180	43,059	351	91,163	737	166,373	..	..	239		

In reducing the money values, £1 is taken = 7 thalers.

The following table shows the proportion of cash payments consequently made to that which was to be expected from the adopted table of mortality:—

TABLE II.—*Claims payable during* 1829–1853.

Year.	Amount.					Expected according to the Table in use.			Difference.			
			Average for one Life.	Per cent. of the Assurances.				Average for one Life.	There died			
									Plus.		Minus.	
	Persons.	£.	£.	Pers.	£.	Persons.	£	£.	Persons.	£.	Persons.	£.
1829	12	2,157	180	0·93	0·63	15·19	3,997	263	..	..	3·19	1,840
1830	16	4,900	306	0·90	1·01	22·84	6,241	273	..	..	6·84	1,341
1831	27	7,186	266	0·90	0·88	32·38	9,036	279	..	..	5·38	1,850
1832	53	14,614	276	1·30	1·33	50·25	13,526	269	2·75	1,088		
1833	46	10,147	221	0·95	0·80	64·81	16,814	259	..	..	18·81	6,667
1834	65	15,471	238	1·16	1·10	77·19	19,415	251	..	..	12·19	3,944
1835	85	23,628	278	1·34	1·51	89·34	22,299	249	..	1,329	4·34	
1836	108	25,600	237	1·47	1·44	103·42	25,462	246	4·58	138		
1837	123	31,059	252	1·50	1·58	117·99	28,880	245	5·01	2,179		
1838	124	30,271	244	1·37	1·40	133·88	32,579	243	...	..	9·88	2,308
1839	123	29,486	239	1·25	1·26	149·74	36,318	243	..	..	26·74	6,832
1840	136	36,600	269	1·29	1·48	164·25	39,562	241	..	..	28·25	2,962
1841	158	39,789	252	1·41	1·53	179·02	42,772	239	..	..	21·02	2,940
1842	191	42,971	225	1·61	1·56	193·93	46,107	238	..	..	2·93	3,136
1843	201	46,257	230	1·60	1·60	208·55	49,317	236	..	..	7·55	3,060
1844	197	48,629	247	1·49	1·60	224·81	52,964	235	..	..	27·81	4,335
1845	235	54,457	232	1·69	1·71	241·13	56,754	235	..	..	6·13	2,297
1846	224	50,543	226	1·54	1·52	257·03	60,284	235	..	..	33·03	9,741
1847	289	67,143	233	1·88	1·93	275·53	64,311	233	13·47	2,832		
1848	326	70,929	218	2·08	1·98	290·27	67,786	234	35·73	3,143		
1849	335	80,929	242	2·09	2·21	302·28	70,685	234	32·72	10,244		
1850	326	79,514	244	1·96	2·10	318·57	74,356	233	7·43	5,158		
1851	293	65,571	224	1·69	1·67	337·52	78,595	233	..	..	44·52	13,024
1852	335	79,271	236	1·83	1·91	359·54	83,493	232	..	..	24·54	4,222
1853	390	90,514	232	2·05	2·10	382·07	88,407	231	7·93	2,107		
Total	4,418	1,047,636	6,047	37·28	37·84	4591·53	1,089,960	6,110	109·62	28,218	283·15	70,499
Average for one year ..	177	41,905	241	1·49	1·51	183·66	43,598	244				

In reducing the money values, £1 is taken = 7 thalers.

If from the number of deaths which was to be expected according to the table in use, 4591·53, the real deaths 4519 are deducted, there results a minus of 72·53 deaths, or 1½ per cent. of the number expected; and if those 101 cases which have not been paid are added thereto, the difference is 173·53, or about 3¾ per cent. of the claims expected. This difference, which is on the favourable side, appears very trifling, and the table of mortality on which the calculations of the Gotha Life Office are founded has therefore proved upon the whole perfectly convenient and safe. Much greater differences exist between other tables, constructed with the greatest care and founded upon an apparently perfectly safe basis, as will be seen hereafter.

It will not be uninteresting to examine what the mortality has been, according to the different ages of the assured. This may be seen in the comparisons which the Gotha Life Office publishes every year, in its reports, of the real number of deaths with the

number expected,* by uniting the comparisons of each year into one, and taking into consideration how many persons have entered the Company and how many have withdrawn at the different ages. Table III., following, shows this combination, together with the consequences resulting from it. This table contains two deaths more than Tables I. and II., which happened before the end of 1853, but were not announced to the Office until after the publication of the report for this year.

Table I. shows that the total number of persons assured in each of the last 25 years was 267,622. The majority of these persons was indeed assured, or exposed to the risk, during the whole course of the corresponding year; but those who entered and withdrew in any year were not in this case. Persons enter and withdraw during all periods of the year in about the same proportion; consequently it may be assumed, for the computation of the rate of mortality, that the accession to the Company and leaving the same take place in the middle of the year, and each of the members who acceded and retired has been assured only six months in the first and last year. The above accession of 27,210 persons, and the withdrawal of 4,264 persons, have taken place in the different groups of age, in the manner shown in Cols. 3 and 4. If we deduct half of this total number (Col. 6) from the persons who were assured (Col. 2), we exhibit (Col. 7) the number of those persons who, upon an average, were assured, or exposed to the risk, for the term of a whole year. Comparing with this number the number of deaths specified in Col. 8, we find the rate of mortality given in Col. 9.

TABLE III.—*Persons assured and Deaths during* 1829–1853.

Ages.	Number of Persons assured.	Number of Persons entered.	Number of Persons gone in lifetime.	Total of 3 and 4.	Half of the numbers in Col. 5.	Number exposed to the risk of Mortality for a whole Year.	Deaths.	Rate of Mortality per cent.
1.	2.	3.	4.	5.	6.	7.	8.	9.
15—25	2,312	998	137	1,135	567	1,745	8	0·46
26—30	11,547	3,731	468	4,199	2,099	9,448	82	0·87
31—35	28,842	5,832	716	6,548	3,274	25,568	234	0·92
36—40	43,075	5,795	736	6,531	3,265	39,810	398	1·00
41—45	48,971	4,296	705	5,001	2,500	46,471	483	1·04
46—50	45,092	3,050	667	3,717	1,858	43,234	627	1·45
51—55	35,987	2,056	432	2,488	1,244	34,743	631	1·82
56—60	25,594	1,237	254	1,491	745	24,849	689	2·77
61—65	15,412	205	105	310	155	15,257	585	3·83
66—70	7,327	10	35	45	22	7,305	444	6·08
71—75	2,692	..	6	6	3	2,689	243	9·04
76—80	697	..	3	3	1	696	79	11·35
81—85	71	..	..	..	..	71	17	23·94
86—90	3	..	..	..	..	3	1	33·33
Total..	267,622	27,210	4,264	31,474	15,733	251,889	4,521	

* Page 60.

If we cast a glance on the last column of this table, we are immediately struck by the insignificant mortality in the ages under 55 years, and the rapid increase of the same beyond that age. Although it is established by the usual laws of the duration of human life that, after passing the first years of life, mortality increases from one stage of life to another, and that especially the 55th year forms an epoch from which this increase takes place in much greater progression than before, yet other tables of mortality do not show in this respect so considerable a contrast as the above table. This contrast becomes still more evident if the common average of mortality under 55 years and that above 55 years be compared in different tables, and if it be computed for every one of them how much per cent. the average above 55 years exceeds that under 55 years.

TABLE IV.—*Rate of Mortality under* 55 *years and above* 55 *years.*

Ages.	Gotha Life Office, 1829-1853.	Equitable Experience, 1762-1829. (Morgan.)[1]	Seventeen English Life Assurance Offices,[2] 1762-1840.	Prussian Widows' Fund, 1776-1845. Males. (Brune.)[3]	England. Males. (Farr, 1841.)[4]	France. Males. 1817-1851.[5] (Demonferrand.)	Belgium. Males. 1844-1845.[6] (Quetelet)	Hanover, 1836-1843. (Tellkampf.)[7]	Saxony. Males. 1834-1850. (Leonhardi.)[8]
26—55	1·18	1·18	1·19	1·32	1·37	1·22	1·98	1·58	1·59
56—85	9·50	8·15	8·20	9·47	8·26	8·81	8·79	9·63	10·69
Increase per cent.	703	590	587	618	502	622	344	509	572

[1] Tables showing the total number of Persons assured in the Equitable Society from its commencement in September, 1762, to January 1, 1829. By Ar. Morgan. London, 1834, page 28, Table C.

[2] Series of Tables calculated from a New Rate of Mortality amongst Assured Lives. By Jenkin Jones. London, 1843, Table X.

[3] Masius Versicherungszeitung für 1847, pp. 187, 196. Assurance Magazine, Vol. III., page 29.

[4] Sixth Annual Report of the Registrar-General. London, 1845, pp. 597, 598.

[5] Journal de l'Ecole Royale Polytechnique. Tome XVI. Paris, 1838, page 306.

[6] Annuaire de l'Observatoire Royal pour 1850, par Quetelet. Brussels, 1849.

[7] Die Verhältnisse der Bevölkerung und der Lebensdauer im Königreich Hannover, von Dr. A. Tellkampf. Hanover, 1846, pp. 120, 121.

[8] Statistische Mittheilungen aus dem Königreich Sachsen, herausgegeben vom Statistischen Bureau des Ministerium des Innern. II. Abtheilung, II. Lieferung. Dresden, 1852. (A most excellent work).

The Gotha Life Office accordingly shows the highest measure of the gradation of mortality from one period of life to the other. Whilst the mortality amongst the assured members is uncommonly low in the former, it is uncommonly high in the latter.

It will not be uninteresting to examine the proportion which

exists between the mortality of the persons assured by the Gotha Life Office and that of the assured by other similar Companies. Those Offices which assure only healthy persons, demanding at least a medical certificate of health of everyone who desires to become a member, must experience a much less mortality amongst the assured than we find in a mixed population, provided the necessary care be taken and proper precautions used. It is not to be denied that life assurance offers peculiar attractions to weak and sickly persons affected with hidden complaints, whom the fear of an untimely death oppresses, more than to healthy vigorous men, and that it impels them much more to seek admission into such a Society. There exists a striking difference between the Offices assuring sums payable at death and those assuring annuities. Whilst it is chiefly healthy and vigorous individuals, who have the prospect of a long enjoyment of the annuities they are about to purchase, who throng to the latter Companies, and such Societies naturally consist of persons in the most perfect state of health, Life Assurance Offices have always to struggle with the opposite elements. In this struggle they are not always successful; notwithstanding all care and precaution, they are sometimes deceived. At all events, the majority of the persons assured by them possess only a moderate measure of health at the time of their acceptance; they are, indeed, free from such diseases and complaints as are attended with a speedy death, but not from slighter affections, which, though not presenting instant danger, may yet lay the germ of death by and by. While perceptibly bad health is excluded, because it would cause an evident loss, persons who form select specimens of health, confiding in a duration of life which they think not likely to be shortened, are but little attracted. Persons enjoying a moderate measure of health, being sometimes reminded of dangerous diseases by little indispositions, are however warned to attend to their health, live in general carefully and regularly, and frequently reach therefore an advanced age. In them the proverb is verified, that creaking waggons often last longest.

The influence which the medical examination has on the diminution of mortality amongst the assured evidently appears greatest at the time of admission, and in the years immediately following it. The admission to the Office being by preference at the younger ages, and particularly in the ages of 30–45 years, mortality must also be but trifling in these and the next classes of age. Afterwards, the state of the health of the assured members becomes by degrees of a more mixed character, and therefore the mortality

increases. A principal reason of this increase is, that a great number of assured persons withdraw during life. The majority of the individuals who drop their assurances during life and suffer the loss connected therewith being quite healthy, but the members who become sickly regularly continuing their assurances, the average state of health of the Society must be deteriorated in the higher ages, and mortality of course increased, in these classes.

The increase of strictness and precaution at the acceptance of new members will therefore principally produce effect on the mortality at those classes of age at which admission principally takes place, and in those which immediately follow, but have less influence on the classes of later ages, in consequence of the above-named circumstance.

Table V., following, shows the rate of mortality among the members assured by the Gotha Life Office and of those in the English Offices. We perceive that the mortality under the age of 55 years is nearly equal in both cases. Beyond this age, on the contrary, the mortality is not so great amongst the members of English Companies as of the Gotha Office. The reason of this difference is, in our opinion, not to be sought in the circumstance that the English Companies are more circumspect in the reception of new members, and that the physicians there, in consequence of their more extensive experience in this field, understand better than the German ones how to estimate the duration of human life by the different condition of their state of health. If this were the case, the effects would be seen more in the ages below 55 years. That this difference does not present itself till the higher ages, is a proof that there must be some other reason. We can only come to the conclusion that, in general, mortality is less at the higher ages in England than in Germany.

In examining the above table, every reader will be struck by the very trifling mortality amongst the members of the Friendly Societies in England. Though these Societies in a great measure comprise only tradesmen, workmen in manufactories, and labouring men, and though we are accustomed to think the mortality in the lower classes of the people greater than in the middle and higher ranks, by whom life assurance is generally resorted to, yet it is just the reverse in England. We cannot doubt the correctness of this observation, on account of the high respectability of the two authorities, Neison and Finlaison, the latter of whom entered on the inquiry of the returns made by order of Parliament. We will, however, limit this observation only to England, and not extend it

TABLE V.—*Mortality of the Assured Persons in the Gotha Life Assurance Company, compared with the Mortality of the Assured Persons in other Companies.*

Ages.	Gotha Life Office.	Equitable Society.					17 English Life Assurance Offices.					Friendly Societies (Neison). [1]					Friendly Societies (Finlaison). [2]					Prussian Widows' Fund. Males (Brune).				
		Mortality per cent.	Difference.				Mortality per cent.	Difference.				Mortality per cent.	Difference.				Mortality per cent.	Difference.				Mortality per cent.	Difference.			
			Plus.		Minus.			Plus.		Minus.			Plus.		Minus.			Plus.		Minus.			Plus.		Minus.	
			Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.
26—30	0·87	0·78	..	..	0·09	10·3	0·81	..	..	0·06	6·9	0·73	..	..	0·14	16·1	0·75	..	..	0·12	13·79	0·70	..	..	0·17	19·54
31—35	0·92	0·88	..	..	0·04	4·4	0·89	..	..	0·03	3·3	0·80	..	..	0·12	13·0	0·80	..	..	0·12	13·04	0·85	..	..	0·07	7·61
36—40	1·00	1·03	0·03	3·0	..	..	0·99	..	..	0·01	1·0	0·89	..	..	0·11	11·0	0·95	..	..	0·05	5·00	1·09	0·09	9·00	..	..
41—45	1·04	1·18	0·14	13·5	..	..	1·13	0·09	8·7	..	..	1·04	..	..	..	..	1·13	0·09	8·65	..	..	1·35	0·31	29·81	..	..
46—50	1·45	1·38	..	..	0·07	4·8	1·43	..	..	0·02	1·4	1·29	..	..	0·16	11·0	1·37	..	..	0·08	5·52	1·74	0·29	20·00	..	..
51—55	1·82	1·85	0·03	1·6	..	..	1·91	0·09	4·9	..	..	1·70	..	..	0·12	6·6	1·77	..	..	0·05	2·75	2·24	0·42	23·08	..	..
56—60	2·77	2·68	..	..	0·09	3·2	2·65	..	..	0·12	4·3	2·24	..	..	0·53	19·1	2·45	..	..	0·32	11·55	3·11	0·34	12·27	..	..
61—65	3·83	3·72	..	..	0·11	2·9	3·79	..	..	0·04	1·0	3·05	..	..	0·78	20·4	3·12	..	..	0·71	18·54	4·60	0·77	20·10	..	..
66—70	6·08	5·48	..	..	0·60	9·9	5·55	..	..	0·53	8·7	4·62	..	..	1·46	24·0	4·75	..	..	1·33	21·87	6 57	0·49	8·06	..	..
71—75	9·04	7·89	..	..	1·15	12·7	8·13	..	..	0·91	10·1	6·85	..	..	2·19	24·2	6·70	..	..	2·34	25·88	10·04	1·00	11·06	..	..
76—80	11·35	11·18	..	..	0·17	1·5	11·88	0·53	4·7	..	..	8·84	..	..	2·51	22·1	10·32	..	..	1·03	9·07	13·18	1·83	16·12	..	..
81—85	23·94	17·97	..	..	5·97	24·9	17·22	..	..	6·72	28·1	11·97	..	..	11·97	50.0	15·31	..	..	8·63	36·05	19·32	..	..	4·62	19·30

[1] Contributions to Vital Statistics, by Neison, London, 1846. Table F, page 28.

[2] Friendly Societies' Return to an Order of the House of Commons. Finlaison, 1853.

to the Continent, especially not to Germany. Respecting the latter country there are still wanting sufficiently sound and extensive observations to answer this question with precision; what, however, the author has observed in smaller circles does not admit the application to Germany of the observations which have been made in England. On the contrary, with us, mortality among the members of Funeral Funds and Funds for Sick Allowances to persons of the less wealthy population, a great number of which exist in Germany, is much greater than in the middle and higher classes; and, in order to enable these Associations to continue in existence, the contributions of the members must be fixed higher for Funeral Funds here than for Life Assurance Companies.

One important circumstance must, however, not be left quite out of consideration. The experience on which the tables of mortality of the Equitable Society and of the seventeen English Life Assurance Companies are founded goes back to the last century, and, as regards the Equitable, as far back as 1762. The experience of the Friendly Societies commences only from a recent period. It is known that mortality has decreased since the restoration of the universal peace in Europe, and the difference would perhaps not be so great as the above table indicates, if we could compare only the more recent experience of the English Life Assurance Companies with that of the Friendly Societies. The experience of the Gotha Office is likewise of the later period—it agrees with that of the Friendly Societies in the class of 41–45 years, but shows a higher mortality for all other classes of age, and this difference is greater for the higher than the lower ages; which perfectly agrees with and confirms the observation before made, that a less mortality is experienced at the higher ages in England than in Germany.

The last of the above columns shows the proportion of the mortality of the Gotha Office to that of the Prussian Widows' Fund at Berlin. This institution, which has been in existence since 1776, was formerly open to everyone in all Germany, but has been limited since 1830 to individuals in civil offices in the kingdom of Prussia, the contributions of the members being too low and great losses having resulted, which the Prussian government must cover. Up to the year 1845 more than 40,000 married couples had been admitted; amongst whom, up to that period, 11,501 deaths of husbands and 12,788 deaths of wives and widows had happened. Relying upon these data, Brune has computed the two celebrated tables of mortality, which deserve great confidence,

as well on account of the accuracy as of the extent of the observations. Though both Companies (Gotha and Berlin) obtain their members from nearly the same classes of population—that is to say, from the middle and higher classes, especially of men in office—yet the mortality is considerably higher in the Prussian Company than in the Gotha Office. The cause of this difference is partly to be found in the circumstance that the Widows' Fund does not proceed with the same strictness as the Gotha Office in the admission of its members, but demands only the production of a general and short testimony from the physician of the party desirous of admission, without entering into a closer and more accurate examination of his health; partly in the circumstance that the experience of the Prussian Widows' Fund extends principally over the end of the last and the beginning of the present century, when the rate of mortality was greater than during the last 40 years. The considerable difference in the ages of 41–55 years, at which the mortality in the Gotha Company is much diminished, in consequence of a greater strictness on admission, shows that the former of those reasons had a great influence and effect.

In order to show more clearly the difference which is produced in mortality by the medical examination of the persons who desire to be assured, we exhibit in Table VI. (*see* p. 336) a comparison of the mortality at the Gotha Office with that in several mixed populations.

We still want in Germany a general table of mortality of some authority. In most of the German States, sufficient attention is not paid to the statistical facts requisite for the computation of such a table, which can only be obtained by accurate enumerations of the people, often repeated, subdivided according to the classes of age, and by keeping accurate registers of deaths. The greatest confidence may be reposed in the tables of mortality computed by Tellkampf for the kingdom of Hanover, and by Leonhardi for the kingdom of Saxony; we have therefore made use of them for comparison. Though all the observations forming the basis of these tables have originated in recent times, yet both tables indicate for all ages an essentially higher mortality than that which rules amongst the members of the Gotha Company; the examination of health to which these members were obliged to submit at their admission has therefore contributed in no slight degree to the diminution of mortality.

The same fact results from a comparison with the mortality in

TABLE VI.—*Mortality of the Assured Persons in the Gotha Life Assurance Company, compared with the Mortality of some mixed Populations.*

Ages.	Gotha Life Office.	England. Males—Farr.					France. Males—Demonferrand.					Belgium. Males—Quetelet.					Hanover. Teilkampf.					Saxony. Males—Leonhardi.				
		Mortality per cent.	Difference.				Mortality per cent.	Difference.				Mortality per cent.	Difference.				Mortality per cent.	Difference.				Mortality per cent.	Difference.			
			Plus.		Minus.			Plus.		Minus.			Plus.		Minus.			Plus.		Minus.			Plus.		Minus.	
			Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.		Absolute.	Per cent.	Absolute.	Per cent.
26—30	0·87	0·97	0·10	11·49	..	..	0·88	0·01	1·15	..	..	1·50	0·63	72·30	..	..	1·11	0·24	27·59	..	..	1·07	0·20	22·99	..	..
31—35	0·92	1·10	0·18	19·57	..	..	0·95	0·03	3·26	..	..	1·58	0·66	71·74	..	..	1·25	0·33	35·87	..	..	0·99	0·07	7·61	..	..
36—40	1·00	1·25	0·25	25·00	..	..	0·94	..	..	0·06	6·00	1·72	0·72	72·00	..	..	1·35	0·35	35·00	..	..	1·15	0·15	15·00	..	..
41—45	1·04	1·42	0·38	36·54	..	..	1·17	0·13	12·50	..	..	2·16	1·12	107·69	..	..	1·48	0·44	42·31	..	..	1·57	0·53	50·96	..	..
46—50	1·45	1·62	0·17	11·72	..	..	1·43	..	..	0·02	1·38	2·43	0·98	67·59	..	..	1·77	0·32	22·07	..	..	2·07	0·62	42·76	..	..
51—55	1·82	1·87	0·05	2·75	..	..	1·93	0·11	6·04	..	..	2·47	0·65	35·71	..	..	2·50	0·68	37·36	..	..	2·71	0·89	48·90	..	..
56—60	2·77	2·71	..	..	0·06	2·17	2·50	..	..	0·27	9·75	3·15	0·38	13·72	..	..	3·61	0·84	30·32	..	..	3·71	0·94	33·94	..	..
61—65	3·83	3·95	0·12	3·13	..	..	4·10	0·27	7·05	..	..	4·24	0·41	10·70	..	..	5·70	1·87	48·83	..	..	5·43	1·60	41·78	..	..
66—70	6·08	5·75	..	..	0·33	5·43	5·60	..	..	0·48	7·89	5·83	..	..	0·25	4·11	7·91	1·83	30·10	..	..	7·37	1·29	21·22	..	..
71—75	9·04	8·32	..	..	0·72	7·96	9·25	0·21	2·32	..	..	8·65	..	..	0·39	4·42	9·34	0·30	3·32	..	..	10·39	1·35	14·93	..	..
76—80	11·35	11·94	0·59	5·20	..	..	12·79	1·44	12·69	..	..	12·86	1·51	13·30	..	..	12·74	1·39	12·25	..	..	15·48	4·13	36·39	..	..
81—85	23·94	16·90	..	..	7·04	29·41	18·62	..	..	5·32	22·22	18·01	..	..	5·93	24·77	18·50	..	..	5·44	22·72	21·74	..	..	2·20	9·19

Belgium—a country the greater part of which belonged to Germany for a considerable time, which is inhabited by a German race, and where German manners and institutions are still to be found. Mortality is much greater here, especially at the younger ages, and somewhat lower at the higher ages, than amongst the persons assured in the Gotha Office. The latter difference would very likely not be observed, or perhaps fall even on the opposite side, if precisely the same ages could be compared with each other; Quetelet, however, has given the mortality in leaps from 5 to 5 years, and for the ages of 25–29, 30–34, 35–39, &c., in the most recent table, which I have made use of, and which seems to deserve the greatest confidence. All these classes of age are therefore a year lower than in the other tables given for comparison, which makes an essential difference for higher ages, and naturally gives the average of mortality too low in comparison with the other tables.

(To be continued.)

Memoir of the late Griffith Davies, Esq., F.R.S.; abridged from a more extended one, by his Nephew, THOMAS BARLOW, *Esq., of the Metropolitan Life Assurance Society.*

MR. GRIFFITH DAVIES was born on the 28th of December, 1788, at the foot of Cilgwyn Mountain, in the parish of Llandwrog, Carnarvon. His father, Owen Davies (or, in Welsh, Owen Dafydd), who was a most industrious and worthy man, held a small tenement, and devoted his spare time to work in the neighbouring slate quarries. His family consisted of four sons and three daughters. A short time previous to the birth of his second child, Griffith, a brother of his died whose name was Gryffyth Dafydd, and out of respect for his memory Mr. Davies was called after him; otherwise he would have been named "Gryffydd Owen," in accordance with the custom of the country, which gives for a surname to the son the Christian name of the father. Mr. Davies, when a child, was exceedingly delicate, so much so that his parents entertained great fears as to whether they would be able to rear him; but his health somewhat improved as he grew to manhood.

The spot on which he was ushered into life was at that time completely isolated. There were no schools of any kind whatever either in the parish of Llandwrog or in any of the parishes adjoin-

ing, and on that account Mr. Davies' father, like most of his neighbours, had never been taught to read in any language; but his mother, who was a delicate woman, possessing nevertheless a large amount of common sense and mental power, had by some means learnt to read the Welsh language tolerably well. The first instruction Mr. Davies ever received was at a Welsh Sunday-school, originated and conducted by a poor man who worked in the slate quarries. This poor fellow met with considerable opposition in this good work from many of the inhabitants, who thought that learning to read was by no means proper employment for Sunday. Mr. Davies, however, as long as he lived, considered himself very much indebted to that worthy man; and by means of this and similar efforts he soon acquired the art of spelling and reading the Welsh language, and also a little writing; but in consequence of his delicate state of health his progress was much retarded.

When about seven years old, a school for teaching English was opened in the adjoining parish, the master of which lodged at a house where an aunt of Griffith Davies was in service. On one occasion this person asked Griffith to perform some little errand for him, and rewarded him with a penny, telling him at the same time to buy a book and come to school. Griffith went home full of joy, and related to his parents all that had passed between him and the schoolmaster; and, the penny book being procured, he was forthwith despatched to school, a distance of two miles across a rough and rugged country, taking with him a little refreshment merely.

At the end of the first week he brought home his penny book, stating that he had learnt it all and that he wanted a spelling-book, which was promptly procured, and he continued in his attendance there for five or six quarters, at the rate of 2*s*. 6*d*. per quarter; but, although he was beginning to acquire the art of reading English tolerably well, yet, from the circumstance of all his relations and his associates being exclusively Welsh, he could make but little progress in understanding the meaning of what he read.

Being now between eight and nine years old, Griffith, or Guto as he was called in the country, was taken home from school to render what little assistance he could upon the land held by his father. When about the age of twelve, he was sent to an English day-school established in the parish where he was born; but, owing to the straitened circumstances of his parents and the dearness of provisions, occasioned by the bad harvests of the years 1800 and 1801, he was again taken from school before six months had elapsed, and hired to a distant relation on a neighbouring farm,

to drive horses, assist in manuring and ploughing land, and for similar duties, his wages being at the time his only support.

At the age of fourteen he became permanently engaged in the slate quarries; and, having passed through the usual ordeal of apprenticeship, he in a short time began to earn a man's wages, support himself, and save a little money. By this means, at the age of seventeen he placed himself again in an English day-school at Carnarvon, and at this comparatively advanced age began to learn for the first time the principles of the numeration table. It is worth observing, that although he had manifested considerable acuteness and ingenuity in the manufacture of various articles of ornament and utility, it had never before been discovered that he had a capacity or liking for figures in any way whatever; but in three months' time he was enabled to go through an elementary system of arithmetic, and made considerable progress in spelling, reading, and writing the English language, although the principles of grammar were as yet entirely unknown to him. On his return to the quarry at the expiration of the three months he became fully conscious that he possessed some power which he was before unconscious of, and which was continually becoming developed. Every moment now that he could spare, he might be seen with his iron pen covering some of the slates which he had to manufacture with arithmetical calculations, and no doubt by this means he made considerable progress in acquiring that expertness and dexterity in computations which in afterlife proved so useful to him.

Having now arrived at the age of nineteen, and finding that almost all those who had learnt a little English succeeded in attaining to something higher than mere manual labour, Griffith began to feel a strong inclination to go in search of some employment in England, so as to have an opportunity of improving himself in the language. Determined as he was upon this adventure, his ideas at the time were not by any means exalted: all he seemed to care for was, to have something to do in any part of England by means of which he could maintain himself whilst he made further progress in the English tongue, and then to return to his native country.

Having collected together the little money which he had managed to save, and procured a few letters of recommendation to persons in the metropolis, he sailed from Carnarvon on or about the 6th of September, 1809. The vessel arrived in London on the 15th September, 1809; and Griffith, having no relations or friends in London, was kindly allowed by the master to remain on board

for a few days, and was taken about by him, so as to learn his way, and seek some employment in the great city.

Before Griffith left home, a neighbouring farmer had requested him to carry half a guinea as a present to his nephew, who was at a school at Cambridge Heath, Hackney Road, which errand Griffith willingly undertook to perform; and on his calling at that school he was first introduced to the lady of the house, who, observing his extreme timidity, and that he was but imperfectly acquainted with the English tongue, called a Welshman who was then engaged at the school, to speak to him. His fellow countryman received him kindly, and, learning what his object was, suggested to him that, if he had the means, the best plan would be to place himself at a school—adding, that he had no doubt he would be able to get a situation as an usher at the then ensuing Christmas vacation. Having performed his errand, Griffith adopted this suggestion at once, and lost no time in seeking lodgings and a school on his way to the vessel that very day.

The assistant at Cambridge Heath School advised Griffith to try to improve himself in writing and grammar; but when he went to school, so little was his delight in the one or the other of those studies, that his mind even recoiled from them, and from every subject whatever except arithmetic: and finding the master at this school unable to be of any great assistance to him in that science, he changed to another, and from that to a third school, till at last he met with a man of the name of Westbrook, in Westminster, much of the same disposition as himself, and who acted towards him more as a father than a stranger: he used to assist him in seeking to obtain situations, and even drew up an advertisement for him with that object, suggesting also that he should leave his address at some of the school stationers, stating himself to be in want of employment.

Failing in these endeavours, and seeing no chance of being employed in any intellectual capacity, he fully made up his mind to seek a situation as a porter or messenger, and went about for that purpose; and it is due to him to remark, that he never shunned work of any kind, whether mental or manual. Having been one whole day thus engaged, and being unsuccessful, he returned home quite broken-hearted, not knowing what to do, unless to return to his own country; but on his reaching his lodgings, a ray of light shone upon him in the shape of a letter from a schoolmaster in the neighbourhood of Sadlers' Wells, of the name of Rainalls, requesting that he would call upon him, which he did, and was

engaged as the arithmetical tutor of his school, at a salary of £20 a year, with his board and residence. This was in January, 1810; and as he had nothing else to do but to attend to the arithmetical and mathematical departments, he had a good deal of time at his disposal, which he assiduously devoted to the cultivation of his own mind, especially as regarded English grammar and mathematics, in the latter of which he soon made such proficiency as to be able to calculate the times of the eclipses and exhibit their mode of occurrence by diagrams.

With this gentleman Mr. Davies remained until the summer of 1811, when, some apprentices of the establishment having become capable of undertaking the duties of it, Mr. Davies' services were required no longer. It now having become known that he was about to leave Mr. Rainalls, a Mr. Birt, schoolmaster in the Barbican, informed him that there was a schoolroom vacant, at a very low weekly rent, in James Street, Old Street, and suggested to him the propriety of hiring that room and beginning on his own account. This Mr. Davies ventured upon, and Mr. Birt was obliging enough to let two of his own sons attend there at the commencement, Mr. Rainalls still employing him to give private lessons at his establishment. In order at the same time to prosecute his own studies, he joined the Mathematical Society, whose meetings were held in Crispin Street, Spitalfields; and he was often heard to say, that the extensive library possessed by that Society, and the assistance rendered him by some of its intellectual members, were of incalculable benefit to him.

In the summer of 1812 he ventured upon a good-sized house in Lizard Street, Bartholomew Square, Saint Luke's, and most of his old scholars followed him thither, and in the month of November he thought his prospects justified his taking a wife; but soon after that important step, most of the younger members of his school left him on account of the streets, which were new and unpaved, becoming almost impassable, and during the greater part of that winter his prospects were materially clouded. He did his utmost to maintain himself and wife by correcting the press of a Welsh magazine then published; and about this time he also commenced writing his *Key to Bonnycastle's Trigonometry*. In the month of October, 1813, his first child, a daughter, was born—his joy for which event was no doubt greatly alloyed, in his then straitened circumstances, by his consciousness of the increased responsibility which it involved.

It was to the first edition of Bonnycastle's Trigonometry that

Mr. Davies originally wrote his *Key*, and which contained in particular one extremely difficult problem he had for a long time laboured to solve, seeking the assistance of a celebrated writer in the *Lady's Diary*, who after several weeks of trial did not seem to be more successful than himself; but at last Mr. Davies, with unwearied application, devised some mechanical means to assist his conception, and succeeded in solving the problem. Unfortunately, however, as he was about to put the *Key* to press, a second edition of the work appeared, in which the problem in question had been omitted, and a number of other alterations made—and this, of course, rendered it necessary for him to write the *Key* afresh; but so poor was he at the time, that he had not a sufficient sum of money to buy himself a copy of the second edition, and he was actually obliged to take some books from his then scanty library, which were almost equally indispensable to him, to a bookseller's shop, in exchange for a copy of the new edition. In spite of all difficulties, the *Key* to the Trigonometry came through the press in the summer of 1814.

Although the publication of this work was of but little pecuniary advantage to him, still it answered the purpose well, in bringing his name before the public and establishing his character as a mathematician. Soon after this he began to receive private pupils; and amongst the first of these was a gentleman connected with an Assurance Office, who, being desirous of studying the theory of life assurance, was recommended by a Mr. Crossley, the then President of the Mathematical Society, to Mr. Davies, as a person likely to be able to give him instruction. This was at a time when Mr. Davies had no knowledge whatever, either theoretical or practical, of the subject of life assurance; but he set his pupil to learn algebra, whilst he procured the necessary books and prepared himself to give lessons on life annuities and assurances; and by the time the pupil had gone through a limited course of pure mathematics, more particularly algebraical equations, the master was quite conversant with the subject of annuities and assurances, having concentrated his whole force for a time on that one point. This is the simple account of the circumstances which led Mr. Davies to a study which it is well known ever afterwards chiefly occupied his attention. This gentleman, being much pleased with Mr. Davies's teaching, introduced a second pupil, and he a third, and so on, several of whom became very useful and influential men in society, and continued, during their lives, on the most friendly terms with Mr. Davies. Amongst others whom he had the honour of teaching was the

celebrated Sir John Franklin, who, after many years of service at sea, came to him to perfect himself in some of the higher branches of the science of navigation. Mr. Davies entertained the most profound regard for that eminent navigator; and in conversation, whenever the melancholy loss of Sir John Franklin was alluded to, would invariably speak of him as one of the most unassuming and gentlemanly men that he had ever met with.

Mr. Davies' prospects now beginning to brighten a little, receiving many pupils, both public and private, he, in the summer of 1815, removed to Bartholomew Square, and for the first time arranged to have his school elsewhere. In 1816 he removed to Cannon Street, where he opened a more select school, for a limited number of pupils, chiefly in mathematics, and during his half holidays he engaged himself in giving lessons to several of the scholars of the Merchant Tailors' School. He received an offer of payment for the whole year, on condition that he would keep himself free to give lessons during college vacation only.

Amongst others, he now began to teach gentlemen holding the appointment of actuaries to Life Offices, and even made calculations for some of those institutions; and as he had, by means of study and tuition, become somewhat familiar with the theory and practice of life assurance, he was encouraged to aim at becoming himself an actuary; but he found it difficult to get anyone to recommend him; and feeling this, he introduced himself to the late venerable William Morgan, of the Equitable Life Assurance Society, stating his intention of becoming a candidate for the situation of an actuary, and requesting that Mr. Morgan would be good enough to examine him on such subjects as an actuary of a Life Office ought to be familiar with, and would favour him with a written opinion as to his acquirements. Mr. Morgan received him most courteously; he did not formally examine him, but elicited from him sufficient to enable him to furnish the required certificate. Mr. Davies was, however, unsuccessful in his first application to become connected with an Assurance Office.

In December, 1819, he was advised to submit to the Society of Arts a sundial of a peculiar kind, engraved on slate by him some years previously, which dial showed the following particulars:—

1. The hour of the day in London.
2. The hour of the day at Pekin.
3. The sun's place in the ecliptic, and the curves traced out by the shadow of the summit of the gnomon at the entrance of the sun into each of the twelve signs of the zodiac.

4. The day of the month.
5. The sun's declination.
6. The time of the sun's rising.
7. The time of the sun's setting.
8. The length of the day.
9. The time from the sun's rising.
10. The sun's altitude.
11. The sun's azimuth.
12. The equation of time to mean noon.
13. The day and hour when the sun is vertical at several places situated between the tropics.
14. The proportional quantity of solar rays falling obliquely on a given surface, as compared with the quantity falling vertically on a like portion of surface.

Of the construction of this dial, a full description will be found, with plates, in the *Transactions of the Society of Arts* for the year 1820, in the summer of which Mr. Davies had the honour of receiving the large silver medal of that Society, through the hands of its then President, the Duke of Sussex.

About the beginning of the year 1821, Mr. Davies, having been engaged to give lessons to a gentleman connected with a certain Life Assurance Office, but who was preparing to become one of the principal officers of another, then about to be formed under the name of the "Guardian Assurance Company," was invited to attend to give his advice and assistance to the projectors of the proposed Company at their preliminary meetings, to discuss and decide upon the nature of its constitution. He was also engaged to construct the requisite tables; and at the close of the same year the Company was fairly established at its present well known house in Lombard Street. Not long after this, he was appointed consulting actuary to that Company; and about the same time he obtained the appointment of actuary to a Company then being established by Sir George Stevens, for the purpose of purchasing reversionary property—a branch of business which, up to that period, had been confined chiefly to individuals. For this Company he constructed most elaborate and useful tables, concerning which more perhaps will be said on future occasions. But in passing, it may be useful to glance at one paragraph of the first of the valuable reports which he made to that Company, dated August, 1823, as it refers to a subject which must be full of interest to all, viz., the gradual diminution of mortality in this country, and shows that Dr. Farr does not stand alone in the opinion which he has

from time to time expressed as to the Northampton Table of Mortality:—

"By laborious investigations, I have ascertained, upon indubitable evidence (which I could not think it prudent at present to disclose, as it is my intention to publish a work on the subject at the earliest opportunity), that a gradual diminution has taken place in the mortality among the inhabitants of this country throughout the last 100 years; and that, taking all ages together, out of the same population, that

	106	persons	died	annually	from	1720	to	1730
Only	104	„	„	„	„	1730	„	1740
„	92	„	„	„	„	1740	„	1750
„	85	„	„	„	„	1750	„	1760
„	84	„	„	„	„	1760	„	1770
„	86	„	„	„	„	1770	„	1780
„	79	„	„	„	„	1780	„	1790
„	75	„	„	„	„	1790	„	1800
„	70	„	„	„	„	1800	„	1805
„	66	„	„	„	„	1805	„	1810
„	61	„	„	„	„	1810	„	1815
and	62	„	„	„	„	1815	„	1820

Supposing therefore, the Northampton Table to have been a correct index of human life at the time it was formed (from 1735 to 1780), it follows as a necessary consequence that it cannot at present be looked upon as giving anything like the real duration of British lives."

At the close of the year 1823, in consequence of the unprecedented success of the Guardian, Mr. Davies was appointed the regular and permanent actuary to that Company; an appointment which he honourably held for nearly a third of a century.

Towards the close of the year 1825 Mr. Davies casually heard that the celebrated Mr. Charles Babbage was conducting through the press a work on life annuities, containing a table of mortality deduced from the experience of the Equitable; and as he considered a table, deduced by himself from the same source, one of the leading features of his own work, which he had no prospect of completing for some time, he immediately had an extra number of the tables comprised in his intended work printed off, to which he prefixed a few pages of practical examples, and a short explanation of his new columnar method of computing the values of annuities and assurances, which he considered both an improvement and an extension of Barrett's method, and published them in the shape of a tract on life contingencies, in the winter of 1825-6, before Mr. Babbage's work made its appearance.

His reputation had now become established as an actuary, and references were made to him from various Assurance and Annuity

Societies in England, Scotland, Ireland, America, and the continent of Europe. In 1829 the Honourable Court of Directors of the East India Company, wishing to have the state of the Bombay Military Fund investigated and reported on, submitted the whole documents concerning it to the attention of Mr. Davies, who made elaborate investigations into its present state and future prospects; and from this period up to about the year 1851 he had, it may be said, constantly under his attention the affairs of some one of the Indian funds. He wrote not less than nine reports on the funds in Bombay, six on those in Madras, and five on those established in Bengal, each of such reports containing extensive tables, grounded on an investigation into the contingencies of life, death, marriage, and fecundity found to prevail in them. He was also engaged from time to time for the Bank of England.

In the year 1831 Mr. Benjamin Gompertz, formerly of the Alliance Life Assurance Society, who had been Mr. Davies' rival candidate for three several appointments as actuary, invited Mr. Davies to accompany him to a meeting of the Royal Society, of which Mr. Gompertz had been a respected member for many years; and soon after, on the personal recommendation of Mr. Gompertz, Mr. Davies was elected a Fellow of that honourable Society. Mr. Davies, in afterlife, used to speak in the most unqualified terms of the generous disposition and kindness of Mr. Gompertz towards him —the more remarkable, after the rivalry which had so frequently occurred between them.

Mr. Davies' knowledge of languages was but limited. He knew his native tongue (the Welsh) well, as it had been the only medium by which he could convey his thoughts to others until he had nearly attained the age of manhood; and not only was he a fluent speaker in that language to the day of his death, but had also studied it grammatically, and had lectured and written on various subjects in it; and although he had but a very slight acquaintance with Latin, and had not had the advantage of knowing Greek, French, or German, still he had, without doubt, made himself, in spite of all difficulties, a good English scholar; and so much had he exercised that language as an instrument to communicate his thoughts, that he could use it with remarkable success. He had a great horror of verbosity, as he knew from experience how easy it is to encumber a train of thought with a multiplicity of words. He had also the same feeling with regard to the language of notation, his aim always being to simplify it as much as possible. He could not bear to have many letters in his formulæ: "they are fatiguing

to the mind," he would say, "and sometimes calculated to mislead."

Mr. Davies was a man of very deeply religious feelings from his youth, and he became still more impressed with a sense of religion on his arrival in the metropolis. He endeavoured to make every worldly pursuit subordinate to that of his eternal welfare. For many years he was a faithful member, and one of the principal supporters, of a chapel in Jewin Street, Cripplegate, belonging to a body of Dissenters called the "Welsh Calvinistic Methodists," the service being performed in the Welsh language, and the tenets very similar to those of the Established Church of England. Mr. Davies, for several years before his death, had absented himself from all society; when in a state of health able to leave the house, the only two places he frequented were his office and his chapel: and the Bible had at that time become his almost exclusive study.

He rendered considerable assistance to his family, and supported his parents for many years—more particularly his father, who lived to the age of 93, and died on the 21st March, 1854, exactly one year prior to his own decease.

Mr. Davies was married twice—the first time in 1812, to an English person, to whose memory it is due to remark that she never murmured at his poverty or want of success; but, on the contrary, devised every possible means to keep up his spirits under their pressure. She died in the year 1836, when Mr. Davies had removed to Palmer Terrace, and was living in very different circumstances from those in which he had commenced his career with her. They had four daughters, of whom the youngest only survived to the age of maturity, and who is now married to a respectable solicitor. The loss of his three children and his beloved wife was a severe trial to him.

He married, the second time, in the year 1841, a widow lady, a native of Wales, who, from her quiet and domesticated habits, was a very suitable partner for him in his less active days. They had one child, a son, who is now grown up to be an intelligent youth, bearing his father's name.

In the winter of 1847 Mr. Davies was afflicted with a severe influenza, which fixed on his lungs a chronic bronchitis, from which he suffered during the remainder of his days. In February, 1853, he had another severe attack of bronchitis, which confined him to his room for several weeks together, but which he gradually recovered from for a time.

On the evening of the 5th December, 1854, after being en-

gaged for several hours in a conversation of a nature tending rather to excite him, he had a paralytic stroke, which was followed by all the consequences of such an attack; and he gradually sank until he died, on the 21st March, 1855, after several weeks of intense suffering. The able medical men who attended him during his illness, and performed a post-mortem examination, reported that he died from the combined effects of bronchitis and paralysis. It may be interesting to the phrenologist to know that both physician and surgeons gave evidence that his brain, though not very large, was one of the most compact and healthy they had ever seen.

Thus ended a career which will be ever memorable as connected with the mathematics of life assurance, and which affords another remarkable instance of the successful pursuit of knowledge under difficulties and disadvantages of no ordinary description. For those who are in the habit of consulting the pages of this *Journal*, the events here recorded will scarcely fail to possess some interest; nor will probably the general reader regard with entire indifference the circumstances of a life distinguished, as was that of the subject of this memoir, by so signal a triumph over the early "encumbrances of fortune."

NOTES AND QUERIES.

Premiums required for Life Assurance when Interest is allowed to the Assured upon them.

We have noticed a proposition put forward of this nature, and it is worth while to inquire what consequences it involves.

If we suppose that all the interest realized is allowed, the conditions are equivalent to the granting assurances in consideration of payments which bear no interest, and the rates of premium for the risk merely will be equal to the sum assured divided by 1 + the number of integral years' expectation at the given age. Thus, at 25, taking the experience mortality, the net annual premium per cent. will be $\frac{100}{38} = 2{\cdot}632$; at 35 it will be $\frac{100}{31} = 3{\cdot}226$; at 45, $\frac{100}{24} = 4{\cdot}167$; and so on; which rates, it will be remembered, afford no provision for anything beyond the sum assured. If more interest be realized than is allowed, then of course less rates will be required.

ED. *A. M.*

Form of Life Assurance Policy in use a Century ago.

"In consideration of five pounds for one hundred pounds for one year, and according to that rate for every greater or lesser sum, received of Mr. George Cockburne, Wee whose names are hereunto subscribed do for ourselves severally and our several heirs, executors, administrators, or assigns, and not one for the other or other of us, or for the other or other of us, or for the heirs, executors, administrators, or assigns of the other or others of us, assume and promise that we respectively or our respective heirs, executors, administrators, or assigns shall and will pay or cause to be paid unto the heirs, executors, administrators, or assigns of the said Mr. George Cockburne the sum and sums of money which wee have hereunto subscribed, with an abatement of £2 per cent. only, In case the Right Reverend William Carmichael, Lord Bishop of Clonfert, shall die or depart this life by any ways or means whatsoever (suicide and the hands of justice excepted), at any time on or between this present 27th day of June, 1754, and the 27th June next, 1755, both days inclusive.

"Provided always, and it is hereby declared to be the true intent and meaning of this policy of assurance and this policy, and accepted by the said Mr. George Cockburne, upon condition that the same shall be utterly void and of no effect in case the aforesaid Right Reverend William Carmichael, Lord Bishop of Clonfert, shall voluntarily go out of his Britanick Majesty's dominions of Great Britain and Ireland within the time above specified, without licence in writing first had and obtained for that purpose of us the insurers on this policy. London, June 27th, 1754."

Relative intensity of Diseases amongst the Lives assured in the Eagle Company, compared with that affecting the general Population of London.

	Registrar-General, London (1845-7).		Eagle Experience.	
	Numbers dying of Diseases specified.	Relative intensity.	Numbers dying of Diseases specified.	Relative intensity.
Zymotic	32,907	20·80	286	14·16
Diseases of variable seat	15,821	10·00	204	10·10
" respiratory organs	47,465	30·00	428	21·19
" brain, &c.	23,935	15·13	373	18·47
" heart & blood-vessels	5,625	3·55	127	6·29
" organs of digestion	12,781	8·08	193	9·55
" kidneys	1,655	1·05	37	1·83
" organs of generation	2,002	1·27	39	1·93
" bones, &c.	1,389	0·89	8	0·39
" skin, &c.	452	0·29	13	0·64
Old age	8,332	5·27	149	7·38
Violence, &c.	5,003	3·16	48	2·38
Not certified	827	0·51	115	5·69
	158,194	100·00	2,020	100·00

CORRESPONDENCE.

ORIGINAL TABLE.—ASSURANCES, JOINT LIVES.

To the Editor of the Assurance Magazine.

SIR,—May I beg the favour of your inserting in your eminently useful *Magazine* the accompanying Tables of Single and Annual Premiums for Joint Life Assurances for all ages between 15 and 60, deduced by me from the Carlisle Table of Mortality, and reckoning the improvement of money at three per cent.? These premiums have been carefully checked, so that their accuracy may be relied upon.

I am not aware that any similar table has ever been published; and, considering that assurances on this principle are so constantly being effected, it cannot fail to be of considerable practical value. By its use, and by means of the table already published, of the Value of Assurances on Single Lives, may be readily deduced the value of an assurance payable on the death of the survivor of two lives, adopting the simple processes of addition and subtraction. For example: Required the present value of an assurance of £100, payable on the death of the longest liver of two lives aged respectively 32 and 43 next birthday, Carlisle 3 per cent.

Value of assurance on single life, 32 = per cent.	41·357
Ditto ditto 43 = per cent.	49·352
	90·709
Deduct value of assurance on joint lives, 32·43 per cent., per table now forwarded .	56·695
Leaves value of assurance of £100, payable on the death of the survivor . . .	34·014

To determine the annual premium payable during the joint continuance, or until the decease of both, the ordinary rules of calculation must be adopted.

I am, Sir,
Your most obedient Servant,

5, *North St. David Street, Edinburgh,*
7th May, 1855.

WILLIAM BRAID.

NOTE.—As our correspondent has been at the trouble of computing these tables, and as they will certainly afford a little additional facility, we print them in extenso (*see* page 363); but, considering the ease with which the quantities given can be obtained from the annuities by means of Orchard's Tables, we fear our readers will think Mr. Braid's labour somewhat superfluous.—ED. *A. M.*

ON ASSURANCES AGAINST ISSUE.

To the Editor of the Assurance Magazine.

SIR,—Following up the suggestion in my letter to you of the 8th of February last, in reference to the publication of practical questions, I wish to draw the attention of your readers to a class of cases now of frequent

occurrence, and which we are unable to solve correctly for want of data that can be relied upon for finding the probability of issue, male and female, of married couples at given ages, together with the probability of a second marriage by the husband, and of issue therefrom.

The cases I allude to are what have been termed "issue cases." Suppose an individual to be entitled to a life interest in certain property if another, now in possession of it, should die without leaving issue: if he desire to raise money on his contingent life interest, the leaving issue by the tenant in possession must be insured against. These contingencies are sometimes very remote; still, premiums of 20*s.*, 30*s.*, or 40*s.* per cent. per annum have been demanded for risks of this description, because near approximation to their actual value cannot at present be obtained.

The following proposal for insurance against issue has recently been offered:—

A, aged 58, has been married for 30 years to B, aged 53, who is stated to be in good health. They have never had any children. C, aged 26, will be entitled to a life interest in certain property if A die without leaving issue male. It is proposed to take out a policy for a sum to become payable at the death of A, provided he shall leave issue male surviving him, or shall have had such issue and it shall have attained 21 during his lifetime, and provided also C shall be living at his death. What rate of premium, single or annual, should be required for this risk?

If we suppose it certain that A will marry again if B should die first, and certain also that there will be male issue of such marriage, the question will resolve itself into finding the present value of £1 to be paid upon A dying second of the three lives A, B, and C, provided B die first. This present value is

$$\mathfrak{A}\mathfrak{B}\mathfrak{C}_{11\,1} = \mathfrak{A}\mathfrak{C}_{1\,1} - \mathfrak{A}\mathfrak{B}\mathfrak{C}_{1\,1} = \mathfrak{A}\mathfrak{C}_{1\,1} - \mathfrak{A}\mathfrak{D}_{1} \text{ suppose;}$$

the joint lives of B and C being taken equal to a single life, D.

Using the Carlisle Tables, interest 3 per cent., and taking 53 & 26 =56, we have

$$\mathfrak{A}\mathfrak{B}\mathfrak{C}_{11\,1} = \cdot 56038 - \cdot 39437 = \cdot 16611.$$

As the death of either A or C at once puts an end to the contract, the equivalent annual premium is

$$\frac{\cdot 16611}{1 + \mathrm{AC}} = \frac{\cdot 16611}{11 \cdot 249} = \cdot 01476.$$

Consequently, the single premium per cent. would be £16. 12*s.* 3*d.*, and the annual premium per cent. £1. 9*s.* 6*d.*

There can, I think, be no doubt that the assumptions here made are very much in favour of the assurers, and that it is probable the actual risk might be covered for half the amount of annual premium, particularly as the contingency is restricted to issue male.

I am, Sir, yours truly,

ROBERT TUCKER.

Lombard Street, 2nd June, 1855.

ON THE SOLUTION OF A PROBLEM RECENTLY GIVEN BY MR. TUCKER.

To the Editor of the Assurance Magazine.

SIR,—At page 255 of your Number for *April* appears a communication from Mr. Tucker embodying a problem and its solution, on which latter I beg to be allowed a few remarks.

First: For the purpose apparently of confining himself to the employment of Mr. Thomson's tables, Mr. Tucker approximates to the value of one of the quantities required in the solution; but I submit whether it is a legitimate mode of approximation which for its application presupposes a knowledge of the *true* value: for how else does Mr. Tucker ascertain that 50 is the age of the single life most nearly equivalent to the joint lives 32, 40, but by first ascertaining the true value of these joint lives? Having done this, does it not seem trifling to throw away this correct value, and to employ an erroneous one instead?

But, secondly: it appears to me that Mr. Tucker is wrong in the value he assigns for the portion of the annuity remaining after the first five years. The value of the n^{th} payment for those years is $p_{32,n}(1-p_{40,5})v^n$, in which it will be noticed that the factor $1-p_{40,5}$ is *independent of* n; it is therefore *constant in all the terms* composing the value of the *deferred* annuity. The value of the latter portion of the annuity is therefore ${}_{5|}a_{32}(1-p_{40,5})$, or ${}_{5|}a_{32} - {}_{5|}a_{32}\cdot p_{40,5}$; and the value of the former portion being ${}_{5|}a_{32} - {}_{5|}a_{32,40}$, the total value is

$${}_{5|}a_{32} - {}_{5|}a_{32,40} + {}_{5|}a_{32} - {}_{5|}a_{32}p_{40,5} \cdot = a_{32} - {}_{5|}a_{32,40} - {}_{5|}a_{32}p_{40,5}.$$

This expression proves itself. It shows that the annuity in question is equivalent to a whole life annuity on 32, the payments of which during the first five years are to be repaid if 40 be alive at the time they are made, and also during the remainder of 32's life if 40 shall have attained the age of 45.

Mr. Tucker's value is, in the notation I use, ${}_{5|}a_{32} - {}_{5|}a_{32,40} + a_{37}\cdot {}_{5|}A_{40}$, ${}_{5|}A_{40}$ denoting the present value of a short term assurance on 40; and I believe it will be found that this expression does not admit of an interpretation in accordance with the conditions of the problem.

The numerical solution is as follows:—

$N_{32,40}$ (Jones)	123087546		
$N_{37,45}$,,	86373791		
	36713755	Log.	7·5648289
$D_{32,40}$	8600330	,,	6·9345151
${}_{5|}a_{32,40}$	4·26889		0·6303138
${}_{5|}a_{32}$ (Thomson)	14·69049	,,	1·1670363
$p_{40,5}$,,		,,	$\bar{1}$·9691496 Table V.
${}_{5|}a_{32}\cdot p_{40,5}$	13·68314	,,	1·1361859
${}_{5|}a_{32,40}$ (above)	4·26889		
	17·95203		
a_{32} (Thomson)	19·13521		
	1·18318; or, for £100 annuity, £118. 6*s.* 4*d.*		

For the annual premium, divide the single premium by $\frac{N_{31,39}-N_{36,44}}{D_{32,40}}$, which gives ·26260, or, for £100, £26. 5*s*. 2*d*.

These values do not differ greatly from those given by Mr. Tucker; but they *might* have differed very much more, since it so happens that his three errors nearly neutralize each other. The problem, of course, admits of ready solution by the commutation method. The formula for this mode of solution is

$$\frac{100[(N_{32}l_{40}-N_{37}l_{45})v^8+N_{37,45}-N_{32,40}]}{N_{31,39}-N_{36,44}},$$

which gives the annual premium. The formula for the single premium has the same numerator as the above, and for denominator $D_{32,40}$.

I am, Sir,

Your most obedient Servant,

Aberdeen, 4th June, 1855. H. A. S.

Note.—We understood Mr. Tucker's communication as proposing merely an approximate method, convenient on the score of its conciseness, and sufficiently accurate for practical purposes. Our correspondent, nevertheless, does well to show that the exact method of solution is attended with not much more labour.—Ed. *A. M.*

WORKS RECENTLY PUBLISHED.

Census of the United States, 1850. Companion to the Almanac, 1855. Occupations of the People. Census of Great Britain, 1851.

Comparative Statement of the different Plans of Decimal Accounts and Coinage which have been proposed by the Witnesses examined before the Committee of the House of Commons, and others, &c. By Theodore W. Rathbone, Esq. London: James Ridgway, Piccadilly, &c. 1854. Price 10*d*.

Manual of the Mercantile Law of Great Britain and Ireland, comprising International Commerce, Restraints of Trade, Patents, Copyright, Joint Stock and Banking Companies, the Law of Partnerships, Bills of Exchange; with Remarks on Partnership with Limited Liability. By Leone Levi, Lecturer on Commercial Law at King's College, London, &c. London: Smith, Elder, & Co., Cornhill. 1854.

The Laws of Shipping and Insurance; with a copious Index, containing the existing Statutes, Pilots' Regulations, &c. By James Lees, Esq., Author of the "Shipmaster's Manual." Sixth Edition, greatly improved and enlarged, and adapted to the present Mercantile Marine Law, and including the Passengers' Act Amendment (15 & 16 Vict., c. 44). Liverpool: George Philip & Son. London: W. Allan, Aldine Chambers, Paternoster Row, &c. 1854.

Report on the Bengal Military Fund. By Francis G. P. Neison, Actuary to the Fund, &c. Thomas Brettell, Rupert Street.

REPORTS OF ASSURANCE COMPANIES.

Equity and Law Life Assurance Society.—The annual general meeting, and also an extraordinary general meeting for declaring the amount of profits to be divided for the period of five years ending on the 31st December, 1854, were held on the 27th April, 1855; George Lake Russell, Esq., in the chair.

The annual report of the directors, and also the report of the results of the operations of the Society during the quinquennial period ending 31st December, 1854 (being the second quinquennial period since the institution of the Society in 1845), were read. The chairman, in his address, congratulated the assured and the shareholders upon the success of the Society. It appeared that the new business of the Society had in each of the five years of the period steadily increased, notwithstanding the competition with which this, as well as all Life Assurance Societies, have had to contend. The new premiums for 1849 were £3,505; for 1854, £4,602. The renewal premiums for 1849 were £11,612; for 1854, £25,528. The income for 1849 was £17,802; for 1854, £35,533. The capital (actually invested) on 31st December, 1849, was 74,958; ditto 1854, £156,024: and as to this capital, there was this important fact, that whereas the surplus capital on 31st December, 1849, after deducting the sums which have been paid up by the subscribers, was only £27,353. 10*s.* 7*d.*, it now amounted, by the accumulations from the business of the last five years, to £107,203. The total losses in the ten years had been only £33,257. 16*s.* 10*d.*, reduced by assurances in other Offices to £28,907. 10*s.* 10*d.*; whereas, according to the calculations of the actuary, the amount which might have been estimated for loss on the average amount at risk exceeded £65,000. The excess of assets over the proprietors' capital, and liabilities of every description, being £107,203, the sum of £81,614 was reserved for future liabilities, and £25,588 would remain to be divided in ready money between the proprietors and the assured, the latter taking four fifths. The effect of this on policies participating in the profits would be an average addition to the amount assured of 55 per cent. on the sums paid for premiums within the last five years, or the immediate payment of upwards of 25 per cent. on such amount. Thus, a policy effected in 1845 for £1,000, on the life of a person of 50, is now increased by the bonuses to £1,201. 10*s.*; and another policy, effected at the same time, for £1,000, on the life of a person at 40, is in like manner increased to £1,157. 10*s.* The dividend payable to the proprietors during the next five years will be 5½ per cent., clear of income tax, being an advance of one per cent. on the dividends previously paid. The actuary stated that he had adopted the most approved and cautious principles of calculation in ascertaining the amount proper to be set apart as a reserve; the rate of interest which the Society would be able to make upon its investments having been taken at £3 per cent. (the present actual rate being very considerably more), and the present value of the income derived under the policies having been estimated, without taking any account of the margin for providing against fluctuations in the average expectation of life.

Reliance Mutual Life Assurance Society. — Report for the Year ending 31st December, 1854. *Presented to the Half-yearly General Meeting of the Members, held January* 30, 1855.—The accounts for the year 1854 show a continued steady increase in the business and funds of the

Society, whilst the ratio of mortality preserves the low average of previous years. The receipts, during the period under review, in respect of assurance premiums, amount to £10,841. 11*s.* 10*d.*, of which £2,359. 11*s.* 10*d.* represents new business. The sum of £1,176. 15*s.* 10*d.* has also been received for an immediate annuity upon an advanced life. The funds of the Society, vested in Government stocks, mortgages, and other securities, have yielded an addition to premium income of £2,210. 5*s.* 7*d.*, a sum more than equal to the difference between the premiums now receivable and those which would be required for reassurance of the existing policies at full Office rates for present ages. The disbursements include the sum of £1,900, in respect of claims by decease of six members, making in the aggregate, from the establishment of the Society to the present date, the sum of £12,640, which, compared with the estimated mortality of £27,252, shows an average exceeding 50 per cent. in favour of the Society. Notices of further claims to the extent of £1,475 have been sent in, which will, in due course of settlement, be brought into the accounts of the current year.

The attention of members has on former occasions been called to the rate of increase of the Society's assets, and it will be satisfactory to bring the subject again under notice. At the first division of profits, for the period ending 31st December, 1846, the cash and invested funds of the Society amounted only to £10,183. 16*s.* 8*d.*; at the next triennial investigation the amount was £22,273. 9*s.* 6*d.*; at the division of profits in 1853, the assets were £36,658. 6*s.* 5*d.*; and at the close of the year they amounted to £44,540. 7*s.* 4*d.* The accounts now before you, of which an abstract will be appended to this report for information of absent members, show a balance under this head of £52,215. 8*s.* 4*d.*, being a further increase, during the past year, of £7,675. These accounts are exclusive of benefits in the shape of bonus allocations, amounting to £9,000 of present value.

The engagements of this Society in respect of its policies for general assurance—599, assuring £306,446—calculated by the Office tables of mortality, are estimated at £139,236. 6*s.*, against a value of £103,899. 0*s.* 8*d.* for true or mathematical risk premiums, leaving a sum of £35,337. 5*s.* 4*d.* as the balance of net liability to be provided for. In this calculation the Office margin, or excess of premiums receivable beyond the risk values, is wholly excluded, as profit in expectancy, to be taken into account only as realized; this margin gives a further value of £31,424. 6*s.* 9*d.*, the total value of the Office premiums amounting to £135,323. 7*s.* 5*d.* To the net present liability above stated must be added the sum of £4,915. 0*s.* 9*d.* as the present value of annuities granted by the Society, making together the sum of £40,252. 6*s.* 1*d.* to be provided for, to meet which the assets in hand, as shown by the accounts on the table, amount to £52,215. 8*s.* 4*d.*, being an excess of £12,000 over the total liability.

Abstract of Annual Account.

Dr.	£.	*s.*	*d.*
Balance of receipts to 31st December, 1853	47,616	16	1
Annual premiums on new assurances	2,359	8	11
Annual premiums on renewed assurances	8,482	2	11
Consideration for immediate annuities	1,176	15	10
Commission on reassurance premiums	9	8	0
Interest on stocks, loans, forborne premiums, and cash deposits	2,210	5	7
	£61,854	17	4

Cr.	£.	s.	d.
Annuities	514	16	6
Bonus	466	1	3
Reassurance premiums	158	16	6
Cancelled policies	994	4	7
Claims	1,900	0	0
Commission, income tax, and miscellaneous charges	327	11	10
Expenses of management, salaries, rent, and law charges	2,067	16	10
Policy stamps, advertisements, and discount	133	2	9
Balance at credit on 31st December, 1854	55,292	7	1
	£61,854	17	4

Specification of Balance.

	£.	s.	d.
Cash balances	7,551	0	2
Consols Stock, £10,091. 9s. 3d.	9,784	2	7
East India Transfer Loan Stock	5,950	13	2
Loans on mortgage and other securities	17,426	19	11
Half premiums forborne	7,646	2	2
Reversionary interests, present value	3,630	8	1
Reserved account of expenses	3,076	8	9
Office furniture, reduced value	291	18	6
	55,357	13	4
Less, rent due by the Society	65	6	3
	£55,292	7	1

Rock Life Assurance Company.—Statement of Receipts and Disbursements for the Half Year ending 30th June, 1854.

As regards the Subscription Capital Stock.

Dr.	£.	s.	d.
Balance brought forward from 31st December, 1853, viz.:—			
Cash at bankers'	3,484	4	1
Do. on deposit with London Joint Stock Bank, at interest	1,111	18	1
Dividend on £23,000, Long Annuity, in April	11,164	11	8
Interest, viz.:—			
On Mortgage	16,942	8	5
Railway debentures	1,893	2	6
Deposits with London Joint Stock Bank	478	19	1
Amount of mortgage paid off	43,000	0	0
	£78,075	3	10

Per Contra.

Cr.	£.	s.	d.
Payment of proprietors' dividends, viz.:—			
On previous year's dividends	2,128	7	6
On the current half year's dividend	21,196	5	0
Cash at bankers, 30th June, 1854	2,750	11	4
Ditto, London Joint Stock Bank, at interest	52,000	0	0
	£78,075	3	10

Investments and assets on this account, on the 30th of June, 1854:—£23,000 per annum Long Annuity, 1860; £100,000, Railway Debentures, average £3. 18s. per cent.; £766,210. 5s. 8d. on mortgage, average £4 per cent.; £2,750. 11s. 4d., cash at bankers'; £52,000, cash at London Joint Stock Bank, at interest.

As regards the Assurance Fund.

Dr.	£.	s.	d.	£.	s.	d.
Balance brought forward from 31st December, 1853, viz.:—						
Cash at bankers'	6,860	18	11			
Do. on deposit with London Joint Stock Bank, at interest	25	12	0			
Petty cash	36	13	10			
Balances due from agents	3,440	18	9			
Carried forward				10,364	3	6

	£.	s.	d.	£.	s.	d.
Brought forward				10,364	3	6
Dividends, viz.:—						
On £257,436. 6*s.*, Consols, January	3,748	18	4			
£3,725. 2*s.* per ann., Government Annuity, Jan.	1,808	4	7			
£65,000, British Guiana Loan, January	1,262	1	8			
£20,000, Canada Debentures, April	3,883	6	8			
£172,255. 3*s.* 8*d.*, 3½ per cent. Reduced Annuities, April	2,717	10	1			
£17,535. 17*s.* 1*d.* per ann., Long Annuity, April	8,512	3	11			
£7,702. 9*s.* per ann., Government Annuities on Lives, April	3,714	13	3			
				25,646	18	6
Interest, viz.:—						
On mortgage	19,153	10	6			
Railway and Dock Debentures	6,292	7	8			
Claims paid under discount	5	2	11			
Loans on policies	931	18	7			
Deposits with London Joint Stock Bank	938	12	6			
				27,321	12	2
Terminable annuities				205	6	1
Premiums, viz.:—						
On £1,803,515. 13*s.*, assured by policies issued prior to 1854	60,591	15	9			
£102,300, assured by policies issued in 1854	4,678	10	10			
Annuities	195	14	6			
				65,466	1	1
Amount of fines				34	7	6
Produce of stock sold, viz.:—						
£257,436. 6*s.*, Consols, sold in June	239,642	13	7			
£85,490. 2*s.* 4*d.*, 3½ per Cents., sold in February £64,509. 17*s.* 8*d.*, ditto ditto March £72,255. 3*s.* 8*d.*, ditto ditto June	207,363	0	4			
				447,005	13	11
Amount of mortgage paid off				143,399	17	6
Loans on policies paid off				1,600	0	0
				£721,044	0	3

Cr. *Per Contra.*	£.	s.	d.	£.	s.	d.
Payment of claims on policies originally assured for	61,825	0	0			
Redemption of £4,600 originally assured	772	8	1			
				62,097	8	1
Payment of bonus additions to claims	19,460	1	3			
Redemption of £1,235. 8*s.* 4*d.*, bonus additions	697	4	3			
				20,157	5	6
Payment for return of extra premiums				173	9	7
„ for annuities				242	10	6
„ for commission				1,188	0	10
„ of half year's pension to Mr. Robert Wakefield				100	0	0
„ of allowance to directors and auditors, and officers' salaries				2,227	8	3
„ of miscellaneous expenses, viz., rent, taxes, stamps, stationery, advertising, &c.				731	15	7
„ of law expenses				841	5	5
Purchase of £150,255. 3*s.* 8*d.*, 3½ per cent. Annuities, in June				140,908	6	1
„ of £17,535. 17*s.* 1*d.* per ann., Long Annuities, 1860				93,836	9	4
„ of £100,000, Railway Debentures				100,000	0	0
„ of £403. 14*s.* 10*d.* per ann., Terminable Annuities				7,497	0	8
Amount advanced on mortgage				166,215	17	9
„ „ on policies				5,156	9	4
Cash at bankers', 30th June, 1854	6,911	5	9			
Ditto London Joint Stock Bank, at interest	112,000	0	0			
Petty cash	31	5	10			
Balances due from agents	728	3	9			
				119,670	15	4
				£721,044	0	3

Investments and assets on this account, on the 30th June, 1854:—£100,000, 3½ per cent. Reduced Annuities; £17,535. 17*s.* 1*d.* per ann., Long Annuity, 1860; £3,725. 2*s.* per ann., Government Annuity, 1880; £7,602. 9*s.* per ann., Government Annuities on Lives; £911. 17*s.* 11*d.* per ann., Terminable Annuities; £200,000, Canada Debentures, 4 per cent.; £65,000, British Guiana Debentures, 4 per cent.; £340,000, Railway Debentures, average £4. 6*s.* per cent.; £50,000, Dock Debentures, 4 per cent.; £974,947. 11*s.* 10*d.* on mortgage, average £4. 4*s.* 10*d.* per cent.; 53,772. 9*s.* 7*d.*, loans on policies, 5 per cent.; £6,911. 5*s.* 9*d.*, cash at bankers'; £112,000, cash at London Joint Stock Bank, at interest; £31. 5*s.* 10*d.*, petty cash; £728. 3*s.* 9*d.*, balances due from agents; £11,352. 12*s.* 4*d.*, house in Bridge Street.

Scottish Equitable Life Assurance Society.—Twenty-second Annual General Meeting.—Report for the Year ending 1st March, 1853.—During the year the business transacted has been as follows:—Policies issued, 603; total sums assured thereby, £276,470; annual premiums and entry money, £9,999. 19*s.* 10*d.*; policies lapsed by death, 88, amounting, including bonuses, to £65,173. Compared with the preceding year, this statement exhibits a small decrease. Eleven fewer policies have been issued; the sum assured is less by £14,380; and the premiums and entry money are diminished by £231. The number of deaths exceeds that of last year by 15, and the amount payable by £4,992.

The following statement exhibits the position of the Society at 1st of March last:—Sums remaining assured, £3,892,031; annual revenue, £144,085; accumulated fund, £763,871.

The gross funds, assets, and property of the Society amounted, at 1st March, 1853, to £2,130,083. 17*s.* 7*d.*, viz.—

	£.	s.	d.	£.	s.	d.
Funds realized				804,468	17	7
Which sum is invested as follows:—						
Loans on heritable securities and mortgages	403,554	8	3			
Ditto to members on the Society's policies	64,349	2	11			
Ditto to railways on debenture	208,150	19	5			
Reversions, policies, and annuities purchased	43,906	18	4			
Government Life Annuities	19,864	18	0			
Outstanding sums, chiefly premiums, due on or immediately before 1st March, 1853, but not falling to be remitted till after that date	46,306	10	9			
Balance due by the Society's bankers	13,095	19	11			
House and furniture, St. Andrew Square, Edinburgh (Head Office)	5,250	0	0			
	£804,468	17	7			
Present value of contributions or premiums of assurance receivable by the Society, after deducting two and a half per cent. for expense of collection				1,325,615	0	0
Gross funds				£2,130,083	17	7

The whole *obligations* of the Society amounted, at 1st March, 1853, to £1,969,070. 8*s.* 9*d.*, viz.—

	£.	s.	d.
Various sums outstanding, including policies which had emerged at 1st March, 1853, but had not been paid at that date	40,597	18	0
Present value of sums contained in and to become due under the Society's policies	1,928,472	10	0
Total obligations	£1,969,070	8	9

	£.	s.	d.
The gross assets of the Society thus amounting to	2,130,083	17	7
And the total obligations to	1,969,070	8	9
There arises a surplus, as at 1st March, 1853, of	£161,013	8	10

The committee have made the foregoing valuation of the funds and obligations of the Society in strict conformity with the law passed on the 1st February, 1838, for regulating the investigations, by which it is declared as a leading principle that the calculation of the value of sums assured by the Society's policies, and of the contributions or premiums receivable thereon, as well as any property or obligations held by the Society dependent on life contingencies, shall be made according to the Northampton Table of Mortality, assuming the improvement of money to be at the rate of four per cent.

Two thirds of the foresaid surplus of £161,013. 8*s*. 10*d*. amount to £107,342. 5*s*. 9*d*.; and it appears, from calculations made by the manager, that an allocation of £84,515 of this sum will afford a vested addition, at 1st March, 1853, at the rate of one and a half per cent. per annum to all policies then of five years' standing. The committee accordingly recommended such vested addition to be made, which will provide for a bonus of £166,109, payable at the death of the parties entitled thereto. After providing for this vested addition, there will still remain £22,827 between the sum allocated and the two thirds of the surplus placed by law at the discretion of the directors for division.

The reporters further recommend that, in terms of the law of the Society passed on 18th November last, prospective additions, at the rate of one per cent. per annum (being two thirds of the rate per annum of the vested additions), shall be made on all policies which may become claims between 1st March last and the next triennial investigation on 1st March, 1856, and which, at the time they become so, shall be of full five years' standing.

View of the progress and situation of the Society.

	Amount Assured.	Annual Revenue.	Accumulated Fund.
	£.	£.	£.
At 1st March, 1853	325,611	11,364	24,661
„ „ 1841	1,569,570	55,536	153,329
„ „ 1847	2,763,381	99,270	400,503
„ „ 1853	4,058,140	144,085	763,871

	£.
Amount assured, exclusive of the present vested additions . . .	3,892,031
Add vested additions to policies at 1st March, 1853, equivalent to £84,515 of present value	166,109
Amount assured, including vested addition	£4,058,140

Scottish Widows' Fund Life Assurance Society.—Report for 1853.—The manager read the report by the directors, from which it appeared that the number of policies passed during the year 1853 was 821, covering assurances to the amount of £534,015. 17*s*. 1*d*., whereof there had been taken up 770 policies, representing £504,513. 15*s*. 6*d*. of insurances—that the annual average amount of assurances effected during the last sixteen years had exceeded half a million sterling—that the capital had increased during the year to the extent of £130,042. 15*s*. 2*d*., and amounted, at 31st December last, to £2,686,585. 5*s*. 3*d*.—that the annual revenue had increased to the extent of £13,371. 4*s*. 3*d*., and amounted, at 31st December

last, to £351,783. 12*s.* 9*d.*—and that the deaths which had occurred during the year numbered 131, covering assurances to the amount of £187,910. 16*s.* 2*d.*

The chairman, after some remarks on the position of the Society, proposed the resolutions and said, "The first of these resolutions is a proposal to increase the amount of the sum to which hitherto an insurance on a single life has been restricted. According to one of the original rules of our constitution, the funds of the Society cannot be charged in favour of any individual for a larger sum than £5,000. But, in the infancy of our Institution, it was provided that no insurance should be taken even at this maximum amount. At first the arrangement was, that until the funds of the Society amounted to £10,000—a sum which certainly looks curious enough when put in juxtaposition with the two and a half millions sterling which we are now possessed of—it was provided that, until the funds of the Society amounted to £10,000, no insurance upon a single life should exceed the sum of £1,000. It seems, according to the numbers which have been given me, that the funds had exceeded that sum in the year 1821, when we had a capital of £15,000 sterling; but the Society did not even then allow that an insurance should be taken on a single life to the full extent of £5,000; but, proceeding in the safe and cautious way which has always distinguished us, it was resolved in 1821 that an insurance might be taken to the extent of only £2,000; in 1822, to the extent of £3,000; in 1827, to the extent of £4,000; and it was not until the year 1830, when the capital had reached £190,000, that an insurance could be taken to the maximum amount of £5,000. Since 1830, up to the year 1854, that has been the maximum. Now, it is proposed to increase it to the extent of £7,000, and I beg your attention to the circumstances in which this proposal is made. It appears that the increase of the sum allowed to be insured has always proceeded upon a consideration of the increase of the capital, and of the total risks of the Society. In 1821, when £2,000 could be taken, the capital was £15,000, and the risks £140,000. I need not detain you by stating the particulars of the respective amount of capital and risks at the different periods between 1821 and 1830, but will merely observe that in 1830 the capital was £190,000, and the risks £915,000. How stands the case now? The risks amount to £7,730,000, being eight to one compared with those in 1830; while the capital, being £2,686,585, is more than fourteen times to one the capital of 1830. Now it appears to me, that if in 1830, when our capital was only £190,000, it was considered safe—and that it was safe, our experience has proved—to take an insurance to the amount of £5,000, it must surely be safe now, with such a capital as we have, increasing in a ratio even beyond the risks, to take insurances to the extent of £7,000."

The report having been unanimously approved of, the chairman moved the adoption of the following resolution:—"Whereas it is provided and declared by Title I. of the Articles of Constitution that no sum of money shall be charged upon the funds of the Society in favour of any individual or his heirs, executors, assignees, nominee or nominees, upon any contingency or contingencies whatever, to a greater extent than £5,000 sterling, to which extent accordingly the amount of insurance upon a single life has been hitherto limited and restricted; and whereas it is no longer expedient to continue such limitation or restriction—'Resolved, that from and after the final passing of this resolution, the funds of the Society may be charged

in favour of any individual, or his heirs, executors, assignees, nominee or nominees, by way of insurance upon any one life, whether such life be taken singly or in conjunction with any other life or lives, to any extent not exceeding the sum of £7,000 sterling, exclusive of such bonus addition or additions as may from time to time be made thereto, in terms of the Society's laws and regulations.'"

On the question being put from the chair, whether the resolution were approved of, a member of the Society rose and moved as an amendment, "that its farther consideration be postponed for twelve months." The amendment not having been seconded, the chairman declared the resolution to be carried.

The chairman then moved the adoption of the following resolution:— "Resolved, that the whole of the 3rd clause of Title VI. of the Articles of Constitution be repealed, and that the following clause be substituted in its place: '3. All payments, whether in respect of claims for capital sums and annuities, of loans to members on the security of their policies, or of surrenders of the policies, or any portion thereof, shall be legally exigible at the Society's head office in Edinburgh only; but the directors may, if they shall think proper, remit the same to any part of Great Britain or Ireland for the convenience of the members, without calling upon them to bear the expense of such remittance, which may, at the discretion of the directors, be defrayed out of the Society's funds.'"

On the question being put from the chair, whether the resolution were approved of, and no amendment being proposed, the resolution was carried unanimously.

The chairman then intimated that, in terms of the Articles of Constitution of the Society, a special general meeting of members would be held early in December, for the purpose of again considering and finally disposing of these resolutions.

The Standard Life Assurance Company.—Extracts from the Report made by the Directors, 1853.—Number of proposals for assurance made to the directors during 1852, 1,026. Number of proposals for assurance accepted, 777; other life transactions proposed and accepted, 45; together, 822. Amount of sums proposed for assurance during the year, £601,404. 7*s.* 7*d.*; amount of sums for which new policies have been issued, £445,799. 6*s.* 6*d.*; corresponding annual premiums, £15,145. 15*s.* 6*d.*; annual income for year to 15th November, 1852, £192,928. 16*s.* 10*d.*; amount of claims by death during the year, £59,322. 13*s.*

Yearly results, 1842 *to* 1852.

	Sums Assured. £. s. d.	Annual Premiums. £. s. d.
1843—New business . .	348,135 17 1	10,638 11 5
1844 Ditto . . .	387,381 0 0	13,461 10 5
1845 Ditto (bonus year) .	446,026 12 10	14,979 8 10
1846 Ditto . . .	368,679 7 10	12,846 4 10
1847 Ditto . . .	443,578 4 11	16,140 0 1
1848 Ditto . . .	395,864 12 5	12,200 9 5
1849 Ditto . . .	429,371 17 1	14,743 4 8
1850 Ditto (bonus year) .	509,147 10 6	17,550 14 9
1851 Ditto . . .	467,499 8 1	15,240 2 11
1852 Ditto . . .	445,799 6 6	15,145 15 6
	£4,241,483 17 3	£142,946 2 10
Annual average for 10 years .	£424,148 7 8	£14,294 12 3

The annual average amount of new business for the last ten years has been upwards of £424,000, and for the last five years £450,000. The Company's funds have been nearly doubled during the last six years, and quadrupled during the last eleven years.

Waterloo Life, Education, Casualty, and Self-Relief Assurance Company.—Third Annual Report, Nov. 13th, 1854.—During the last year, 632 proposals have been made to the directors, of which 488 have been completed, assuring the sum of £128,620, and producing the annual income of £4,238. Of the remaining proposals for assuring £45,534, several are in course of completion; others the directors have deemed it desirable altogether to decline. The annual income of the Company, from premiums alone, exceeds £11,400; the total number of policies granted is 1,091; and the amount assured is £342,883—no claim having arisen from death during the year.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

Fifth Ordinary Meeting, Session 1854-55.—*Monday, 26th March*, 1855.

E. J. FARREN, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The following candidates, duly nominated at the last ordinary meeting, were elected Associates of the Institute:—

C. G. Collins, | William Lethbridge, M.A.

Mr. Jellicoe, Vice President, read a paper entitled "An examination of the objections urged against the plan of decimal coinage proposed by the Royal Commissioners and by the Select Committee of the House of Commons."

Sixth Ordinary Meeting, Session 1854-55.—*Monday, 30th April*, 1855.

CHARLES JELLICOE, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced numerous donations to the library.

The undermentioned candidates, duly nominated at the last ordinary meeting, were elected Associates of the Institute:—

James Dix, | C. W. Dixon.

Mr. Farren, Vice President, read a paper "On the valuation of Government securities."

Seventh Ordinary Meeting, Session 1854-55.—*Monday, 28th May*, 1855.

E. J. FARREN, Esq., Vice President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

Dr. Guy, one of the Honorary Secretaries of the Statistical Society, read a paper "On the analogy existing between the aggregate effects of the operations of the human will and the results commonly attributed to chance."

ORIGINAL TABLES.*

ASSURANCES.—JOINT LIVES.

TABLE I., *showing the Single Premium required to secure an Assurance of £100 payable on the Death of* EITHER *of Two Assigned Lives, according to the Carlisle Table of Mortality, and assuming Interest at the rate of Three per cent.*

Younger Ages.	OLDER AGES.										
	15.	16.	17.	18.	19.	20.	21.	22.	3.	24.	25.
15	42·015	42·295	42·563	42·837	43·115	43·428	43·751	44·101	44·467	44·855	45·260
16	..	42·566	42·828	43·096	43·379	43·676	43·990	44·333	44·695	45·076	45·478
17	..	..	43·081	43·344	43·620	43·911	44·223	44·555	44·914	45·286	45·682
18	..	..	..	43·597	43·868	44·153	44·459	44·785	45·132	45·502	45·892
19	..	..	..	..	44·130	44·409	44·707	45·027	45·371	45·729	46·114
20	..	..	..	..	..	44·680	44·971	45·286	45·621	45·973	46·346
21	..	..	..	..	..	..	45·251	45·560	45·886	46·233	46·600
22	..	..	..	..	..	..	..	45·860	46·181	46·518	46·876
23	..	..	..	..	..	..	..	..	46·489	46·821	47·171
24	..	..	..	..	..	..	..	..	..	47·141	47·485
25	..	..	..	..	..	..	..	..	..	..	47·817

TABLE I.—*continued.*

Younger Ages.	OLDER AGES.										
	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
15	45·685	46·128	46·574	46·999	47·389	47·785	48·202	48·641	49·128	49·626	50·151
16	45·889	46·329	46·771	47·188	47·572	47·966	48·377	48·816	49·288	49·783	50·302
17	46·090	46·515	46·955	47·366	47·745	48·129	48·536	48·971	49·437	49·926	50·439
18	46·291	46·716	47·139	47·549	47·919	48·301	48·700	49·128	49·588	50·072	50·578
19	46·507	46·923	47·342	47·741	48·106	48·481	48·875	49·297	49·751	50·230	50·728
20	46·736	47·145	47·558	47·948	48·306	48·676	49·061	49·477	49·926	50·395	50·890
21	46·979	47·381	47·788	48·173	48·522	48·883	49·262	49·672	50·113	50·576	51·065
22	47·249	47·639	48·042	48·417	48·761	49·114	49·483	49·888	50·322	50·776	51·257
23	47·534	47·919	48·310	48·680	49·015	49·361	49·725	50·118	50·546	50·995	51·466
24	47·841	48·216	48·601	48·959	49·288	49·626	49·984	50·369	50·785	51·228	51·691
25	48·166	48·534	48·907	49·259	49·579	49·909	50·258	50·636	51·045	51·475	51·933
26	48·501	48·863	49·227	49·571	49·882	50·203	50·544	50·913	51·312	51·738	52·183
27	..	49·212	49·567	49·903	50·205	50·517	50·849	51·211	51·601	52·015	52·454
28	..	..	49·914	50·241	50·532	50·838	51·158	51·510	51·892	52·297	52·728
29	..	..	..	50·555	50·841	51·135	51·449	51·790	52·163	52·559	52·979
30	..	..	..	..	51·114	51·402	51·706	52·040	52·401	52·789	53·200
31	..	..	..	..	..	51·676	51·973	52·297	52·652	53·028	53·430
32	..	..	..	..	..	..	52·256	52·573	52·917	53·284	53·674
33	..	..	..	..	..	..	..	52·874	53·211	53·567	53·948
34	..	..	..	..	..	.	..	..	53·532	53·878	54·251
35	..	..	..	..	..	..	..	..	..	54·213	54·574
36	..	..	..	..	..	..	..	..	..	..	54·921

* *See* letter from Mr. William Braid, page 350.

TABLE I.—*continued.*

Younger Ages.	OLDER AGES.											
	37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	48.
15	50·689	51·245	51·816	52·384	52·935	53·479	54·027	54·601	55·201	55·827	56·500	57·213
16	50·834	51·385	51·950	52·515	53·060	53·599	54·143	54·711	55·302	55·934	56·596	57 307
17	50·969	51·513	52·073	52·632	53·174	53·706	54·248	54·811	55·399	56·022	56·681	57·388
18	51·103	51·641	52·198	52·751	53·287	53·817	54·350	54·910	55·495	56·112	56·768	57·470
19	51·246	51·778	52·329	52·879	53·406	53·933	54·464	55·017	55·597	56·208	56·861	57·560
20	51·403	51·927	52·471	53·016	53·541	54·059	54·582	55·126	55·707	56·316	56·960	57·656
21	51·568	52·087	52·626	53·165	53·681	54·196	54·714	55·259	55·827	56·430	57·071	57·758
22	51·755	52·267	52·801	53·331	53·841	54·350	54·863	55·401	55·964	56·561	57·196	57·881
23	51·956	52·460	52·987	53·512	54·015	54·516	55·023	55·556	56·112	56·704	57·333	58·008
24	52·175	52·678	53·185	53·704	54·202	54·697	55·198	55·722	56·273	56·858	57·478	58·152
25	52·408	52·897	53·404	53·911	54·403	54·892	55·384	55·903	56·445	57·024	57·639	58·303
26	52·649	53·130	53·631	54·132	54·610	55·093	55·579	56·089	56·625	57·196	57·805	58·463
27	52·909	53·383	53·873	54·365	54·839	55·309	55·789	56·290	56·820	57·383	57·985	58·635
28	53·174	53·637	54·120	54·600	55·066	55·530	55·999	56·494	57·015	57·572	58·166	58·807
29	53·418	53·873	54·344	54·819	55·274	55·731	56·191	56·675	57·190	57·737	58·323	58·958
30	53·628	54·073	54·539	55·002	55·451	55·897	56·351	56·829	57·330	57·874	58·451	59·080
31	53·849	54·284	54·737	55·195	55·635	56·072	56·517	56·986	57·481	58·011	58·585	59·202
32	54·085	54·510	54·956	55·402	55·833	56·261	56·695	57·158	57·644	58·166	58·725	59·339
33	54·347	54·764	55·198	55·632	56·054	56·471	56·899	57·350	57·828	58·341	58·891	59·491
34	54·638	55·043	55·466	55·891	56·301	56·709	57·126	57·566	58·032	58·535	59·077	59·671
35	54·950	55·343	55·754	56·171	56·567	56·966	57·371	57·801	58·259	58·751	59·281	59·864
36	55·285	55·650	56·068	56·470	56·858	57·243	57·638	58·058	58·501	58·984	59·505	60·074
37	55·638	56·008	56·395	56·785	57·161	57·537	57·918	58·326	58·783	59·229	59·738	60·298
38	..	56·366	56·739	57·117	57·482	57·843	58·216	58·609	59·031	59·488	59·986	60·534
39	..	..	57·106	57·467	57·817	58·169	58·527	58·912	59·319	59·765	60·249	60·782
40	..	..	..	57·822	58·157	58·495	58·838	59·209	59·608	60·039	60·551	61·029
41	..	..	..	..	58·483	58·807	59·138	59·494	59·878	60·301	60·755	61·262
42	..	..	..	..	..	59·118	59·436	59·778	60·149	60·554	61·003	61·492
43	..	..	..	..	..	..	59·741	60·071	60·426	60·819	61·250	61·731
44	..	..	..	..	..	..	..	60·388	60·729	61·105	61·521	61·984
45	..	..	..	..	..	..	..	..	61·055	61·416	61·815	62·264
46	..	..	..	..	..	..	..	..	..	61·763	62·147	62·578
47	..	..	..	..	..	..	..	..	..	..	62·514	62·928
48	..	..	..	..	..	..	..	..	..	..	..	63·327

TABLE I.—*continued.*

Younger Ages.	OLDER AGES.											
	49.	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
15	58·090	58·841	59·741	60·656	61·588	62·535	63·511	64·504	65·520	66·531	67·495	68·388
16	58·090	58·926	59·823	60·735	61·661	62·604	63·575	64·575	65·576	66·583	67·544	68·439
17	58·166	58·999	59·890	60·799	61·722	62·660	63·627	64·614	65·622	66·625	67·585	68·476
18	58·244	59·072	59·960	60·863	61·780	62·715	63·679	64·664	65·669	66·668	67·623	68·511
19	58·329	59·153	60·033	60·933	61·847	62·762	63·737	64·716	65·718	66·714	67·667	68·549
20	58·419	59·237	60·115	61·009	61·917	62·846	63·799	64·774	65·773	66·767	67·714	68·596
21	58·518	59·331	60·205	61·093	61·996	62·919	63·868	64·838	65·832	66·822	67·766	68·645
22	58·632	59·442	60·307	61·189	62·089	63·007	63·953	64·917	65·905	66·892	67·830	68·738
23	58·757	59·558	60·420	61·297	62·191	63·103	64·041	65·002	65·986	66·965	67·903	68·771
24	58·891	59·686	60·543	61·413	62·302	63·208	64·139	65·095	66·074	67·049	67·978	68·844
25	59·037	59·826	60·674	61·539	62·421	63·321	64·251	65·200	66·169	67·142	68·066	68·925
26	59·188	59·971	60·813	61·669	62·546	63·441	64·364	65·308	66·271	67·236	68·156	69·012
27	59·354	60·129	60·965	61·815	62·683	63·571	64·486	65·424	66·385	67·343	68·258	69·109
28	59·520	60·286	61·113	61·958	62·820	63·700	64·609	65·541	66·493	67·448	68·357	69·202
29	59·663	60·423	61·244	62·081	62·936	63·811	64·713	65·639	66·586	67·535	68·438	69·281
30	59·776	60·531	61·343	62·174	63·021	63·892	64·789	65·710	66·653	67·594	68·497	69·333
31	59·893	60·638	61·445	62·267	63·111	63·973	64·864	65·779	66·717	67·655	68·552	69·385
32	60·011	60·758	61·556	62·371	63·207	64·064	64·949	65·858	66·790	67·722	68·613	69·440
33	60·167	60·895	61·684	62·491	63·318	64·168	65·045	65·948	66·874	67·800	68·686	69·510
34	60·330	61·052	61·830	62·631	63·449	64·288	65·159	66·053	66·973	67·894	68·774	69·592
35	60·516	61·221	61·993	62·782	63·592	64·422	65·284	66·169	67·081	67·996	68·869	69·679
36	60·714	61·412	62·167	62·948	63·749	64·571	65·421	66·301	67·203	68·109	68·974	69·779
37	60·924	61·612	62·357	63·123	63·912	64·725	65·567	66·435	67·332	68·226	69·085	69·884
38	61·149	61·821	62·555	63·313	64·087	64·888	65·721	66·581	67·463	68·354	69·202	69·991
39	61·387	62·046	62·768	63·511	64·277	65·063	65·885	66·732	67·606	68·485	69·327	70·108
40	61·620	62·269	62·977	63·705	64·460	65·234	66·044	66·881	67·747	68·613	69·446	70·221
41	61·838	62·474	63·169	63·886	64·626	65·392	66·190	67·014	67·868	68·727	69·548	70·314
42	62·057	62·677	63·359	64·064	64·789	65·543	66·327	67·142	67·984	68·832	69·644	70·401
43	62·278	62·884	63·551	64·241	64·958	65·695	66·467	67·271	68·101	68·936	69·740	70·489
44	62·520	63·111	63·764	64·439	65·138	65·864	66·624	67·410	68·229	69·053	69·845	70·582
45	62·782	63·359	63·994	64·655	65·339	66·050	66·796	67·571	68·375	69·187	69·935	70·593
46	63·079	63·638	64·259	64·900	65·569	66·263	66·994	67·754	68·541	69·338	70·104	70·818
47	63·411	63·951	64·553	65·179	65·829	66·508	67·221	67·964	68·739	69·519	70·271	70·969
48	63·790	64·311	64·894	65·500	66·132	66·790	67·486	68·211	68·969	69·732	70·469	71·153
49	64·236	64·737	65·299	65·884	66·496	67·134	67·809	68·514	69·254	70·000	70·719	71·386
50	..	65·217	65·756	66·321	66·912	67·530	68·175	68·867	69·586	70·311	71·010	71·663
51	..	..	66·274	66·819	67·384	67·978	68·611	69·275	69·968	70·675	71·354	71·988
52	..	..	..	67·335	67·879	68·453	69·062	69·700	70·373	71·057	71·715	72·327
53	..	..	..	..	68·400	68·951	69·537	70·151	70·801	71·465	72·100	72·691
54	..	..	..	..	..	69·475	70·038	70·629	71·255	71·893	72·508	73·078
55	..	..	..	..	..	..	70·573	71·142	71·744	72·359	72·950	73·500
56	..	..	..	..	..	..	..	71·686	72·263	72·851	73·422	73·941
57	..	..	..	..	..	..	..	..	72·813	73·378	73·923	74·427
58	..	..	..	..	..	..	..	..	..	73·920	74·439	74·922
59	..	..	..	..	..	..	..	..	..	..	74·937	75·400
60	..	..	..	..	..	..	..	..	..	..	..	75·840

TABLE II., *showing the Annual Premium required to secure an Assurance of* £100 *payable on the Death of* EITHER *of Two Assigned Lives, according to the Carlisle Table of Mortality, and assuming Interest at the rate of Three per cent.*

Younger Ages.	OLDER AGES.										
	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.
15	2·1104	2·1348	2·1583	2·1826	2·2079	2·2359	2·2654	2·2978	2·3322	2·3691	2·4082
16	..	2·1586	2·1818	2·2058	2·2313	2·2585	2·2875	2·3196	2·3538	2·3904	2·4295
17	..	..	2·2045	2·2282	2·2534	2·2802	2·3092	2·3405	2·3747	2·4107	2·4495
18	..	..	..	2·2513	2·2762	2·3027	2·3314	2·3624	2·3957	2·4317	2·4703
19	..	..	..	..	2·3005	2·3267	2·3549	2·3856	2·4189	2·4542	2·4924
20	..	..	..	..	..	2·3524	2·3803	2·4107	2·4435	2·4784	2·5159
21	..	..	..	..	..	..	2·4073	2·4375	2·4697	2·5044	2·5417
22	..	..	..	..	..	..	..	2·4671	2·4992	2·5333	2·5700
23	..	..	..	..	..	..	..	..	2·5304	2·5644	2·6006
24	..	..	..	..	..	..	..	..	..	2·5975	2·6336
25	..	..	..	..	..	..	..	..	..	..	2·6689

TABLE II.—*continued.*

Younger Ages.	OLDER AGES.										
	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
15	2·4498	2·4938	2·5389	2·5827	2·6235	2·6655	2·7104	2·7589	2·8128	2·8693	2·9301
16	2·4700	2·5141	2·5592	2·6024	2·6428	2·6849	2·7294	2·7789	2·8308	2·8874	2·9480
17	2·4901	2·5330	2·5782	2·6211	2·6611	2·7025	2·7468	2·7900	2·8476	2·9040	2·9641
18	2·5103	2·5536	2·5972	2·6404	2·6798	2·7211	2·7650	2·8127	2·8650	2·9211	2·9807
19	2·5321	2·5749	2·6186	2·6608	2·7000	2·7409	2·7843	2·8318	2·8837	2·9394	2·9988
20	2·5556	2·5980	2·6413	2·6830	2·7217	2·7623	2·8052	2·8523	2·9040	2·9590	3·0182
21	2·5806	2·6226	2·6658	2·7072	2·7453	2·7853	2·8278	2·8747	2·9258	2·9804	3·0394
22	2·6088	2·6500	2·6930	2·7338	2·7717	2·8111	2·8529	2·8996	2·9503	3·0044	3·0628
23	2·6388	2·6798	2·7221	2·7627	2·8000	2·8391	2·8807	2·9264	2·9769	3·0309	3·0886
24	2·6714	2·7119	2·7540	2·7938	2·8308	2·8693	2·9107	2·9560	3·0055	3·0593	3·1165
25	2·7065	2·7466	2·7879	2·8275	2·8640	2·9020	2·9428	2·9877	3·0369	3·0897	3·1468
26	2·7431	2·7831	2·8239	2·8630	2·8989	2·9370	2·9766	3·0209	3·0696	3·1223	3·1786
27	..	2·8222	2·8627	2·9013	2·9366	2·9735	3·0132	3·0571	3·1053	3·1571	3·2133
28	..	..	2·9026	2·9408	2·9752	3·0119	3·0507	3·0940	3·1417	3·1931	3·2488
29	..	..	..	2·9780	3·0122	3·0479	3·0865	3·1289	3·1760	3·2268	3·2816
30	..	..	..	..	3·0454	3·0807	3·1183	3·1604	3·2065	3·2567	3·3109
31	..	..	..	..	..	3·1147	3·1519	3·1931	3·2389	3·2881	3·3416
32	..	..	..	..	..	..	3·1879	3·2287	3·2735	3·3221	3·3746
33	..	..	..	..	..	..	..	3·2678	3·3124	3·3601	3·4120
34	..	..	..	..	..	..	..	..	3·3552	3·4024	3·4539
35	..	..	..	..	..	..	..	..	..	3·4550	3·4992
36	..	..	..	..	..	..	..	..	..	..	3·5485

TABLE II.—*continued.*

Younger Ages.	OLDER AGES.											
	37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	48.
15	2·9349	3·0613	3·1322	3·2043	3·2758	3·3483	3·4228	3·5029	3·5889	3·6810	3·7830	3·8947
16	3·0114	3·0786	3·1490	3·2211	3·2923	3·3644	3·4390	3·5186	3·6036	3·6971	3·7978	3·9096
17	3·0276	3·0944	3·1645	3·2363	3·3074	3·3790	3·4535	3·5327	3·6177	3·7102	3·8109	3·9226
18	3·0440	3·1103	3·1804	3·2518	3·3225	3·3940	3·4690	3·5468	3·6318	3·7239	3·8245	3·9356
19	3·0614	3·1274	3·1972	3·2685	3·3385	3·4100	3·4836	3·5623	3·6468	3·7384	3·8391	3·9503
20	3·0807	3·1461	3·2155	3·2865	3·3565	3·4273	3·5006	3·5786	3·6632	3·7549	3·8546	3·9658
21	3·1012	3·1664	3·2355	3·3062	3·3754	3·4462	3·5190	3·5908	3·6810	3·7723	3·8720	3·9824
22	3·1245	3·1894	3·2583	3·3283	3·3973	3·4677	3·5402	3·6181	3·7015	3·7924	3·8919	4·0025
23	3·1498	3·2140	3·2827	3·3526	3·4212	3·5100	3·5632	3·6408	3·7239	3·8146	3·9137	4·0235
24	3·1775	3·2416	3·3089	3·3786	3·4471	3·5165	3·5884	3·6654	3·7482	3·8386	3·9371	4·0459
25	3·2073	3·2709	3·3381	3·4068	3·4751	3·5443	3·6156	3·6923	3·7745	3·8647	3·9630	4·0725
26	3·2385	3·3016	3·3687	3·4374	3·5042	3·5732	3·6442	3·7204	3·8023	3·8919	3·9901	4·0995
27	3·2724	3·3353	3·4017	3·4698	3·5368	3·6045	3·6754	3·7590	3·8327	3·9217	4·0197	4·1286
28	3·3074	3·3695	3·4357	3·5028	3·5694	3·6370	3·7068	3·7821	3·8633	3·9522	4·0497	4·1580
29	3·3400	3·4017	3·4670	3·5340	3·5994	3·6666	3·7358	3·8100	3·8910	3·9791	4·0759	4·1841
30	3·3683	3·4292	3·4943	3·5602	3·6254	3·6915	3·7602	3·8341	3·9133	4·0015	4·0975	4·2052
31	3·3984	3·4577	3·5223	3·5880	3·6525	3·7178	3·7857	3·8587	3·9375	4·0240	4·1202	4·2266
32	3·4309	3·4902	3·5535	3·6182	3·6819	3·7464	3·8132	3·8859	3·9639	4·0497	4·1440	4·2506
33	3·4673	3·5261	3·5884	3·6520	3·7151	3·7785	3·8450	3·9165	3·9939	4·0789	4·1725	4·2774
34	3·5083	3·5661	3·6276	3·6906	3·7526	3·8154	3·8808	3·9512	4·0274	4·1117	4·2047	4·3096
35	3·5527	3·6096	3·6702	3·7327	3·7933	3·8555	3·9130	3·9895	4·0652	4·1484	4·2404	4·3442
36	3·6011	3·6554	3·7172	3·7785	3·8386	3·8993	3·9630	4·0318	4·1059	4·1886	4·2800	4·3823
37	3·6529	3·7081	3·7669	3·8272	3·8864	3·9465	4·0087	4·0764	4·1539	4·2312	4·3216	4·4236
38	..	3·7624	3·8200	3·8794	3·9376	3·9963	4·0579	4·1241	4·1967	4·2769	4·3664	4·4674
39	..	..	3·8776	3·9352	3·9920	4·0501	4·1103	4·1760	4·2471	4·3264	4·4144	4·5140
40	..	..	..	3·9929	4·0482	4·1049	4·1634	4·2277	4·2982	4·3760	4·4631	4·5612
41	..	..	..	..	4·1029	4·1580	4·2154	4·2780	4·3468	4·4241	4·5090	4·6061
42	..	..	..	..	..	4·2118	4·2676	4·3289	4·3962	4·4712	4·5562	4·6510
43	..	..	..	..	..	..	4·3294	4·3818	4·4473	4·5212	4·6038	4·6983
44	..	..	..	..	..	..	..	4·4403	4·5041	4·5757	4·6568	4·7490
45	..	..	..	..	..	..	..	..	4·5662	4·6362	4·7151	4·8058
46	..	..	..	..	..	..	..	..	..	4·7046	4·7829	4·8706
47	..	..	..	..	..	..	..	..	..	..	4·8573	4·9440
48	..	..	..	..	..	..	..	..	..	..	..	5·0216

TABLE II.—*continued.*

Younger Ages.	OLDER AGES.											
	49.	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
15	4·0221	4·1639	4·3221	4·4903	4·6700	4·8616	5·0694	5·2927	5·5347	5·7898	6·0479	6·3018
16	4·0371	4·1785	4·3369	4·5051	4·6844	4·8760	5·0834	5·3077	5·5482	5·8034	6·0615	6·3157
17	4·0497	4·1911	4·3489	4·5173	4·6965	4·8876	5·0950	5·3184	5·5597	5·8147	6·0728	6·3269
18	4·0627	4·2037	4·3616	4·5295	4·7079	4·8992	5·1065	5·3300	5·5712	5·8254	6·0835	6·3371
19	4·0769	4·2179	4·3750	4·5428	4·7215	4·9090	5·1194	5·3422	5·5835	5·8377	6·0955	6·3483
20	4·0921	4·2327	4·3898	4·5573	4·7355	4·9267	5·1331	5·3558	5·5972	5·8516	6·1085	6·3620
21	4·1088	4·2491	4·4064	4·5735	4·7513	4·9421	5·1485	5·3709	5·6118	5·8662	6·1232	6·3766
22	4·1281	4·2687	4·4252	4·5920	4·7702	4·9607	5·1675	5·3895	5·6300	5·8847	6·1412	6·3972
23	4·1495	4·2893	4·4462	4·6122	4·7909	4·9812	5·1871	5·4096	5·6504	5·9041	6·1617	6·4139
24	4·1725	4·3122	4·4690	4·6356	4·8135	5·0037	5·2094	5·4319	5·6725	5·9267	6·1831	6·4357
25	4·1977	4·3374	4·4936	4·6602	4·8381	5·0282	5·2346	5·4569	5·6968	5·9517	6·2080	6·4603
26	4·2240	4·3637	4·5200	4·6861	4·8640	5·0542	5·2606	5·4830	5·7228	5·9770	6·2340	6·4867
27	4·2532	4·3925	4·5489	4·7151	4·8925	5·0828	5·2887	5·5112	5·7521	6·0063	6·2633	6·5160
28	4·2826	4·4214	4·5775	4·7437	4·9213	5·1111	5·3171	5·5397	5·7800	6·0350	6·2920	6·5445
29	4·3081	4·4467	4·6027	4·7684	4·9458	5·1356	5·3415	5·5640	5·8042	6·0591	6·3158	6·5687
30	4·3284	4·4669	4·6219	4·7874	4·9638	5·1538	5·3593	5·5813	5·8217	6·0753	6·3329	6·5849
31	4·3495	4·4870	4·6419	4·8064	4·9831	5·1720	5·3771	5·5987	5·8385	6·0923	6·3491	6·6011
32	4·3720	4·5096	4·6636	4·8278	5·0037	5·1924	5·3971	5·6183	5·8577	6·1109	6·3672	6·6183
33	4·3994	4·5356	4·6890	4·8526	5·0276	5·2160	5·4199	5·6408	5·8800	6·1329	6·3888	6·6402
34	4·4295	4·5656	4·7180	4·8816	5·0561	5·2433	5·4471	5·6673	5·9064	6·1593	6·4148	6·6659
35	4·4641	4·5982	4·7507	4·9132	5·0873	5·2576	5·4773	5·6968	5·9353	6·1882	6·4435	6·6934
36	4·5013	4·6355	4·7861	4·9483	5·1220	5·3083	5·5105	5·7304	5·9682	6·2205	6·4752	6·7250
37	4·5411	4·6746	4·8249	4·9856	5·1583	5·3443	5·5462	5·7649	6·0032	6·2541	6·5089	6·7585
38	4·5841	4·7163	4·8658	5·0264	5·1977	5·3826	5·5843	5·8027	6·0391	6·2912	6·5445	6·7932
39	4·6305	4·7613	4·9102	5·0694	5·2406	5·4242	5·6248	5·8424	6·0786	6·3295	6·5831	6·8311
40	4·6763	4·8069	4·9545	5·1123	5·2827	5·4653	5·6651	5·8816	6·1176	6·3672	6·6202	6·8682
41	4·7197	4·8645	4·9955	5·1525	5·3212	5·5034	5·7021	5·9173	6·1519	6·4009	6·6521	6·8989
42	4·7637	4·8912	5·0364	5·1924	5·3593	5·5404	5·7371	5·9517	6·1848	6·4323	6·6824	6·9278
43	4·8087	4·9348	5·0783	5·2326	5·3992	5·5777	5·7732	5·9865	6·2181	6·4637	6·7128	6·9570
44	4·8585	4·9831	5·1253	5·2779	5·4430	5·6198	5·8141	6·0246	6 2549	6·4991	6·7463	6·9883
45	4·9132	5·0365	5·1766	5·3279	5·4907	5·6666	5·8592	6·0688	6·2972	6·5400	6·7819	7·0151
46	4·9762	5·0975	5·2366	5·3854	5·5468	5·7207	5·9119	6·1199	6·3457	6·5866	6·8301	7·0683
47	5·0478	5·1669	5·3042	5·4520	5·6110	5·7838	5·9730	6·1791	6·4044	6·6430	6·8845	7·1204
48	5·1311	5·2485	5·3840	5·5297	5·6873	5·8577	6·0455	6·2498	6·4735	6·7101	6·9502	7·1842
49	5·2313	5·3470	5·4809	5·6248	5·7807	5·9494	6·1354	6·3380	6·5606	6·7961	7·0346	7·2665
50	..	5·4611	5·5920	5·7356	5·8900	6·0575	6·2408	6·4428	6·6640	6·8979	7·1345	7·3659
51	..	..	5·7236	5·8649	6·0175	6·1832	6·3664	6·5669	6·7857	7·0197	7·2559	7·4834
52	..	..	..	6·0040	6·1551	6·3201	6·5018	6·7000	6·9183	7·1508	7·3849	7·6125
53	..	..	..	..	6·3047	6·4682	6·6485	6·8453	7·0624	7·2945	7·5268	7·7528
54	..	..	..	..	..	6·6293	6·8084	7·0040	7·2200	7·4491	7·6817	7·9063
55	..	..	..	..	..	..	6·9853	7·1802	7·3954	7·6247	7·8550	8·0787
56	..	..	..	..	..	..	..	7·3743	7·5882	7·8157	8·0462	8·2608
57	..	..	..	..	..	..	..	..	7·8008	8·0282	8·2568	8·4769
58	..	..	..	..	..	..	..	..	..	8·2555	8·4820	8·7017
59	..	..	..	..	..	..	..	..	..	..	8·7085	8·9273
60	..	..	..	..	..	..	..	..	..	..	..	9·1428

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END OF VOL. V.

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CONSTITUTION AND LAWS

OF THE

INSTITUTE OF ACTUARIES

OF

GREAT BRITAIN AND IRELAND.

SECTION I.

THE OBJECTS OF THE INSTITUTE AND ITS CONSTITUTION.

1. The INSTITUTE OF ACTUARIES of Great Britain and Ireland is an Association founded for the purpose of elevating the attainments and status, and promoting the general efficiency, of all who are engaged in occupations connected with the pursuits of an Actuary; and for the extension and improvement of the data and methods of the science which has its origin in the application of the Doctrine of Probabilities to the affairs of life, and from which Life Assurance, Annuity, Reversionary Interest, and other analogous Institutions, derive their principles of operation. It embraces as its peculiar province of inquiry, all monetary questions involving a consideration of the separate or combined effects of Interest and Probability.

2. The Institute consists of contributing Members, viz., Fellows and Associates (those of the latter who are, or may become, Principal Officers of any Life Assurance, Annuity, or Reversionary Interest Society, being distinguished by the title of "Official Associates"), and non-contributing, viz., Honorary, Foreign, and Corresponding Members.

3. It is governed by a President and Council.

SECTION II.

OF THE MEMBERS OF THE INSTITUTE, AND THE MANNER OF THEIR ELECTION AND DISQUALIFICATION.

4. Every Candidate for future admission as an Associate must be approved by the Council and proposed by two or more Members, who shall certify from their acquaintance with him or his works that he is a fit person to be admitted. The Certificate to that effect, having been read, shall be suspended in some conspicuous place in the Meeting Room of the Institute during an Ordinary Meeting, there to remain until the following Ordinary Meeting, at which a ballot shall be taken among all the Members there present; and in the event of not less than three-fourths of the votes given being in the candidate's favour, he shall be declared duly elected; otherwise, not; and in the latter case he shall remain ineligible for renomination till the next Session.

5. Any Associate of the Institute who shall hereafter obtain the appointment of Actuary to the Government, or to any Life Assurance, Annuity, or Reversionary Interest Society, and who shall previously to such appointment have obtained a Certificate of Competency from the Institute, shall be at once admitted a Fellow without ballot, subject to such regulations as may from time to time be appointed.

6. Upon the nomination of the Council a ballot may be taken among the Fellows and Official Associates for the admission, as a Fellow of the Institute, of any person who shall previously to the 8th of July, 1848, have been Actuary to the Government, or to any Life Assurance, Annuity, or Reversionary Interest Society, or who, although not an Actuary, may be eminent for scientific attainments connected with the occupations of one; or, lastly, of any Associate who shall have obtained the Certificate of Competency under the provisions of Articles 40 and 41, and whom the Council shall deem especially deserving of such distinction; and in the event of at least sixteen voting, and a majority of the votes given being in favour of the admission of any such person, he shall be declared duly elected a Fellow; otherwise, not; provided always, that no person be elected under this Rule until a Certificate of his nomination by the Council shall have been suspended in the manner provided under Rule 4.

7. Every Associate or Fellow elect residing within twenty miles of London shall attend in person to be admitted at one of the three next Ordinary Meetings subsequent to his election, or within such further time as shall be specially appointed by the Council, otherwise his election shall become void.

8. Every Associate and Fellow, having first paid his subscription for the current year, or a composition in lieu thereof, shall, before his admission, sign an undertaking to the following effect, in a book to be called the Obligation Book:—

"*We who have written our names hereunder, do hereby engage, each for himself, that we will endeavour to further the good of the Institute of Actuaries of Great Britain and Ireland, and promote the objects for which the same has been founded; that we will be present at its Meetings as often as we conveniently can, and submit to be governed by its Laws, as they are now framed, or as they may hereafter be amended or altered; provided, that having paid up all arrears, and delivered up all books and papers, or other property belonging to the Institute, and having signified in writing to one of the Secretaries that we are desirous of withdrawing therefrom, we shall thereupon be released from this engagement.*"

9. The Admission of every Associate and Fellow shall be in manner and form following:—The President of the Meeting, taking him by the hand, shall say: "*A B, By the authority and in the name of the Institute of Actuaries of Great Britain and Ireland, I do hereby admit you an Associate* [Official Associate or Fellow] *thereof.*"

10. Upon their admission as above, Fellows shall have the right of attaching to their names the letters F.I.A.; and Associates, the letters A.I.A.

11. Upon the recommendation of three-fourths of the Council, Honorary and Foreign Members may be elected by a majority of votes at a ballot to be taken among the Members of the Institute generally. The number of each of these classes not to exceed twenty.

12. Corresponding Members may be elected according to the provisions of Article 30.

13. All Members of the Institute shall have the privilege to attend its Meetings, and avail themselves of the accommodation afforded by its Rooms. The right of voting belongs to contributing Members only. Any contributing Member residing more than twenty miles from London shall have the right to vote by proxy on elections, or any other question before the Institute of which notice shall have been given by the Council, such proxy to be held by a Member qualified to vote, and

no Member to hold more than five proxies. The proxies to be in a form to be determined by the Council; and to be addressed to the Honorary Secretaries, and delivered at the Rooms of the Institute three clear days before the day of meeting.

14. Until a Member shall have paid up all his arrears, and given notice of resignation in writing to one of the Secretaries, he shall be considered liable for all subscriptions due.

15. There shall be an annual list drawn up of the Members of the Institute, in which the classes shall be distinguished; and a special enumeration shall be made of those Associates who have submitted to Examination, and obtained Certificates of Competency.

16. In the event of improper conduct on the part of any Member of the Institute, the Council shall be specially summoned to consider the case; and if they shall be of opinion that his remaining a Member would be discreditable to the Institute, their decision shall be reported to the next Ordinary Meeting thereof; and at the Meeting subsequent thereto a ballot shall be taken among all the Members present, and if three-fourths of the votes then given are for his expulsion, the President of the Meeting shall say: "*By the authority and in the name of the Institute of Actuaries of Great Britain and Ireland, I do hereby declare that A B* [naming him] *is no longer a Member thereof;*" and his name shall forthwith be erased from the list of Members.

SECTION III.

OF THE PAYMENTS OF CONTRIBUTING MEMBERS.

17. The subscriptions are payable yearly, in advance, on the 1st of October in each year, and must be paid within the month.

18. The annual subscription for Members residing *within* twenty miles of London, shall be as follows, viz.:—

Fellows	Three Guineas.
Official Associates	Three Guineas.
Associates	Two Guineas.

19. The annual subscription for Members residing *beyond* twenty miles of London, shall be as follows, viz.:—

Fellows	Two Guineas.
Official Associates	Two Guineas.
Associates	One Guinea.

20. Any Member whose annual subscription shall be in arrear shall be applied to in writing by one of the Secretaries, and shall be incapacitated from voting at the Meetings of the Institute, or exercising any other of the rights of a Member, until such subscription be paid; and if the same be not discharged on or before the 30th of November next following, he shall be declared a defaulter, and shall cease to be a Member of the Institute.

SECTION IV.

OF THE ANNUAL GENERAL MEETING AND ELECTION OF COUNCIL AND OFFICERS.

21. There shall be held a General Meeting of the Members on the first Saturday of June in each year, to confirm the audit of the receipts and expenditure for the financial year ended the 31st March preceding, to elect a President, Vice Presidents, and Officers, for the year then immediately ensuing; and to consider questions relating to the laws and management.

22. No motion relating to the laws and management shall be considered, unless fourteen days' notice shall have been given by the Council to the Members of the Institute; and it shall be imperative on the former to give such notice on receiving from any Member, at least one month before the General Meeting, a requisition so to do.

23. The Council shall consist of the Fellows and Official Associates, out of whom shall be elected, by ballot taken among the Members generally, a President, four Vice Presidents, a Treasurer, and two Secretaries.

24. The Council shall, a fortnight previously to the General Meeting, send to each Member a list containing the names of those whom they recommend for election as President, Vice Presidents, and Officers, for the ensuing year; but Members shall have the right to erase any of the names so recommended, and to substitute others.

25. The General Meeting shall also elect for the ensuing year three Associates as Auditors, by ballot taken among the Members generally.

SECTION V.

POWERS AND DUTIES OF THE COUNCIL.

26. The Council shall have power to make from time to time, *for*

their own government, such regulations as may be found necessary, which shall be entered as resolutions upon their Minutes.

27. The Council shall also have power to make from time to time, *for the government of the Institute*, such regulations, not inconsistent with these laws, as may be found necessary, which shall be entered as above, and remain in force until the next Annual General Meeting, when they shall be either affirmed or annulled, and, if annulled, shall not be revived unless under the immediate sanction of another General Meeting.

28. Any vacancy occurring in the Presidentship or among the Vice Presidents or Officers of the Institute, shall, as soon as possible, be filled up by ballot at some Meeting of the Council to be specially summoned for that purpose, and the appointment so made shall continue in force until the next Annual General Meeting.

29. Five Members of the Council shall be required to constitute a quorum.

30. The Council, upon notice being first given to each of its Members, shall have power to elect persons resident out of the United Kingdom as Corresponding Members of the Institute.

31. The Council shall appoint three Trustees, not necessarily Members of the Institute, in whose names its funds shall be invested.

32. The Council shall have the management of the funds of the Institute, but shall not at any time contract engagements exceeding the amount which would remain after satisfying all debts and liabilities.

33. The Council shall, from time to time, appoint from among the Members a Mathematical Committee, a Statistical Committee, a Legal, Parliamentary, and Commercial Committee, and such other Committees as may be deemed necessary.

SECTION VI.

DUTIES OF THE OFFICERS.

34. The President shall preside at all Meetings of the Institute, or Council and Committees, at which he may be present, and in case of an equality of votes shall have a second, or casting vote. He shall sign the Diplomas of admission of Foreign and Honorary Members, and act as the organ and representative of the Institute on all public occasions.

35. In the absence of the President from any Meeting one of the Vice Presidents shall act for him, and for the time being be invested with the same powers. In the absence of the President and Vice Presidents,—the Treasurer,—or if he also be absent, some other Member of the Council, shall be called upon to preside.

36. The Treasurer shall receive all moneys due to the Institute, and pay in the same to the Bankers, reserving a sufficient balance to meet any demands upon the Institute, and shall keep an account of its receipts and disbursements.

37. The Secretaries shall conduct the correspondence of the Institute under the direction of the Council; and it shall be the duty of one or both of them to attend all the Meetings of the Institute and Council, and take Minutes thereof.

38. The Auditors shall examine the accounts of the Institute, to the 31st of March in each year (and, where they think fit, investigate the items of expenditure), and shall, previously to the 30th of April, present their Report thereon to the Council for confirmation at the ensuing Annual General Meeting.

SECTION VII.

EXAMINATIONS FOR CERTIFICATES OF COMPETENCY.

39. The Council, at their first Meeting subsequent to the Annual General Meeting, shall appoint four or more Members of the Institute, of whom a moiety at least shall be Fellows, as Examiners, whose duty it shall be to examine candidates for Certificates of Competency, in accordance with a plan to be approved of by the Council, and to report to the Council the result of such examination.

40. The Examination shall comprise at least the four following branches, viz.:—Mathematical Theory; Vital Statistics; Computation and Construction of Tables; Book-keeping and Office routine; and it shall consist of three parts, separated from each other by an interval of at least one year, and no Member shall be eligible for the final examination unless he shall be of the full age of twenty-one years.

SECTION VIII.

ORDINARY AND SPECIAL MEETINGS.

41. The Ordinary Meetings of the Institute shall be held Monthly during the period extending from the 1st of November to the 1st of May, in every year, on such day and at such hour as the Council shall from year to year appoint.

42. The business of the Ordinary Meetings shall be the election and admission of Members. Reports, letters, and papers, on subjects interesting the Institute, shall be read, and conversation on points of interest, theoretical and practical, may be introduced by or with the sanction of the President of the Meeting. No question shall be considered at these meetings, as to the rules or management of the Institute, except such as may arise out of the confirmation of the Minutes of the Annual General Meeting, or of any Special General Meeting immediately prior thereto.

43. Strangers may be introduced at the Ordinary Meetings by any Member, with the leave of the President of the Meeting.

44. All papers, letters, or other communications proposed to be read at the Ordinary Meetings, shall be first submitted to the Council, who shall decide on their fitness to be read, and shall also subsequently decide on their fitness to be inserted in the Journal of the Institute.

45. Papers and memoirs presented for reading at the Ordinary Meetings shall (unless in special cases) be received upon the understanding that they thereby become the absolute property of the Institute; subject, however, to the condition, that if the Council shall not, within three months after application has been made to them to that effect, undertake to publish the same within a further period of three months, then and in such case the right of publication shall revert to the author.

46. The Council may at any time call a Special General Meeting; and it shall be imperative upon them so to do, at the requisition of any ten Members, communicated to one of the Secretaries, specifying the subjects to be brought forward thereat. A fortnight's notice shall be given to each Member of the day and business for which any Special Meeting is summoned, and no business except what is named in the notice shall be introduced at such Meeting.

LIST OF MEMBERS

OF THE

INSTITUTE OF ACTUARIES

OF

GREAT BRITAIN & IRELAND.

(*Corrected to July*, 1855.)

No. 12, ST. JAMES'S SQUARE, LONDON.

LONDON:
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1855.

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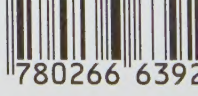
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Statistical Society.
Thomson, W. T., F.R.S.E.
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